

## Fast Computation of Sommerfeld Integrals in Planar Multilayered Media

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Modeling of electromagnetic wave propagation in stratified media is an active research topic, which finds practical applications in many areas, such as microstrip printed antennas or circuits, remote sensing of underground buried objects, *etc.*, to name just a few. A thorough understanding of wave phenomena in layered structures is still challenging both theoretically and computationally, especially at high frequency applications. An example is the design and analysis of antennas on platforms for the fifth-generation (5G) applications, e.g., mobile phones operating at millimeter waves, which pose considerable challenges both in terms of CPU time and memory when one attempts to simulate them by using legacy Computational Electromagnetics (CEM) codes designed to handle layered media at microwave frequencies. CEM tools typically employ the Method of Moments (MoM) to solve the Mixed Potential Integral Equation (MPIE) to analyze planar antennas and circuits, since the MoM is often far more computationally efficient than Finite Methods for the type of problem at hand. The solution of MPIE requires the calculation of Green's functions (GFs) associated with the layered media. GFs in a multilayered structure are usually constructed in the Fourier transform domain (also referred to as the spectral domain), reducing the original problem to that of solving an equivalent transmission line network [1,2]. Once the spectral-domain GFs have been constructed, they are transformed back in the spatial domain by using the inverse Fourier transform. This Fourier-Bessel transformation reduces to the well-known Sommerfeld Integral (SI), which is a one-dimensional integral of the complex spectral variable  $k_\rho$  over a semi-infinite interval. It is well known that the computation of the SI—and its variants—is very time-consuming, owing to the oscillatory and slowly decaying nature of its integrand, as well as due to the existence of singularities on and/or near the integration path in the complex spectral  $k_\rho$  plane. Various methods have been proposed in the literature to tackle these challenges, the Discrete Complex Image Method (DCIM) [3] among them, for instance. However, the convergence of DCIM becomes an issue when the horizontal distance  $\rho$  between the source and observation points becomes large in terms of the operating wavelength, which is the case in the millimeter wave regime. Additionally, the DCIM does not perform well when the source and observation points are in different layers, and the radial distance  $\rho$  is electrically large.

In the past, the authors have developed a GF module for the MoM-based commercial CEM solver, utilizing the DCIM for the computation of GFs for layered media. However, although the module works well for lower bands of 5G frequencies, it becomes computationally expensive both in terms of time and memory when the platform size becomes electrically large at higher 5G frequency bands, e.g., at 26 GHz and above. To mitigate these problems and to overcome the computational difficulties arising in the computation of SIs, we propose a novel method in this work for fast and accurate evaluation of SIs, which departs from the traditional DCIM technique in several ways. First, we divide the integral into multiple regions during the integration, and handle these regions to the left and right of the branch point singularity as separate sub-regions. Second, we extract the oscillatory part of the integrand in each region, and only deal with the corresponding envelope function numerically, taking advantage of its smooth behavior and representing it with just a few exponentials, enhancing the computational efficiency of the evaluation of the SIs considerably in the process without sacrificing the accuracy. Work is currently in progress to implement this novel technique in a robust CEM solver, with applications in diverse areas of electromagnetic modeling of devices embedded in layered media.

## References

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