URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

A model of transionospheric radio wave propagation in a randomly inhomogeneous medium in the DWFT approximation

S. I. Knizhin (1), M. V. Tinin (2)
(1) Physics dept. Irkutsk State University, 20 Gagarin Blvd, Irkutsk, 664003, Russia, e-mail: sergeiknizhin@mail.ru
(2) Applied Physics Institute. Irkutsk State University, 20 Gagarin Blvd, Irkutsk, 664003, Russia, e-mail: mtinin@api.isu.ru

Abstract

The paper continues research into the behavior of amplitude and phase of the frequency coherence function, found in the approximation of the double weighted Fourier transform (DWFT) under conditions of transionospheric radio signal propagation without reflection with due regard to the curvature of the ionosphere. A simulation has been carried out for random irregularities with the Shkarofsky spectrum, which are distributed in a background medium specified by Chapman's model.

1 Introduction

In earlier papers [1–6], we have proposed to use the DWFT approximation for describing statistical characteristics of a radio signal that propagates without reflections through a randomly inhomogeneous medium. As a result, we found a frequency coherence function characterizing wave packet spreading [1–2]. Through numerical simulation, we calculated modulus and argument of the frequency coherence function for irregularities described by the Gaussian spectrum and Shkarofsky's model [3–5], distributed in the background medium, which has a constant electron density and is specified by the Chapman function for the Gaussian spectrum [6]. The calculation results have shown that the DWFT approximation can describe diffraction effects during radio signal propagation through random irregularities with a correlation radius smaller than the Fresnel radius. In this case, the manifestation of diffraction effects in frequency coherence amplitude curves appears as channel bandwidth narrowing as compared to the amplitude obtained in the geometrical optics approximation. Moreover, the influence of diffraction effects in the amplitude curves in the case of the Gaussian spectrum for random irregularities with identical inner scales is stronger than that for Shkarofsky's model.

In this paper, we continue research into the behavior of the frequency coherence function modulus and argument in the DWFT approximation under conditions of transionospheric radio signal propagation with due regard to the curvature of the ionosphere. For this purpose, we examine the influence of diffraction effects on the frequency coherence function for radio wave propagation through random irregularities with the Shkarofsky spectrum, which are distributed in a background medium prescribed by Chapman's model.

2 The frequency coherence function in the DWFT approximation

As is known, for radio wave propagation in a medium with irregularities having scales larger than the wavelength, the determination of the wave field $\mathrm{E}(\mathbf{r}) = E(\boldsymbol{\rho}, z)$ in a small-angle approximation reduces to the solution of a parabolic equation. One of the methods for solving this equation can be the DWFT approximation [1]:

$$\mathrm{E}(\boldsymbol{\rho},\boldsymbol{\rho_0},z,\omega) = E_0(\boldsymbol{\rho},\boldsymbol{\rho_0},z,\omega)\left[k\left(z-z_0\right)/\pi\right]^2 \times\exp\left\{i2k\boldsymbol{\rho}\boldsymbol{\rho_0}/(\mathrm{z}-\mathrm{z}_0)\right\}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} d^2s d^2p\exp\{-2ik \times\left[\mathbf{p}\mathbf{s}(z-z_0)+\mathbf{s}\boldsymbol{\rho_0}-\mathbf{p}\boldsymbol{\rho}\right]+i\tilde{\varphi}(\mathbf{s},\mathbf{p},\omega)\right\}, \quad (1)$$

where

$$\tilde{\varphi}(\mathbf{s},\mathbf{p},\omega) = 0.5\,\mathrm{k}\int\limits_{z_0}^{z}\tilde{\varepsilon}\left[\bar{\boldsymbol{\rho}}(\mathbf{p},\mathbf{s},z'),z'\right]dz' \quad (2)$$

is the partial wave phase fluctuation, the first approximation of the solution of the respective eikonal equation with $\bar{\boldsymbol{\rho}}(\mathbf{p},\mathbf{s},z') = \mathbf{p}(z'-z)+\mathbf{s}(z'-z_0)$.

