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A site diversity model for non-geosynchronous systems: development and preliminary results

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Abstract

An analytical site diversity model for non-geosynchronous (GSO) systems is presented. The model allows predicting the joint rain attenuation statistics along Earth-space links at different elevation angles, from which the site diversity gain can be also calculated. The main advantage offered by the proposed prediction model is that it takes into account the variability of rain attenuation correlation function as a function of the elevation angle (besides the obvious one on the site separation distance), which, in turn, allows its application also to non-GSO system. The proposed model, fully described by analytical expressions, offers satisfactory accuracy when applied to predict the joint rain attenuation CCDFs along non-GSO links with ground station in a tropical site.

1 Introduction

Satellite communication systems (SatCom), Earth-observation and space research have a growing importance nowadays. Satellites are equipped more and more with sophisticated and advanced instrumentation, which allows collecting high-resolution data. The massive quantity of data gathered needs to be transmitted routinely and reliably to ground stations: indeed, the Earth-space communication link is key to the success of scientific missions and to an excellent operation of SatCom systems.

The high amount of data to be transmitted to ground stations require high data rates, i.e. wide bands. As lower bands are more and more congested, the shift towards higher bands is necessary and unavoidable (e.g. [1]). However, by increasing the operational frequency, the tropospheric constituents impair more and more the electromagnetic waves along the Earth-space path [2]. For this reason, it is essential to study electromagnetic wave propagation at high frequencies, and to propose alternative solutions in order to enable a high reliability and availability links, despite the high operational frequency.

Ka-band (26-40 GHz) is the frequency band which will be widely used for the next generation satellites. The advantages of Ka-band are mainly two: that portion of the spectrum is less congested (i.e. there are less interference issues) and it accommodates a larger bandwidth (i.e. higher data rates); the disadvantage instead is the increased tropospheric attenuation. Given the criticality of the link in the Ka-band, further solutions are required to achieve a good performance of the link.

This work offers a contribution in this research field by investigating the advanced techniques that need to be implemented in order to cope with the extremely high atmospheric impairments at Ka-band.

This contribution concerns LEO systems in the equatorial region operating at Ka-band; in particular, the site diversity technique is proposed as a solution to the detrimental tropospheric effects [3]. To this aim, a new analytical site diversity model for non-GSO systems is developed showing satisfactory results.

2 ATM PROP: dual site diversity applications to non-GSO links

Given that the atmospheric attenuation measurements for non-GSO links are not yet available nowadays, the only way to estimate the total atmospheric attenuation (and the correspondent complementary cumulative distribution function, hereinafter referred to as CCDF) for non-GSO links is to resort to complex models, which aim at simulating the interaction between the meteorological environment and the communication system. To this aim the ATMospheric simulator for PROPagation applications (ATM PROP) is taken into consideration: a comprehensive methodology to evaluate the tropospheric effects affecting high-frequency Earth-space communication systems [4]. The total attenuation affecting Earth-space communication systems estimated using ATM PROP is obtained by simulating the interaction of the reference system with the full tropospheric environment. The latter is generated by synthesizing three-dimensional maps of rain, clouds and water vapor, whose ensemble reflects the main statistical properties of each meteorological constituent (e.g. CCDF and spatial correlation). The attenuation maps are obtained by integration of the specific attenuation due to the single components along the path [4].

In the framework of this contribution, for the computation of the CCDFs for LEO satellites operating at Ka-band frequencies, the orbital parameters and the additional input data in Table 1 will be considered. The minimum elevation angle chosen is θ_{min} = 20°, since for lower elevation angles the link is expected to be severely impaired by the tropospheric attenuation along the path, especially in the equatorial site considered in this contribution (the Italian Space Agency ground station in Malindi, Kenya).

Given the low elevation angles of the chosen orbit and the expected high fade events induced by rain at Ka-band, the application of Fade Mitigation Techniques (FMTs) is mandatory to achieve the target system availability and the

throughput necessary to download the huge amount of data. Among the possible FMTs, ATM PROP allows investigating the effectiveness of site diversity, which consists on using two (or more stations) to simultaneously receive the space-borne signal: the CCDFs for dual site diversity can be easily computed considering the joint probability that the attenuation on the first path and on the second path exceeds a given threshold. The concept of dual site diversity is clarified in Figure 1, in which it is clear that while one of the paths (the green line) is impaired by a tropospheric event (rain), the other one is not.

