



## Smart Rewritings of the Inverse Scattering Equations: a Comparison in term of the Degree of Nonlinearity

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Nonlinear inverse scattering [1],[2] is a fascinating area for at least two different reasons. First, successful solution approaches to inverse scattering can be of help in very many different applications such as biomedical imaging, subsurface prospecting and non-destructive testing. Second, because of its non-linearity and ill-posedness, it is very challenging, thus stimulating the efforts and (partial) success of very many researchers and scientists since many years.

In this contribution, attention is devoted to non-linearity of inverse scattering problems with respect to the parameters describing the e.m. characteristics of the target to be retrieved. Notably, it is well known that non-linearity of the problem can induce solution algorithms towards ‘false solutions’ actually different or even very different from the ground truth. Of course, exploitation of any available a priori information (or even partial information arising from some pre-processing) can considerably reduce the false solution occurrence through convenient starting points or regularization techniques.

The non-linearity of inverse scattering problems can be mathematically explored through the state equation, that is the Lippmann–Schwinger integral equation, which relates the induced current/total field inside the investigation domain to the unknown target properties. In particular, the degree of non-linearity (DNL) has been proved to be strictly connected to the norm of the radiation operator adopted in the state equation, which takes into account multiple scattering effects [3]. In this respect, scattering models are of interest such to reduce as much as possible the DNL of inverse scattering problems, and, hence, counteract the false solutions problem.

In this contribution, three different (recent or anyway unusual) rewritings of the Lippmann–Schwinger equation are reviewed and discussed under a unitary perspective, that are the Contrast Source Extended Born (CS-EB) model (and the related family of equations) [3], the family of new integral equations, known as NIE model [4], and the very recent Y0 model [5]. In particular, their DNL, and hence the corresponding difficulties in inversion, are analyzed and compared in term of the function encoding the electromagnetic properties of the targets as well as the norm of the corresponding operator entering the state equation. Then, some considerations are given in order to evaluate which model can be more convenient depending on the scenario at hand. Finally, new effective possibilities and hybridizations between the above models are discussed and introduced.

## References

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