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Head-on Collision and Overtaking of Multi-Solitons in Planetary rings

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Abstract

In this investigation, the head-on collision of dust acoustic (DA) multi-solitons in a magnetized space dusty plasma consisting of negative dust, Maxwellian electrons and q-nonextensively distributed ions under the influence of polarization force is studied. The presence of the q-nonextensive ions yields eloquent alteration in the polarization force. An increase in the nonextensive parameter (via q) lead to escalate the polarization parameter. Two sided KdV equations are obtained by adopting Poincaré-Lighthill-Kuo (PLK) method. Further, the direct Hirota method is employed to carry out multi-solitons solutions of KdV equations. The q-nonextensive polarization force has great impact on the phase shifts after interaction of single, double, and triple-(DA) solitons. It is found that magnetic field alters the polarization effect which leads to modify the phase shifts. The findings of our investigation may be helpful to explore the interaction of multi-solitons in a magnetized space dusty plasma such as planetary rings and comet tails where nonextensively distributed ions, negative dust and Maxwellian electrons are prevalent.

1 Introduction

The observations of Voyager spacecraft [1] in Saturn's radial spokes has engendered a great interest in the fascinating and attracting world of dusty plasmas to explore the various nonlinear excitations in the frame work of various distributions due to its omnipresence in the space environments viz. planetary ring, comets tails and interstellar clouds[1] but also its applications in semiconductor devices and ion implantation, etc [2]. Numerous theoretical and experimental investigations [3] have been found that charged dust react with electromagnetic and gravitational fields and gives rise to new low frequency modes like dust ion acoustic waves (DIAWs), dust acoustic waves (DAWs), etc. In low frequency dust acoustic waves, dust mass provides the inertia and electrons/ions provide the restoring force. Over the last few years, Many researchers have studied the various DA nonlinear structures such as solitons, shocks, double layers, rogue waves, etc, in magnetized and unmagnetized dusty plasmas in the frame work of non-Maxwellian distributions [4]. Hamaguchi and Farouki [5] explored one of such force as polarization force arises because of the distorted Debye sphere around the dust. They found that difference in positive ion density on either side of negative dust leads to occurrence of polarization force. The direction of polarization force is opposite to the electrostatic force and independent of dust charge. Further, the polarization force has great impact on the salient features of DA waves. Numerous investigations have been reported to demonstrate the influence of polarization force on different nonlinear waves in the framework of Maxwellian and non-Maxwellian distribution in various space plasma environments [6, 7, 8]. Singh et al. [6] illustrated that the combined effects of superthermal ions and polarization force have profound influence on the characteristics of DA periodic (cnoidal) waves in superthermal polarized dusty plasma. Very recently, Singh & Saini [8] examined the evolution of DA breathers including, Akhmediev breathers (AB), KM breathers and Peregrine solitons (rogue waves) and higher order rogue waves in nonthermal polarized dusty plasmas.

In statistical mechanics has extensive and nonextensive systems, depending upon their order of inter-particle forces. On the other hand, nonextensive system holds for long range inter-particle forces, needs generalized Boltzmann-Gibbs-Shannon (BGS) entropy and also includes dissipative systems having non-vanishing thermodynamic currents. Tsallis [9] introduced q-nonextensive (Tsallis) distribution which is used to model astrophysical regions (i.e., gravitational systems, stellar polytrops) [10] where Maxwellian distribution becomes inadequate to describe such regions. In nonextensive distribution, the parameter q is used to characterize BGS entropy of system. Owing to the importance of Tsallis statistics, theoretical and simulation evidence of the nonextensive distribution have been incorporated in plasma physics to study various kinds of nonlinear structures.