$$E_0(\boldsymbol{\rho},\boldsymbol{\rho_0},z,\omega) = -A_0\left(4\pi(\mathrm{z}-\mathrm{z}_0)\right)^{-1} \times\exp\{i0.5k(\left(\boldsymbol{\rho}-\boldsymbol{\rho_0}\right)^2/(\mathrm{z}-\mathrm{z}_0)+\int\limits_{z_0}^{z}\varepsilon_b(0,\mathrm{z}')dz')\} \quad (3)$$

is the field of an incident harmonic wave with an amplitude of A_0, with a frequency of ω; $\mathbf{r_0} = \{x_0, y_0, z_0\} = \{\boldsymbol{\rho_0}, z_0\}$ is the transmission point, $\mathbf{r} = \{x, y, z\} = \{\boldsymbol{\rho}, z\}$ is the receiving point, $k = \omega/c$ is the wave number, c is the speed of light. We represent the relative permittivity of a medium $\varepsilon(\mathbf{r})$ as $\varepsilon(\mathbf{r}) = 1+\varepsilon_b(\mathbf{r})+\tilde{\varepsilon}(\mathbf{r})$, where $1+\varepsilon_b(\mathbf{r}) = \langle\varepsilon(\mathbf{r})\rangle$ is the permittivity of the background medium, $\tilde{\varepsilon}(\mathbf{r})$ is the quasi-homogeneous random field with zero mean and correlation function of the electron density fluctuations $\tilde{N}(\boldsymbol{\rho},\mathrm{z})$, $\Psi_N(\boldsymbol{\rho_1},\boldsymbol{\rho_2},z_1,z_2)=\Psi_N(\Delta\boldsymbol{\rho},\xi,\boldsymbol{\rho}_\eta,\eta)=$ $=\langle\tilde{N}(\boldsymbol{\rho_1},\mathrm{z}_1)\tilde{N}(\boldsymbol{\rho_2},\mathrm{z}_2)\rangle$, $\boldsymbol{\rho}_\eta=(\boldsymbol{\rho_1}+\boldsymbol{\rho_2})/2$, $\Delta\boldsymbol{\rho}=\boldsymbol{\rho_1}-\boldsymbol{\rho_2}$, $\xi=z_1-z_2$, $\eta=(z_1+z_2)/2$. Given that background medium scales greatly exceed transverse trajectory variations, in (3) we set $\varepsilon_b(\bar{\boldsymbol{\rho}}(\mathbf{p},\mathbf{s},z'),z')\approx\varepsilon_b(0,\mathrm{z'})$.

For the mutual coherence function, (1) can yield

$$
\begin{aligned}
&\Gamma(\boldsymbol{\rho_1},\boldsymbol{\rho_2},\boldsymbol{\rho_{01}},\boldsymbol{\rho_{02}}z,k_1,k_2)\\
&=\langle E(\boldsymbol{\rho_1},\boldsymbol{\rho_{01}},z,k_1)\mathrm{E}^*(\boldsymbol{\rho_2},\boldsymbol{\rho_{02}},z,k_2)\rangle \qquad (4)\\
&=E_0(\boldsymbol{\rho_1},\boldsymbol{\rho_{01}},z,k_1)E_0^*(\boldsymbol{\rho_2},\boldsymbol{\rho_{02}},z,k_2)\hat{\Gamma}(\boldsymbol{\rho_-},\boldsymbol{\rho_{0-}},z,\mathrm{X},k_0),
\end{aligned}
$$

where

$$
\begin{aligned}
&\hat{\Gamma}(\boldsymbol{\rho}_-,\boldsymbol{\rho}_{0-},z,\mathrm{X},k_0)=(\hat{k}_0(z-z_0)/(2\pi))^2\\
&\times\exp(i\hat{k}_0\boldsymbol{\rho}_-\boldsymbol{\rho}_{0-}/(z-z_0))\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2p\,d^2s \qquad (5)\\
&\times\exp\{-i\hat{k}_0[\mathbf{sp}(z-z_0)+\mathbf{s}\boldsymbol{\rho}_{0-}-\mathbf{p}\boldsymbol{\rho}_-]-D(\mathbf{p},\mathbf{s})/2\},
\end{aligned}
$$

$$
\begin{aligned}
&D(\mathbf{p},\mathbf{s})=(0.5/[k_0(1-X^2)]^2)\int_{z_0}^{z}(k_p^4\sigma_N^2(\eta)/\bar{N}^2)\\
&\times[(1+X^2)\bar{\Psi}_N(0,\eta)-(1-X^2)\bar{\Psi}_N(\bar{\boldsymbol{\rho}}(\mathbf{p},\mathbf{s},\eta),\eta)]d\eta. \qquad (6)
\end{aligned}
$$

