URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

High-Frequency Wave Field Local Focusing on Ionospheric Reflection Paths of Propagation

Nikolay Zernov[(1)], Mikhail Bisyarin[(1)] and Vadim Gherm[(1)]
[(1)]St. Petersburg State University, St. Petersburg, 199034, Russia, http://spbu.ru, e-mail: n.zernov@spbu.ru, m.bisyarin@spbu.ru, v.germ@spbu.ru

Abstract

Earlier developed interpretations of the effects of local focusing the high-frequency wave field on the ionospheric reflection paths of propagation are discussed. Consideration is based on the integral representation of the high-frequency field, propagating in the inhomogeneous background ionosphere above the Earth, which also contains local mid-scale inhomogeneities of the electron density.

1 Introduction

Recently the effects of local focusing of the high-frequency wave field due to mid-scale ionospheric inhomogeneities in the ionospheric reflection, or transionospheric channels became a subject of interest again (see, e.g., [1, 2]). The same type of problem has been considered in [3 - 6] for the HF reflection, or transionospheric channel of propagation in the 2D geometry.

Further developing this technique, we present here the treatment of the problem for the realistic fully 3D case with the spherically symmetric Earth and inhomogeneous spherically symmetric background ionosphere, also containing local mid-scale inhomogeneities of the electron density, which may give rise to additional local focusing of the field.

Constructing the solution to the spherically symmetric EM field, generated by the vertical electric dipole in the spherical coordinates (r, θ, φ), traditionally employs Debye series in variable θ and the consequent Watson transformation in order to obtain the field integral representation. The latter is further treated in terms of the WKB approximation.

Following the briefly described above procedure and omitting the details, the solution to the problem under consideration is asymptotically represented as follows:

$$E_r(r,\theta,\varphi) = \frac{\mathcal{P}k(kR_e)^{\frac{5}{2}}}{\pi\varepsilon_0\varepsilon_m^{\frac{3}{4}}(r)r^2}\frac{e^{-i\frac{3\pi}{4}}}{\sqrt{2\pi\sin\theta}} \times \tag{1}$$

$$\int_{-\infty}^{\infty}\frac{\alpha\left((kR_e\alpha)^2-\frac{1}{4}\right)}{\sqrt{kR_e\alpha-\frac{1}{2}}}\frac{\exp[ik\psi_0(r,\vartheta,\alpha)+ik\psi_1(r,\vartheta,\varphi;\alpha)+ikS_e\alpha]}{\sqrt[4]{\varepsilon_m(r)-\frac{R_e^2}{r^2}\alpha^2}\ \sqrt[4]{1-\frac{R_e^2}{r^2}\alpha^2}}d\alpha.$$

Here

$$\psi_0(r,\vartheta,\alpha) = \int_{R_e}^{\tilde{r}}\sqrt{\varepsilon_m(r')-\frac{R_e^2}{r'^2}\alpha^2}\,dr' + \int_{r}^{\tilde{r}}\sqrt{\varepsilon_m(r')-\frac{R_e^2}{r'^2}\alpha^2}\,dr' \ ; \tag{2}$$

$\mathcal{P}$ is the dipole moment of the source, S_e is the distance between the corresponding points along the Earth's surface, which are placed at $r = R_e$. The dielectric permittivity of the background spherically-symmetric smoothly inhomogeneous ionosphere is given by $\varepsilon_m(r)$. In the case of transionospheric propagation at higher frequencies, in (2) there is only one integral with the limits of integration (R_e, r).

The additional phase $\psi_1(r,\vartheta,\varphi;\alpha)$ in (1) is represented as the following sum

$$\psi_1(r,\vartheta,\varphi;\alpha) = \frac{1}{2}\int_0^S\left\{\frac{\varepsilon_{loc}\big(\boldsymbol{r_0}(s,\alpha),\vartheta_0(s,\alpha)\big)}{\varepsilon_m\big(r_0(s,\alpha)\big)^{1/2}} + \frac{\nabla\varepsilon_m\big(r_0(s,\alpha)\big)\cdot\boldsymbol{r_1}(s,\alpha)}{\varepsilon_m\big(r_0(s,\alpha)\big)^{1/2}}\right\}ds \tag{3}$$

Representation (1 – 3) is written for the case of one-hop path of propagation in the ionospheric reflection channel. This solution extends the classic results in the problem of radio wave propagation over the open spherical Earth, or the spherical vacuum waveguide [7 – 11] to the case of extended spherically symmetric background ionosphere with local disturbances of the electron density.