The concept of interaction of different waves is the most exciting nonlinear activity that occurred in the most of plasma environments. The collision of two or more waves occur as they propagate towards each other and transfer their energies. During this process, the solitary waves maintain their shapes as well as sizes and get positive or negative phase shift. An interaction can be of two types viz., inverse scattering or head-on collision. Numerous researchers investigated the collision among two or more solitary waves considering various plasma models by adopting extended PLK method [11, 12, 13]. Xue [11] formulated the collision of dust acoustic solitary waves (DASWs) in a dusty plasma by using PLK method. It was illustrated that dust charge variation significantly modifies the phase shift occurred during the collision of DA solitary waves. The main aim of our present study is to explore the head-on collisions among the DA single-and multi-solitons and their associated phase shifts in polarized dusty plasmas having of massive negative charged dust fluid and nonextensive ions. Although prolific literature on the collision of *DA* multi-solitons has been reported, yet the role of q-nonextensively modified polarization force and magnetic field on the head-on collision of *DA* multi-solitons has not been demonstrated. The paper is organized as follows: Sec. 2 provides the fluid model for given plasma system. The derivation of two different types of KdV equations and single as well as multi-soliton solutions of KdV equations and their corresponding phase shifts are displayed in Sec. 3. Numerical analysis and discussion are illustrated in Sec. 4. The last Sec. 5 is devoted to the conclusions.

2 Fluid Model

To elaborate the effects of nonextensive distribution of ions on the polarization force, we assume that the ions are featuring q-nonextensive distribution, so the number density is given as

$$n_i = n_{i0}\left[1 - h_1\frac{e\psi}{K_BT_i} + h_2\left(\frac{e\psi}{K_BT_i}\right)^2\right], \qquad (1)$$

where $h_1 = \frac{q+1}{2}$ and $h_2 = \frac{(q+1)(3-q)}{8}$. The electrons obey the Boltzmannian distribution function in slow time regime and the number density is $n_e = n_{e0}\exp\left(\frac{e\psi}{K_BT_e}\right)$. It is important to mention here that the electron-electron thermalization time (Maxwellization time) is much shorter than the electron-ion and ion-ion thermalization time due to the large mass difference between electrons and ions that reduces the energy transfer in each collision. Consequently, the electrons thermalize rapidly into a population with Maxwellian distribution whereas the ions does not thermalize rapidly and obeys nonextensive/non-thermal distribution [6].

We consider magnetized polarized dusty plasma composed of negative dust, nonextensive ions and Maxwellian electrons to examine the head-on collision of *DA* multisolitons. The charge neutrality condition is $n_{e0} + Z_{d0}n_{d0} = n_{i0}$, where n_{j0} for $(j = e, i, d)$ are unperturbed number density, respectively. The dynamics of *DAWs* is characterized by the following normalized fluid equations:

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_du_{dx})}{\partial x} + \frac{\partial(n_du_{dy})}{\partial y} + \frac{\partial(n_du_{dz})}{\partial z} = 0, \qquad (2)$$

$$\frac{\partial u_{dy}}{\partial t} + u_{dx}\frac{\partial u_{dx}}{\partial x} + u_{dy}\frac{\partial u_{dx}}{\partial y} + u_{dz}\frac{\partial u_{dx}}{\partial z} = \chi_p\frac{\partial \psi}{\partial x} - \Omega u_{dy}, \qquad (3)$$

$$\frac{\partial u_{dy}}{\partial t} + u_{dx}\frac{\partial u_{dy}}{\partial x} + u_{dy}\frac{\partial u_{dy}}{\partial y} + u_{dz}\frac{\partial u_{dy}}{\partial z} = \chi_p\frac{\partial \psi}{\partial y} + \Omega u_{dx}, \qquad (4)$$

$$\frac{\partial u_{dz}}{\partial t} + u_{dx}\frac{\partial u_{dz}}{\partial x} + u_{dy}\frac{\partial u_{dz}}{\partial x} + u_{dz}\frac{\partial u_{dz}}{\partial z} = \chi_p\frac{\partial \psi}{\partial z}, \qquad (5)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = n_d + n_e - n_i \qquad (6)$$

where n_d is the dust density normalized by n_{d0} and u_d by $C_d = (Z_{d0}K_BT_i/m_d)^{1/2}$, and ϕ is the electrostatic wave potential normalized by, K_BT_i/e. The space and time variables are normalized by dust Debye radius $\lambda_{Dd} = (K_BT_i/4\pi e^2Z_{d0}n_{d0})^{1/2}$ and plasma frequency $\omega_{pd} = (4\pi Z_{d0}^2e^2n_{d0}/m_d)^{1/2}$ respectively. The magnetic field is assumed to be along the z-axis (i.e., $\mathbf{B} = B_0\hat{\mathbf{z}}$), and the dust cyclotron frequency $\Omega(= \frac{Z_deB_0}{m_d})$ is the normalized by ω_{pd}. Here, $\sigma = T_i/T_e$, $\delta_e = n_e/Z_{d0}n_{d0} = 1/(\rho - 1)$ and $\delta_i = n_i/Z_{d0}n_{d0} = \rho/(\rho - 1)$ and $\rho = n_{i0}/n_{e0}$. Here $\chi_p = 1 - R\Big(h_1 - (2h_2 + \frac{h_1^2}{2})\Big)$ and R is polarization factor.