The function $\sigma_N^2(\eta)\bar{\Psi}_N(\boldsymbol{\rho},\eta)$ is defined by $\sigma_N^2(\eta)\bar{\Psi}_N(\boldsymbol{\rho},\eta)\approx\int_{-\infty}^{\infty}\Psi_N(\boldsymbol{\rho},\xi,\boldsymbol{\rho}_\eta(0,\eta),\eta)d\xi$, where $\sigma_N^2(\eta)=\Psi_N(0,0,\boldsymbol{\rho}_\eta(0,\eta),\eta)$ is the variance of electron density, $\boldsymbol{\rho}_{0-}=\boldsymbol{\rho_{01}}-\boldsymbol{\rho_{02}}$, $\hat{k}_0=k_0(\mathrm{X}^{-1}-X)$, $\mathrm{k}_0=(\mathrm{k}_1+\mathrm{k}_2)/2$, $X=(k_2-k_1)/(2k_0)==(k_2-k_1)/(k_2+k_1)$.

In what follows, we consider only the reduced frequency coherence function when in (5) $\boldsymbol{\rho}_-=0,\boldsymbol{\rho}_{0-}=0$

$$
\begin{aligned}
&\hat{\Gamma}(z,\mathrm{X},k_0)=(\hat{k}_0(z-z_0)/(2\pi))^2\times\\
&\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2p\,d^2s\exp\{-i\hat{k}_0[\mathbf{sp}(z-z_0)]-D(\mathbf{p},\mathbf{s})/2\}. \qquad (7)
\end{aligned}
$$

As has been shown previously [3], to simplify numerical calculations of multiple integral (7), instead of the integral representation for the field in the DWFT approximation we can use the expression for the field in the DWFT approximation for remote irregularities. In this approximation, we utilize new variables $\boldsymbol{\rho}_\mathbf{b}$, $\mathbf{p_s}'$:

$$
\begin{cases}
\mathbf{s}=(\boldsymbol{\rho}_\mathbf{b}-(\mathrm{z}-\mathrm{z}_b)(\mathbf{p_s}'+\mathbf{p_{s0}}))/(\mathrm{z}-\mathrm{z}_0),\\
\mathbf{p}=-(\boldsymbol{\rho}_\mathbf{b}+(\mathrm{z}_b-\mathrm{z}_0)(\mathbf{p_s}'+\mathbf{p_{s0}}))/(\mathrm{z}-\mathrm{z}_0).
\end{cases} \qquad (8)
$$

As a result, Equation (7) for the reduced frequency coherence function takes the following form:

$$
\begin{aligned}
&\hat{\Gamma}(z,\mathrm{X},k_0)=(\hat{k}_0/2\pi)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2p_s'd^2\rho_b\times\\
&\times\exp\{i\hat{k}_0(-\mathbf{p_s}'^2(\mathrm{z}-\mathrm{z}_b)(\mathrm{z}_b-\mathrm{z})/(\mathrm{z}-\mathrm{z}_0)+ \qquad (9)\\
&+\boldsymbol{\rho}_\mathbf{b}^2(\mathrm{z}-\mathrm{z}_0)/(4(\mathrm{z}_b-\mathrm{z}_0)(\mathrm{z}-\mathrm{z}_b)))-D(\mathbf{p_s}',\boldsymbol{\rho}_\mathbf{b})/2\},
\end{aligned}
$$

where

$$
\begin{aligned}
&\mathbf{p_{s0}}=\frac{\boldsymbol{\rho}_\mathbf{b}}{2}\left(\frac{1}{z-z_b}-\frac{1}{z_b-z_0}\right),\\
&\bar{\boldsymbol{\rho}}(\eta)=\frac{\boldsymbol{\rho}_\mathbf{b}}{2}\left[\frac{z-\eta}{z-z_b}+\frac{\eta-z_0}{z_b-z_0}\right]+\mathbf{p_s}'(z_b-\eta).
\end{aligned} \qquad (10)
$$