Phases $\psi_0(r,\vartheta,\alpha)$ of the α-component waves of the integral representation (1), given by (2), are written in terms of the WKB approximation. The effects of local inhomogeneities of the ionosphere are taken into account by the additional term (3) in the phase of each component wave of the integral representation (1). In (3) the function ε_{loc} stands for a model of a local ionospheric inhomogeneity. In the following consideration, the local inhomogeneities will be assumed to be small disturbances as compared to the background ionosphere $\varepsilon_m(r)$ in order to apply the appropriate perturbation theory to obtain additional phases $\psi_1(r,\vartheta,\varphi;\alpha)$ given by (3). The details of calculating the additional phases will be discussed below.

Meanwhile, when the local inhomogeneities are not included, then according to (1, 2), the field in the undisturbed waveguide structure is constructed in the form of the integral representation in terms of α - component waves of the Geometrical Optics type. Phases, given by two terms in (2), correspond to up- and down-going waves in the undisturbed spherically symmetric problem. The points $\tilde{r}$ are the turning points of the component waves depending on the parameter of integration α . The undisturbed field is cylindrically symmetric (does not depend on φ).

To reach the receiver, each α-component wave propagates along its own path shown in Fig. 1. There they are shown for the undisturbed component waves, which correspond to the spherically symmetric background ionosphere, described by $\psi_0(r, \vartheta, \alpha)$ from equation (2).

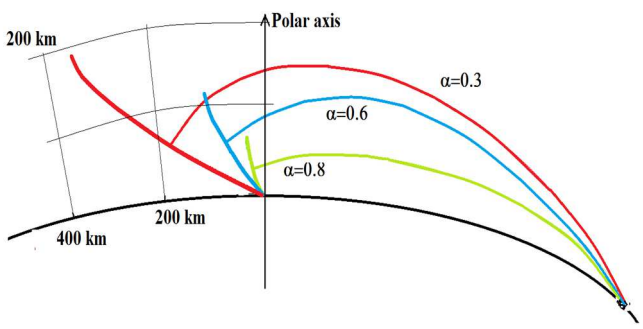

Figure 1. Intersections of the wavefronts of several α-component waves, containing the source point, with the plane of the communicating points, and corresponding ray paths reaching the point of observation.

These ray paths have been found when solving the appropriate ray equations, corresponding to phases (2), of the form as follows:

$$\frac{d\vartheta}{dr} = \pm \frac{\alpha R_e}{r^2 \sqrt{\varepsilon_m(r) - \frac{R_e^2}{r^2}\alpha^2}}. \qquad (4)$$

The trajectories found when solving (4) are further used for constructing additional phases $\psi_1(r, \vartheta, \varphi; \alpha)$, which take account of the local inhomogeneities presence. They are given by the relationship (3).

2 Calculation of $\psi_1(r, \vartheta, \varphi; \alpha)$

Here calculations of $\psi_1(r, \vartheta, \varphi; \alpha)$ will be confined by the case, where the diffraction parameter D for the local mid-scale ionospheric inhomogeneities is equal to zero. Taking into account that the characteristic scale of the mid-scale inhomogeneities is not less than 10 *km,* or higher, this seems reasonable for the frequencies of HF band propagating on the one-hop paths. In this case, the main Fresnel zone size is evaluated by the quantities not greater, than about 3 *km*, or so. It is even more true in the case of transionospheric propagation at higher frequencies.

In principle, diffraction effects for the case of non-zero values of the diffraction parameters D of local inhomogeneities can also be taken into account in the same style as described in [3 - 6]. Here, however, we leave it beyond the scope of consideration.

In the conditions as described above, the effects of local inhomogeneities into the full field (1) are predominantly into the phases of α-component waves. This is why in (1) there is the additional term (3) in the phases of each component wave of the integral representation. When calculating $\psi_1(r, \vartheta, \varphi; \alpha)$ the traditional perturbation theory is employed for the eikonal equation and ray equations of the Geometrical Optics approximation. At this, it should be additionally mentioned that once the perturbation theory is developed for phases, standing in the exponential, this automatically means that the effects of multiple scattering are partially automatically taken into account.

In the relationship (3) for additional phases, there are the two items, where the first one represents the contribution into the phases of the component waves in (1) due to the local inhomogeneities ε_{loc}, and the second one takes account of the change of trajectories, shown in Fig. 1, due to local inhomogeneities ε_{loc}. In both items integration is performed along the undisturbed ray paths in the background spherically symmetric ionosphere, found when solving ray trajectory equations (4). Both the items in (3) are the first-order corrections to the undisturbed phases (2) in terms of small disturbances of the dielectric permittivity $\varepsilon_{loc}(\boldsymbol{r})$ as compared to the dielectric permittivity of the background ionosphere $\varepsilon_m(r)$.