3 KdV EQUATIONS AND PHASE SHIFTS OF MULTI-SOLITONS

Two *DA* solitary waves in a magnetized polarized dusty plasma travel towards each other. We have applied an extended PLK method in order to explore the effects of collision. So, we use the stretching coordinates as [13]:

$$\xi = \varepsilon(l_xx + l_yy + l_zz - v_0t) + \varepsilon^2M_0(\eta,\tau) + \varepsilon^3X_1(\xi,\eta,\tau) + \ldots, \qquad (7)$$

$$\eta = \varepsilon(l_xx + l_yy + l_zz + v_0t) + \varepsilon^2N_0(\xi,\tau) + \varepsilon^3Y_1(\xi,\eta,\tau) + \ldots, \qquad (8)$$

and

$$\tau = \varepsilon^3t, \qquad (9)$$

where ε is a small parameter for the strength of nonlinearity. ξ and η denote the trajectories of two *DASWs* traveling towards each other. v_0 is unknown phase velocity of *DASWs*. The perturbed quantities are described as

$$\Upsilon = \Upsilon_0 + \sum_{r=1}^{\infty}\varepsilon^{r+1}\Upsilon_r, \quad \Gamma = \sum_{r=1}^{\infty}\varepsilon^{r+2}\Gamma_r, \qquad (10)$$

where $\Upsilon = (n_d, u_{dz}, \psi)$ and $\Upsilon_0 = (1,0,0)$, $\Gamma = (u_{dx}, u_{dy})$. Using Eqs. (7)-(10) in Eqs. (2)-(6) and set the powers of ε equal to zero, we get thephase velocity of *DASWs* $v_0 = l_z\sqrt{\frac{(1-Rh_1)}{a_1}}$ is obatined. It is noticed that at first instance, there are two *DA* solitary waves, one of which represented by $\phi_{1\xi}(\xi,\tau)$ is traveling to the right and other one by $\phi_{1\eta}(\eta,\tau)$ is traveling to the left. In order to prevent factitious resonance, one has to remove these two secular terms. Hence, we obtain the following nonlinear KdV equations.

$$\frac{\partial \psi_\xi}{\partial \tau} + A\psi_\xi\frac{\partial \psi_\xi}{\partial \xi} + B\frac{\partial^3\psi_\xi}{\partial \xi^3} = 0, \qquad (11)$$

$$\frac{\partial \psi_\eta}{\partial \tau} - A\psi_\eta\frac{\partial \psi_\eta}{\partial \eta} - B\frac{\partial^3\psi_\eta}{\partial \eta^3} = 0. \qquad (12)$$

In above equations we have considered $\psi_{1\xi} = \phi_\xi$ and $\psi_{1\eta} = \phi_\eta$ for simplicity. Here, $A = \frac{-3l_z^2(1-Rh_1)}{2v_0} + \frac{v_0 b}{a_1} + \frac{v_0(R(2h_2+\frac{h_1^2}{2}))}{2(1-Rh_1)}$, $B = \frac{(1-l_z^2)(1-Rh_1)v_0}{2a_1\Omega^2} + \frac{v_0}{2a_1}$, and $D = \frac{v_0 b}{a_1} + \frac{(1-Rh_1)l_z^2}{2v_0} + \frac{v_0(R(2h_2+\frac{h_1^2}{2}))}{2(1-Rh_1)}$. where $b = \delta_e\sigma^2/2 - \delta_i h_2$. The Hirota bilinear method is employed to construct the single and multi-soliton solutions of the KdV equations (11) and (12).

$$\phi_\xi = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}\ln(\Theta_1) \quad and \quad \phi_\eta = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}\ln(\Xi_1) \tag{13}$$