Next, make a new change of variables:

$$
\boldsymbol{\rho}_\mathbf{b}=2\mathbf{p}_1\sqrt{\frac{(\mathrm{z}_b-\mathrm{z}_0)(\mathrm{z}-\mathrm{z}_b)}{\hat{k}_0(\mathrm{z}-\mathrm{z}_0)}}=2\mathbf{p}_1\sqrt{\frac{z_w}{\hat{k}_0}}, \qquad (11)
$$

$$
\mathbf{p_s}'=\mathbf{p_2}/\sqrt{\frac{(\mathrm{z}_b-\mathrm{z}_0)(\mathrm{z}-\mathrm{z}_b)\hat{k}_0}{(\mathrm{z}-\mathrm{z}_0)}}=\mathbf{p_2}/\sqrt{z_w\hat{k}_0}, \qquad (12)
$$

where

$$
z_w=\left(\frac{1}{z_b-z_0}+\frac{1}{z-z_b}\right)^{-1}. \qquad (13)
$$

Then the reduced frequency coherence function takes the following form:

$$
\begin{aligned}
&\hat{\Gamma}(z,\mathrm{X},k_0)=(\pi)^{-2}\\
&\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2p_1d^2p_2\exp\{(\mathbf{p_1^2}-\mathbf{p_2^2})-D(\mathbf{p_1},\mathbf{p_2})/2\},
\end{aligned} \qquad (14)
$$

$$
\bar{\boldsymbol{\rho}}(\mathbf{p_1},\mathbf{p_2},\eta)=\mathbf{p_1}\sqrt{\frac{z_w}{\hat{k}_0}}\left(\frac{\eta-z_0}{z_b-z_0}+\frac{z-\eta}{z-z_b}\right)+\frac{\mathbf{p_2}(z_b-\eta)}{\sqrt{z_w\hat{k}_0}}. \qquad (15)
$$

In transionospheric radio wave propagation conditions, we should take into account the curvature of the ionosphere, hence in Equation (10) we turn to polar coordinates $\mathbf{p_1} = \{\mathrm{p}, \varphi_1\}, \mathbf{p_2} = \{\mathrm{s}, \varphi_2\}$, and given that $\varphi = \varphi_1 - \varphi_2$ we get:

$$\hat{\Gamma}(z, \mathrm{X}, k_0)$$
$$= (2/\pi)\int_0^\infty ds \int_0^\infty dp \int_0^{2\pi} d\varphi \exp\{i(\mathrm{p}^2 - \mathrm{s}^2) - D(\mathrm{p}, \mathrm{s}, \varphi)/2\}, \quad (16)$$

where

$$\bar{\mathbf{\rho}}^2(\mathrm{p}, \mathrm{s}, \varphi, \eta) = \frac{1}{\hat{k}_0}\left[\mathrm{p}^2 z_w \left(\frac{\eta - z_0}{z_b - z_0} + \frac{z-\eta}{z-z_b}\right)^2 + 2\,\mathrm{ps}\cos\varphi(\mathrm{z}_b - \eta) + \mathrm{s}^2 \frac{(\mathrm{z}_b - \eta)}{z_w}\right]. \quad (17)$$

3 Numerical calculations

Consider how the modulus and argument of reduced frequency coherence function (16) behave under conditions of radio wave propagation through the irregularities distributed in the background medium. As a model for irregularities, we take the spectral density specified by the Shkarofsky function [3]

$$\Phi_N(\mathbf{\kappa}) = \frac{\sigma_N^2 (\kappa_0 l_m)^{(p-3)/2} l_m^3 K_{p/2}\left(l_m\sqrt{\kappa_0^2 + \kappa^2}\right)}{(2\pi)^{3/2}\left(l_m\sqrt{\kappa_0^2 + \kappa^2}\right)^{p/2} K_{(p-3)/2}(\kappa_0 l_m)}, \quad (18)$$

where $\kappa_0 = 2\pi / L_0$, l_m and L_0 are the inner and outer turbulence scales respectively.

