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Beam Frames: A New Alternative to the Plane Wave and Green Function Representations

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Abstract

We review the progress of the BS approach, starting from the original Greens function formulations in the frequency and time domains; the Gabor-series formulations for aperture-source distributions; the UWB phase-space formulations; and up to the beam frame formulations with applications to propagation in fluctuating medium and local inverse scattering.

1 Introduction

Wave propagation in complicated media is typically described by ray methods. The difficulties of this algorithmically intuitive representation stem from the loss of most of the spectral flesh that is structured upon the ray skeleton. As a result, the ray solutions fail in many complicated scenarios such as near caustics; short-pulse propagation where the ray solution describes only the first impact of the signal; rough or fluctuating media where the ray solutions may address only the lowest order moments, etc. Furthermore, proper coverage of the propagation domain requires the tracking of a dense lattice of rays, which becomes computationally intensive, and often chaotic.

The loss of spectral flesh may be addressed by constructing the local spectrum above the ray skeleton (e.g., Maslov's formulation), but this process is computationally expansive and often unstable since each plane wave constituent is, by definition, a global object.

2 Beam Summation

The difficulties discussed in the preceding paragraph regarding the difficulties in constructing the spectral propagators are circumvented in the beam summation (BS) approach since the beam propagators are a priori localized. The BS is a local-spectrum representation that is structured upon the ray skeleton in the propagation domain, and thus it combines the asymptotically uniform features of the spectral representation with the algorithmic ease of the ray representation. Furthermore, unlike the ray representations that are highly sensitive to the local fatures of the medium along the ray path, and therefore tend to be chaotic, the beam propagators are more stable since they depend on the local average of the medium properties along the beam path (e.g., in a fluctuating medium). Finally, the local spectrum representation also resolves the local features of the sources and the local propagation-physics, thus describing the overall field interaction with the medium using only a few beam-basis-functions. Here and henceforth we use the generic term "beam waves" for both the frequency-domain and the time-domain formulations, where the propagators are iso-diffracting Gaussian beam (ID-GB) or iso-diffracting pulsed beams (ID-PB), respectively.

3 Beam Frames

So far, the BS methods were based on beam expansions of *point sources* or of *aperture source*s. The *beam frame (BF)* is a new concept where a properly constructed phase-space set of beam waves constitutes a frame *everywhere in the propagation domain* thus can be used for local expansion not only of the sources but also of the medium. This transforms the problem of tracking waves in complicated media into a self-consistent *local-spectrum diagrammatic* formulation where the same beam-set is used to expand both the sources, the medium, and the local interaction of the field with the medium.

This presentation reviews the progress of the BS approach,

- *(i)* starting from the original Greens function formulations in the frequency [1-4] and time [5-7] domain;
- *(ii)* the Gabor-series formulations for aperture-source distributions [8,9];
- *(iii)* the UWB phase-space formulations [10-12];
- *(iv)* and up to the beam frame formulations [13-15] with applications to fluctuating medium scattering [13,14] and local inverse scattering [16-18].

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7 References

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