Table 1. LEO orbital parameters and reference ground station parameters.

Parameter	Value
Orbital parameters	
Semi-major axis	6928 (km)
Orbital height	550 (km)
Eccentricity	0
Inclination	0°
Right ascension of the ascending node	0°
Argument of perigee	0°
True anomaly	0°
Reference ground station parameters	
Frequency	26.5 (GHz)
Latitude	2° 59'45.925"S
Longitude	40° 11'40.785"E
Altitude	-15.906 (m)
Minimum elevation angle	20°

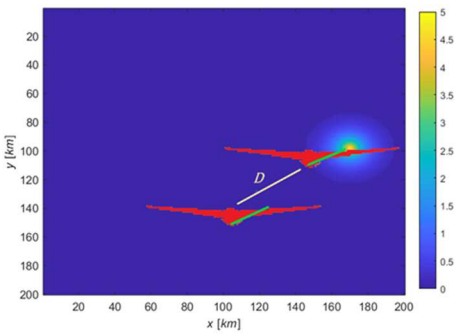

Figure 1. Dual site diversity: aerial view of an ATM PROP rain field (color bar expressing the rain rate in mm/h) and of the portions of the field intersected by the two Earth-LEO links (red triangle-like shape) during a contact window (the distance between the ground stations is D (km), the minimum elevation angle is 20°).

The resulting total tropospheric attenuation CCDFs for dual site diversity are shown in Figure 2, which allows appreciating the effectiveness of site diversity: a system consisting of two stations at 50 km distance would require a fade margin of 20 dB to guarantee availability for 99.99% of the time, while using the same margin with a single receiver would reduce the availability to roughly 99.8%. As is clear from Figure 2, the diversity gain is significant only for exceedance probabilities lower than 1%, which are associated to the effects induced by rain: for this reason, the following Sections will focus only on the impairments caused by precipitation.

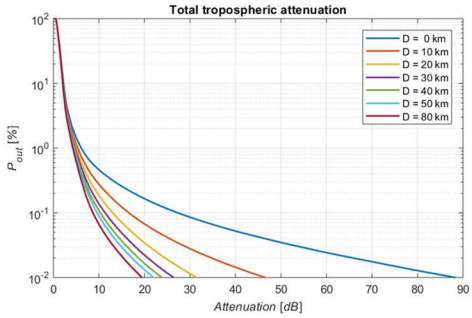

Figure 2. Total tropospheric attenuation CCDFs for single and dual site diversity systems with distance D (km) between the ground stations (reference ground station parameters as in Table 1).

3 The ITU-R model for non-GSO links

Besides the use of simulators such as ATM PROP, the CCDF of total tropospheric attenuation for non-GSO links can also be calculated using the analytical approach proposed by ITU-R in recommendation P.618-13 [5].
The ITU-R model for the evaluation of long-term statistics for non-GSO links suggests the combination of the attenuation CCDFs obtained for fixed elevation angles, weighted by the percentage of the time when the satellite is visible at a specific elevation angle, specifically:

$$P_{non-GSO}(A) = \sum_{j=\theta_{min}}^{\theta_{max}} \frac{P^j(A) f(\theta_j) \Delta\theta}{100} \qquad (1)$$

where θ_{min} and θ_{max} are the minimum and the maximum elevation angle at which the system will operate, $\Delta\theta$ is the angular increment, $f(\theta)$ is the probability density function in percentage of the elevation angles ($\theta \in [\theta_{min}, \theta_{max}]$), $P^j(A)$ is the attenuation CCDFs in percentage for each elevation angle increment (j is the fixed elevation angle in the range $[\theta_{min}, \theta_{max}]$ and A (dB) is the impairment threshold). The mathematical framework of this analytical model will be employed to devise another more complex model, which aims at predicting the site diversity joint CCDFs for non-GSO links without the need of resorting to complex simulators such as ATM PROP: the rationale and the development of such a model is explained in Section 4.