Again, double-soliton solutions of Eqs. (11) and (12) can be written as [13]

$$\phi_\xi = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}\ln(\Theta_2) \quad and \quad \phi_\eta = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}\ln(\Xi_2) \tag{14}$$

Finally, triple-soliton solutions of Eqs. (11) and (12) can be written as [13]

$$\phi_\xi = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}\ln(\Theta_3) \quad and \quad \phi_\eta = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}\ln(\Xi_3)$$

In order to obtain phase shifts after head-on collision, the two solitons A and B are considered asymptotically, far from each other at the initial time ($t \to -\infty$) i.e, before collision, soliton $A(B)$ is at $\xi = 0(\eta = 0)$ and $\eta = -\infty(\xi = \infty)$. After collision ($t \to \infty$), soliton $A(B)$ is at $\xi = 0(\eta = 0)$ and $\eta = \infty(\xi = -\infty)$. Finally, the phase shifts after head-on collisions between two-sided multi-solitons are given as [13]

$$\Delta M_0 = \varepsilon^2\frac{B^{2/3}D}{Av_0}\sum_{i=1}^{n}k_i \quad and \quad \Delta N_0 = -\varepsilon^2\frac{B^{2/3}D}{Av_0}\sum_{i=1}^{n}k_i \tag{15}$$

The phase shifts are sensitive to the variation of different plasma parameters. The phase shift physically means that energy consumption by solitons due to collision without changing its shape and size. It is remarkable to study the influence of all these parameters on the phase shifts occurring due to head-on collision of *DA* solitary waves travelling in opposite directions.

4 NUMERICAL ANALYSIS AND DISCUSSION

In this investigation, the head-on collision of DA single-, double-, triple- solitons in a magnetized dusty plasma having nonextensive ions and Maxwellian electrons in the presence of polarization force by using the extended PLK method is presented. The numerical values of the typical physical parameters of planetary rings [6]: $n_{i0} = 5\times10^7 cm^{-3}, n_{e0} = 4\times10^7 cm^{-3}, Z_d = 3\times10^3, n_d = 10^7 cm^{-3}, T_e = 50eV, T_i = 0.05eV, R = 0-0.14$ are used for analysis. Numerically, it is seen that the nonlinear coefficient A is negative (i.e., $A < 0$) and the dispersion coefficient B is positive (i.e., $B > 0$) for the chosen set of parameters. It implies that $AB < 0$, as a result, negative potential solitary structures are formed. Since the various physical parameters such as nonextensivity of ions, polarization force, obliqueness, magnetic field strength, and other plasma parameters have profound influence on the characteristics of DA multi-solitons, so it is of paramount importance to carry out numerical analysis.

4.1 Time evolution of negative potential DA multi-solitons:

(a) Single-soliton collision: When two sided single-solitons interact, a new complex structure is evolved in their interaction region, and both amplitude and width are larger than those of the colliding solitons. This composite structure decreases gradually and then separated into single-solitons at a later time. In Figs. 1 (a-f), the variation of negative potential DA single-solitons profiles ϕ_ξ and ϕ_η for the different values of time τ under the influence of nonextensively modified polarization force is illustrated (arrows show that direction of propagation). This whole scenario of propagation of single-soliton is shown in Fig. 1(f).

(b) Double-solitons collision: Figs. 2(a-h) display the variation of negative potential DA double-solitons profiles ϕ_ξ and ϕ_η for the different values of time τ under the influence of nonextensive polarization effect. Two sided double-solitons travel towards each other and collide. A new composite structure with larger amplitude is formed at $\tau = 0$ and at a later time, the composite structure decreases and gradually separates into double-solitons (arrows show that direction of propagation). It is also observed that greater amplitude (with higher velocity) solitons overtake the smaller (with lower velocity) one. This whole scenario of propagation of double-solitons is shown in Fig. 2(h).

(c) Triple-solitons collision: Figs. 3(a-h) show the variation of negative potential DA double-solitons profiles ϕ_ξ and ϕ_η for the different values of time τ under the influence of nonextensive polarization effect. Two sided triple-solitons collide, a new composite structure with larger amplitude is formed at $\tau = 0$ and at a later time amplitude decreases and gradually separates into triple-solitons (arrows show that direction of propagation). It is also observed that greater amplitude (with higher velocity) solitons overtake the smaller (with lower velocity) one This whole scenario of propagation of triple-solitons is shown in Fig. 3(a-h) .