Let the variance $\sigma_N^2(\eta)$ be proportional to the background-medium density $\sigma_N^2(\eta) = \sigma_0^2 \bar{N}(\eta)$. Account for the spherical inhomogeneity of the background plasma density through the Chapman layer

$$\bar{N}(\eta) = f_c^2 \exp\{0,5(1 - \chi - \exp\{-\chi\})\}/80.6, \quad (19)$$

$$\chi = \left(\sqrt{\eta^2 + \mathrm{R}_e^{\,2} + 2\,\mathrm{R}_e\,\eta\sin(\alpha)} - h_m - \mathrm{R}_e\right)/H, \quad (20)$$

where f_c is the critical frequency, h_m is the height of the maximum electron density, H is the characteristic layer scale, $\mathrm{R}_e = 6370 km$ is the Earth radius, α is the angle at which we can see the source from the observation point. Integration in (16) is performed provided that the inhomogeneous layer is bounded by heights h_1 and h_2 $(h_1 < h_2)$.

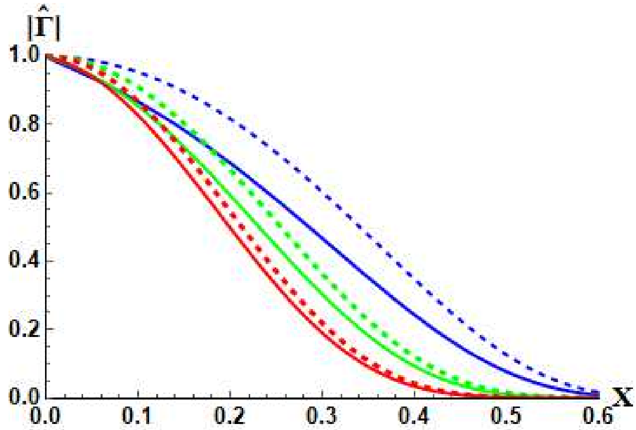

Figure 1. Modulus of coherence function versus the relative mismatch X for the Shkarofsky spectrum (18) for models (18)–(20) with $\sigma_0^2 = 0.05$, inner scale $l_m = 70mm$ and outer scales: 5 km (blue lines), 10 km (green lines), and 15 km (red lines) calculated from Eqs. (16) (solid lines) and GO (dashed lines).

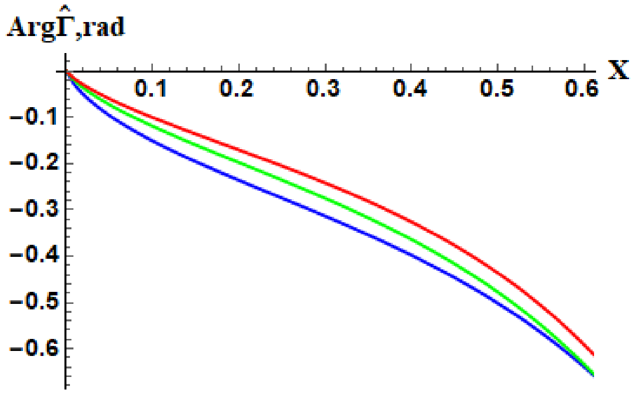

Figure 2. Argument (phase) of the coherence function versus the relative mismatch. Designations and parameters are the same as in Figure 1.

In Figures 1, 2 are curves of the modulus and argument of reduced frequency coherence function (16) for the following parameters: $f_0 = 100 MHz$, $\sigma_0 = 0.05$, $l_m = 70mm$, $h_1 = 150km$, $h_2 = 400km$, $H = 200km$, $h_m = (\mathrm{h}_1 + \mathrm{h}_2)/2 = 325\,\mathrm{km}$, $\mathrm{z}_0 = 0$, $\mathrm{z} = 800\,\mathrm{km}$, $\mathrm{R}_e = 6370km$, $f_c = 6MHz$. The calculations have been made for different outer scales L_0. In Figure 1, the manifestation of diffraction effects in curves of modulus of the reduced frequency coherence function (solid lines) is characterized by channel bandwidth narrowing as compared to the curves derived in the geometrical optics approximation (dashed lines). As in the case of radio wave propagation in random irregularities with the Shkarofsky spectrum, which are distributed in the background medium with a constant electron density [3–5], the curves in Figure 1 demonstrate that the diffraction effects increase with decreasing outer scale L_0. In general, however, the channel bandwidth in the curves found for random inhomogeneous medium with background (18) is wider.