4 Site diversity model for non-GSO links: an analytical approach

ATM PROP allows accurate simulations also for dual site diversity systems (see Figure 1) for different Earth-space systems, but, on the other side, this approach is quite complex and time consuming. As it is not always possible to resort to complex simulators like ATM PROP for the estimation of the attenuation CCDFs for dual site diversity, an analytical site diversity model for non-GSO links represents an appealing alternative. This section describe the development of such a model, which will build upon the most acknowledged analytical methodology to

calculate joint site diversity CCDFs for rain attenuation, which plays the relevant role in the design of the link.

The rain attenuation site diversity model proposed in [6] was developed for GSO links and tested on data collected from GSO satellites, mostly using ground stations at mid-latitudes, subject to temperate climate. This model, whose statistical version is now included in [5], assumes to model with a log-normal distribution, valid both in time and space, the rain attenuation. Specifically, the model aims to predict the joint probability:

$$P_{joint}(A_1 \geq a_1, A_2 \geq a_2) = 100 \times P_r P_a \ (\%) \qquad (2)$$

where:

- P_r is the joint probability that it is raining at both sites;
- P_a is the conditional joint probability that the attenuations exceed a_1 and a_2, respectively, given that it is raining at both sites.

The probabilities P_r and P_a are computed as [6]:

$$P_r = \frac{1}{2\pi\sqrt{1-\rho_r^2}} \int_{R_1}^{\infty} \int_{R_2}^{\infty} exp\left[-\left(\frac{r_1^2 - 2\rho_r r_1 r_2 + r_2^2}{2(1-\rho_r^2)}\right)\right] dr_2\, dr_1 \qquad (3)$$

$$\rho_r = 0.7 \exp\left(-\frac{d}{60}\right) + 0.3 \exp\left[-\left(\frac{d}{700}\right)^2\right] \qquad (4)$$

$$P_a = \frac{1}{2\pi\sqrt{1-\rho_a^2}} \int_{\frac{\ln a_1 - m_{\ln A_1}}{\sigma_{\ln A_1}}}^{\infty} \int_{\frac{\ln a_2 - m_{\ln A_2}}{\sigma_{\ln A_2}}}^{\infty} exp\left[-\left(\frac{b_1^2 - 2\rho_a b_1 b_2 + b_2^2}{2(1-\rho_a^2)}\right)\right] db_2 db_1 \qquad (5)$$

$$\rho_a = 0.94 \exp\left(-\frac{d}{30}\right) + 0.06 \exp\left[-\left(\frac{d}{500}\right)^2\right] \qquad (6)$$

P_a and P_r are bivariate distributions, ρ_r and ρ_a are the rain and attenuation correlation functions, respectively. The parameter d (km) is the separation between the two sites. The thresholds R_1 and R_2 indicate the rain threshold ("rainy-/non rainy-" condition) and they are solution of:

$$P_k^{rain} = 100 \times Q(R_k) = 100 \times \frac{1}{\sqrt{2\pi}} \int_{R_k}^{\infty} exp\left(-\frac{r^2}{2}\right) dr \qquad (7)$$

$$R_k = Q^{-1}\left(\frac{P_k^{rain}}{100}\right) \qquad (8)$$

where P_k^{rain} is the annual probability of rain and R_k is the rain threshold for the k-th site, respectively, Q is the complementary cumulative normal distribution.

The values of the parameters $m_{\ln A_1}, m_{\ln A_2}, \sigma_{\ln A_1}$ and $\sigma_{\ln A_2}$ are determined by fitting the log-normal CCDF in (9) to the single site rain CCDF (A_i as a function of exceedance probability P_i):

$$P_i = P_k^{rain} Q\left(\frac{\ln A_i - m_{\ln A_i}}{\sigma_{\ln A_i}}\right) \qquad (9)$$

To estimate the joint rain attenuation CCDFs for a LEO system, the model described above is applied to fixed elevation angles separately and then the CCDFs are combined as explained in Section 3.