4.2 Variation of phase shifts of multi-solitons:

Fig. 4(a) illustrates the variation of phase shift of single-soliton with nonextensive parameter (q) for different values of polarization parameter (R) in the presence of the magnetic field (via $\Omega = 0.25$). It is seen that magnitude of phase shift of single-soliton reduces with rise in both nonextensive parameter (q) and polarization force (via R). This is because of the energy consumption by solitons suppresses with the increment in R and q. Fig. 4(b) shows that opposite trend of variation is seen in the absence of magnetic field (i.e., $\Omega = 0$). It is found that the phase shift of single-soliton is enhanced (reduced) with increase in R (q)

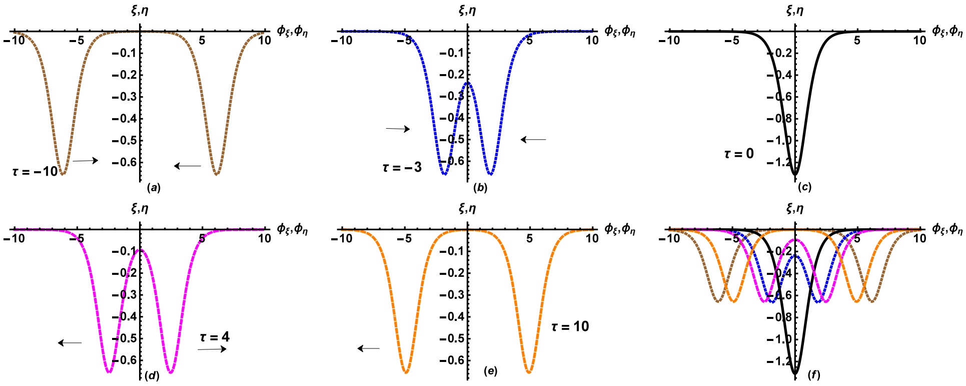

Figure 1. (Color online) Interaction of rarefactive DA single-soliton profiles ϕ_ξ with ξ and ϕ_η with η for different values of τ, whereas other parameters $\sigma = 0.03$, $R = 0.12$, $q = 0.33$, $\Omega = 0.25$, $l_z = 0.6$ and $\rho = 1.11$ are fixed.

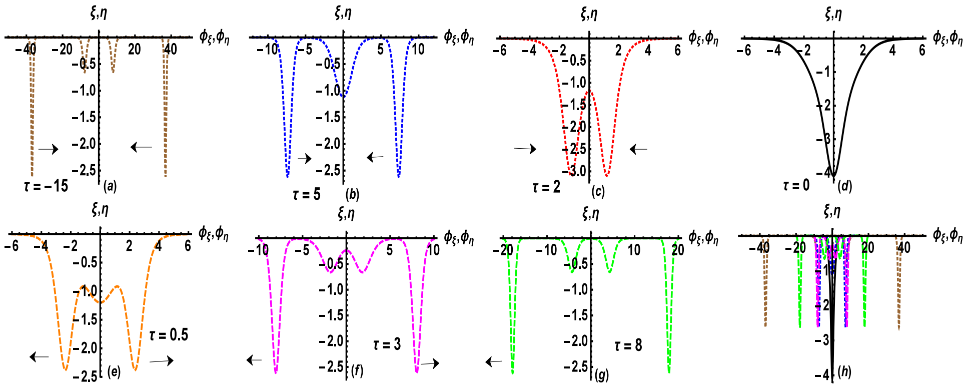

Figure 2. (Color online) Interaction process of negative potential DA double-soliton profiles ϕ_ξ with ξ and ϕ_η with η for different values of τ, whereas other parameters $\sigma = 0.03$, $R = 0.12$, $q = 0.33$, $\Omega = 0.25$, $l_z = 0.6$ and $\rho = 1.11$ are fixed.