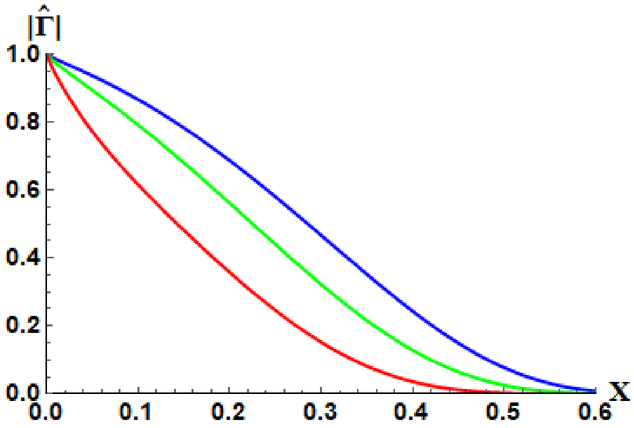

Figure 3. Modulus of coherence function (16) for models (18)–(20) for $\alpha = \pi / 2$ (solid line), $\alpha = \pi / 4$ (green line), $\alpha = \pi / 9$ (red line). Designations and parameters are the same as in Figure 1.

Figure 3 illustrates how with decreasing angle of radio wave incidence relative to vertical propagation at $\alpha = \pi / 2$ (blue line) the channel bandwidth decreases in the curves for the modulus of the reduced frequency coherence function at $\alpha = \pi / 4$ (green line) and $\alpha = \pi / 9$ (red line).

4 Conclusion

Using the DWFT method for remote irregularities, we have obtained a well-behaved expression for the reduced frequency coherence function that takes into account the sphericity of a background slightly inhomogeneous medium.
Through numerical calculations, we have derived curves of the modulus and phase of the reduced frequency coherence function for radio wave propagation through irregularities with the Shkarofsky spectrum, which are distributed in a background medium specified by the Chapman function. The simulation results have shown that diffraction effects narrow the channel bandwidth when the wave propagates in a random inhomogeneous medium, with scales of the spectrum smaller than the Fresnel radius. We have also demonstrated that at the deviation of the angle of incidence on an inhomogeneous medium with respect to vertical propagation, the curves of the modulus of the reduced frequency coherence function will get narrow. These findings are in good agreement with earlier results for radio wave propagation in a randomly inhomogeneous medium with a background having a constant electron density.

5 Acknowledgements

The reported study was funded by RFBR according to the research project № 19-05-00185 and was supported by the Government Assignment for Scientific Research from the Ministry of Science and Education, Russia (no. FZZE-2020-0017). We are very grateful to O. A. Kulish for her assistance in preparing the English version of the manuscript.

6 References

1. M. V. Tinin, “Higher Approximations to Study Statistical Characteristics of Waves in Multiscale Inhomogeneous Media,” *Advances in Mathematical Physics*, V. 2018, Article ID 1570407, 2018, pp. 1-13, https://doi.org/10.1155/2018/1570407.

2. M. V. Tinin, “Integral Representation for Two-Frequency Coherence Function of Ionospheric Radio Signal,” *2018 2nd URSI Atlantic Radio Science Meeting (AT-RASC)*, Meloneras, 2018, pp. 1-4. doi: 10.23919/URSI-AT-RASC.2018.8471506.

3. M. V. Tinin, “Two-Frequency Coherence Function for the Field of a Wave Propagating Through a Multiscale Randomly Inhomogeneous Medium,” *Radiophysics and Quantum Electronics*, **61**, 2018, pp. 259–271, https://doi.org/10.1007/s11141-018-9887-2.

4. M. V. Tinin, “Features of Radio Wave Propagation in a Multiscale Randomly Inhomogeneous Ionosphere,” *2019 Russian Open Conference on Radio Wave Propagation (RWP)*, Kazan, Russia, 2019, pp. 148-151. doi: 10.1109/RWP.2019.8810334.

5. M. V. Tinin, S. I. Knizhin, “Statistical Characteristics of a Radiowave Propagating in a Multiscale Randomly Inhomogeneous Plasma,” *2019 International Conference on Electromagnetics in Advanced Applications (ICEAA)*, Granada, Spain, 2019, pp. 0266-0269. doi: 10.1109/ICEAA.2019.8879233.

6. M. V. Tinin, S. I. Knizhin, I. V. Bezler, “Modeling of the frequency coherence function in a randomly inhomogeneous medium,” Proc. SPIE 11208, 25th International Symposium on Atmospheric and Ocean Optics: Atmospheric Physics, 112080S (18 December 2019); https://doi.org/10.1117/12.2538956.