The model presented so far is not suitable for LEO dual site diversity systems, mainly because of the attenuation correlation function in (6). In fact, as it can be easily guessed, the attenuation correlation function ρ_a will also depend on the elevation angle: the lower the elevation angle, the longer will be the portion of the link crossing the troposphere, the higher will be correlation function: as the elevation angle decreases, a less and less steep decrease in ρ_a with the distance is expected.

The main idea is therefore to derive a new analytical expressions of ρ_a for the various elevation angles, so as to predict more accurately the site diversity rain attenuation CCDFs using the approach in [6] for a given elevation angle. Eventually, the joint rain attenuation CCDFs for fixed elevation angles will be combined using the approach described in Section 3 to obtain a joint CCDF of rain attenuation for a site diversity LEO system.

To this aim, let us consider a balanced site diversity system, i.e. two identical ground stations with the same attenuation thresholds, rain thresholds and the same single site rain CCDF (this assumption typically holds for distances roughly up to 100 km). The proposed model aims to predict the diversity curves $P_{joint,non-GSO}(A_1 \geq u, A_2 \geq u)$, by evaluating the diversity CCDFs for every elevation angle $\bar{\theta}$ in the range $[\theta_{min}, \theta_{max}]$ using the associated attenuation correlation function $\rho_a(d, \bar{\theta})$.

The analytical expression for the attenuation correlation function $\rho_a(d, \theta)$ taken into consideration for its derivation is follows the one defined in (6):

$$\rho_a(d,\theta) = a(\theta) \exp\left(-\frac{d}{k_1(\theta)}\right) + (1 - a(\theta)) \exp\left[-\left(\frac{d}{k_2(\theta)}\right)^2\right] \qquad (10)$$

where $a(\theta)$, $k_1(\theta)$ and $k_2(\theta)$ are the parameters to be derived.

In order to derive the attenuation correlation function as in (10), the joint rain attenuation curves for a single elevation angle $\bar{\theta} \in [\theta_{min}, \theta_{max}]$ obtained using (2) are fitted to the diversity curves estimated by ATM PROP for the same elevation angle. The fitting consists in solving an optimization problem (for instance with genetic algorithms), having three parameters with the following constraints: $0 < a(\theta) < 1$; $k_1(\theta) > 0$ and $k_2(\theta) > 0$.

In this way for a single elevation angle $\bar{\theta}$, a set of optimum parameters $[a(\bar{\theta}), k_1(\bar{\theta})$ and $k_2(\bar{\theta})]$ are derived.

From the values of the parameters a, k_1 and k_2 obtained as explained so far, the following analytical expressions are derived (valid in the elevation angles range 20°- 60°):

$$a(\theta) = 0.01217 \times \theta^{0.6296} + 0.7083, \ 20° \leq \theta \leq 60° \qquad (11)$$

$$k_1(\theta) = 33.31 \times \theta^{-0.515} + 9.631, \ 20° \leq \theta \leq 60° \qquad (12)$$

$$k_2(\theta) = 733.58 \times \theta^{-0.32167} - 14.4657 \times \theta^{0.32164} + 36.05, \quad 20° \leq \theta \leq 60° \qquad (13)$$

As it can be noted in Figure 3, the trend of the parameters are quite well fitted by the analytical expressions shown above.

The validity of the analytical expression for the parameters $a(\theta), k_1(\theta)$ and $k_2(\theta)$ is for elevation angles between 20° and 60°, the range where the system in Ka-band is expected to operate, as the model was developed based on the specific site coordinates (Malindi, Kenya) and the orbit on the equatorial plane.