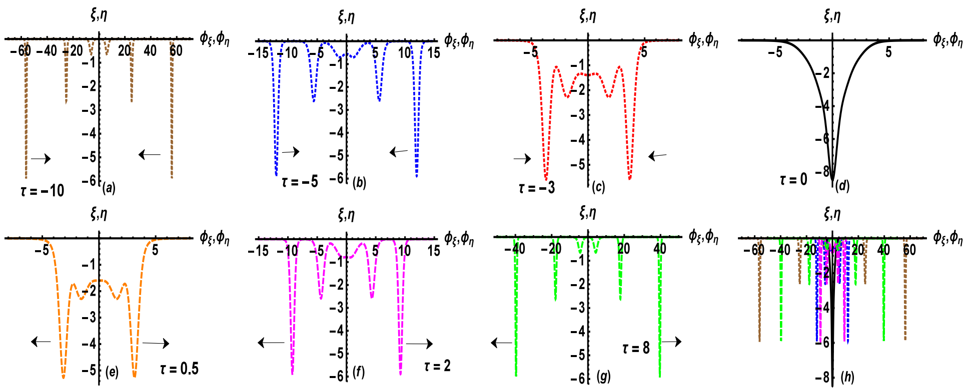

Figure 3. (Color online) Interaction process of negative potential DA triple-soliton profiles ϕ_ξ with ξ and ϕ_η with η for different values of τ, whereas all other parameters $\sigma = 0.03$, $R = 0.12$, $q = 0.33$, $\Omega = 0.25$, $l_z = 0.6$ and $\rho = 1.11$ are fixed.

but the magnitude of phase shift is lower in the absence of magnetic field clearly seen from the colorbars. The results of the present investigation (at $R = 0$, $q = 1$ and $\Omega = 0$) agree with the Maxwellian case [11]. It is also found that magnitude of phase of double-solitons is more than single-solitons. It is remarked that magnetic field altered the effect of polarization force. It is also found that magnitude of phase of double-solitons is more than single-solitons. It is found that the four-solitons solution provides that taller and faster solitons overtake the smaller and slower one during collision. It is found that the phase shift of triple solitons is enhanced with increase in both R and q but the magnitude of phase shift is lower in the absence of magnetic field. It is concluded that phase shifts of DA multi-solitons have significantly affected under the influence of polarization force (via R), nonextensivity parameter (via q) and magnetic field (via Ω) and other plasma parameters.

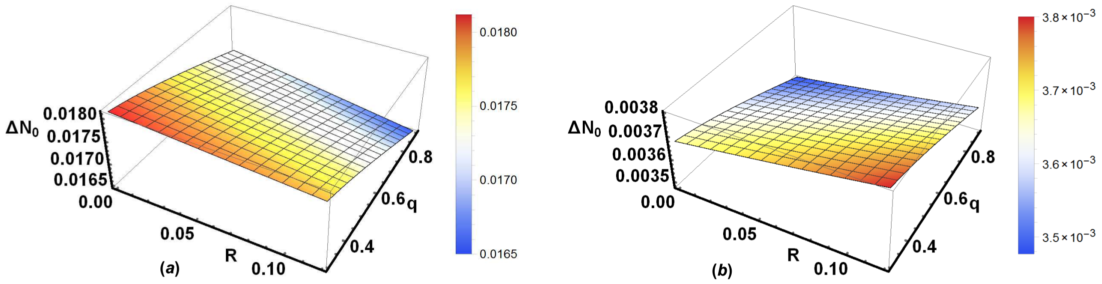

Figure 4. (Color online) The variation of single soliton phase shift ΔN_0 in R - q plane (a) in the presence of magnetic field (via $\Omega = 0.25$) (b) in the absence of magnetic field (via $\Omega = 0$) with $R = 0.12$ and $\rho = 1.11$

5 Conclusions

In present investigation, we have studied the effects of nonextensively modified polarization force on the head-on collision between DA multi-solitons in a magnetized dusty plasma composed of negatively charged dust with q-nonextensive ions. By using extended Poincaré-Lighthill-Kuo method, two opposite directional KdV equations are obtained. Further, Hirota method is employed to obtain the multi-solitons solution and expressions for their phase shifts after collision. It is found that magnetic field drastically modifies the impact of polarization force. All physical parameters have significant influence on the head-on collision of DA multi-solitons and their phase shifts. In the last, we summarized that the results of our investigation might be helpful for the study of nonlinear structures in magnetized dusty plasma [6, 13] (e.g., planetary rings and cometary environment).

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