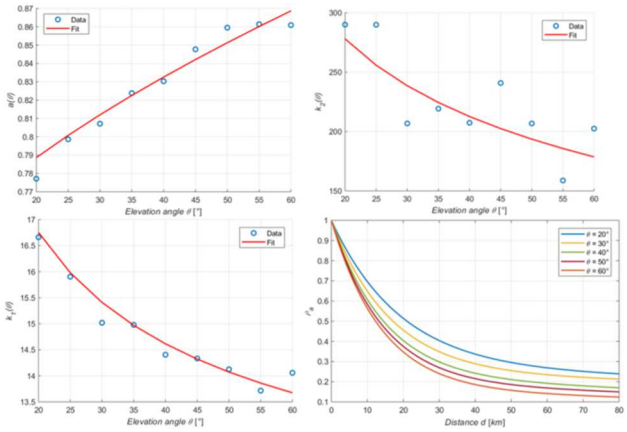

Figure 3. Top left $a(\theta)$: RMSE = 0.78%. Bottom left $k_1(\theta)$: RMSE = 1.5%. Top right $k_2(\theta)$: RMSE = 11.84%. Bottom right $\rho_a(d,\theta)$: resulting rain attenuation correlation function for different elevation angles.

The proposed analytical site diversity model for non-GSO systems is summarized in Figure 4. The results are shown in Figure 5, which compares the output of the new site diversity model for non-GSO link (dashed lines) with the numerical simulations obtained using ATM PROP (solid lines); also reported in Figure 5 are the results produced by the same analytical site diversity model but using the fixed trend for ρ_a defined in (6) [5]. Results clearly indicate the effectiveness of using different rain attenuation correlation functions that are function of the elevation angle. To give an idea of the goodness of the new model, consider the following definition of the root mean square error (RMSE):

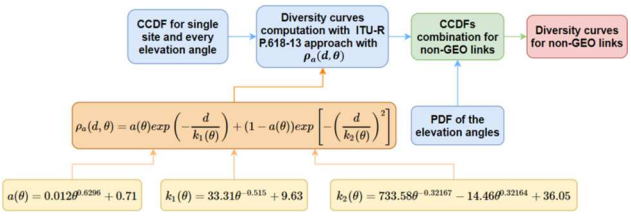

Figure 4. Flow chart of the analytical site diversity model for non-GSO systems.

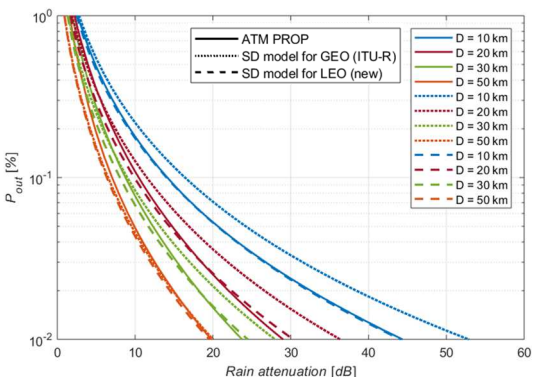

Figure 5. Rain attenuation CCDFs for dual site diversity with distance D (km) between the ground stations: ATM PROP simulation for LEO link in dual site diversity (solid line), proposed site diversity model for LEO (dashed line), same site diversity model but using the constant function to model the rain attenuation correlation in (6) (dotted line).

$$RMSE = \sqrt{E[\epsilon]^2 + \sigma_\epsilon^2} \ (\%) \tag{14}$$

$$\epsilon = 100\ \frac{\tilde{x}-x}{x} \ (\%) \tag{15}$$

where x is the simulation result, $\tilde{x}$ is the model result (estimation of x), ϵ is the error, $E[\epsilon]$ is the mean error and σ_ϵ^2 is the error variance. The mean RMSE between the diversity curves obtained with ATM PROP and those obtained with the model is 20.8%.

5 Conclusions

Given that the existing models aimed at estimating joint rain attenuation statistics for site diversity systems are suitable only for GSO links, a new analytical site diversity model for non-GSO links was developed. The model is of statistical nature, but it has a solid physical foundation: the spatial correlation of rain attenuation, which can be calculated analytically, varies with the elevation angle. Although the proposed site diversity model for LEO links offers good results, this work is the starting point of a generalized site diversity model for LEO links, suitable for very low elevation angles (e.g. 5°) up to 90°. Possible future works are: the extension of this model to include the contribution to attenuation of all the tropospheric constituents, i.e. also clouds and gases; the application and test of the model in sites with different climatic features (i.e. temperate and/or cold regions).

7 References

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