The perturbation $\boldsymbol{r}_1(s, \alpha)$ to the ray path of a given α-component wave, obtained from equation (4), is found according to the standard perturbation theory [12], taking into account that the full dielectric permittivity is of the form $[\varepsilon_m(r) + \varepsilon_{loc}(\boldsymbol{r})]$, where the second item is a small perturbation to the first one.

It is important to point out that in the scalar product of two vectors in the second term in (3) it is solely the radial component of the vector $\boldsymbol{r}_1$ of the perturbation of the ray path of each α-component wave, which contributes into the scalar product. This is because of the spherical symmetry of the background ionosphere $\varepsilon_m(r)$, so that its gradient solely obeys the radial component. As the result, the disturbed ray paths are still in the same plane $\varphi = const$ as the undisturbed ray paths in the background ionosphere.

To conclude this Section, the presence of the ionospheric local inhomogeneities breaks the symmetry of the propagation problem under consideration. This is because the local inhomogeneities are really the 3D bodies, which also explicitly depend on variable φ.

3 Asymptotic evaluation of integral (1)

Once functions $\psi_0(r,\vartheta,\alpha)$ and $\psi_1(r,\vartheta,\varphi;\alpha)$ have been constructed, the integral in (1) is calculated by the steepest descent (stationary phase) technique. The stationary points are found from the equation

$$\frac{\partial}{\partial\alpha}[\psi_0(r,\vartheta,\alpha)+\psi_1(r,\vartheta,\varphi;\alpha)]=0\,. \qquad (5)$$

According to (2) $\psi_0(r,\vartheta,\alpha)$ is defined by a given vertical profile of the electron density of the background ionosphere through $\varepsilon_m(r)$. As far as the second item is concerned, it is specified by a given model of a local mid-scale ionospheric inhomogeneity.

Having the explicit representation of ψ_0, given by (2), this transforms (5) into the form as follows:

$$S_e-2\int_{R_e}^{\tilde{r}}\frac{\alpha R_e^2 dr'}{r'^2\sqrt{\varepsilon_m(r')-\frac{R_e^2}{r'^2}\alpha^2}}+\frac{\partial\psi_1(r,\vartheta,\varphi;\alpha)}{\partial\alpha}=0. \qquad (6)$$

In the absence of the local ionospheric inhomogeneities (when $\psi_1=0$), the reduced equation (6) presents the well-known angle-distance curve for a regular spherical waveguide with the inhomogeneous background ionosphere. The last term in (6) for some analytic models of the local inhomogeneities of the ionosphere can be evaluated almost analytically. One of such models is

$$\varepsilon_{loc}(s,n,t)=A(t)\exp\left[-\frac{(s-s_0)^2+(n-n_o)^2}{R^2(t)}\right]. \qquad (7)$$

It presents evolving in time local inhomogeneity with the parameters depending on "slow time" in quasi-stationary approximation (see, e.g., [6]). Here variables (s,n) are the local ray variables, associated with a ray path of a given α- component wave (see Fig.1); (s_0,n_0) is the position of its center; $A(t)$ and $R(t)$ are the time-varying amplitude and scale of the inhomogeneity, respectively.

For this model, calculations of $\psi_1(r,\vartheta,\varphi;\alpha)$ can be performed "almost" analytically. Thus, the first term in (3) is evaluated as follows [4]:

$$\psi_{11}(r,\vartheta,\varphi;\alpha)=\frac{\sqrt{\pi}A(t)R(t)}{2\sqrt{\varepsilon_0(s_0,0)}}(1-iD^2)^{-\frac{1}{2}}\exp\left[-\frac{n_0^2}{R^2(1-iD^2)}\right]. \qquad (8)$$

According to (8), the contribution of the mid-scale ionospheric inhomogeneity of the type (7) is complex-valued, once the wave parameter D is non-zero. In turn, this makes the stationary points, defined by equation (6), to be complex-valued in the general case. As was already mentioned, the consideration here is, however, confined by the case $D=0$, when ψ_1 is pure real. Finally, the dependence of ψ_1 on α is expressed through the dependence of (s_0,n_0) on α. The second term in (3) is generally computed numerically.

4 Applications of the theory for ionospheric reflection channel

In the following examples, a typical vertical electron density profile with E- and F-layers generated by the NeQuick model was used for calculations, with the total electron content of 50 TEC units and the critical frequency 8.22 MHz; the transmission frequency was 10 MHz. The local spherical disturbance was placed on the height of 200 km and taken of the form (7) with $R=15$ km.

In Fig. 2, the fragments of the angle-distance curves $S_e(\alpha)$ are shown for different intensities of the local spherical inhomogeneity (parameter A in (7)). The values of α corresponding to a given distance S_e are obtained by numerically solving equation (6). They represent the stationary points of the integral (1).

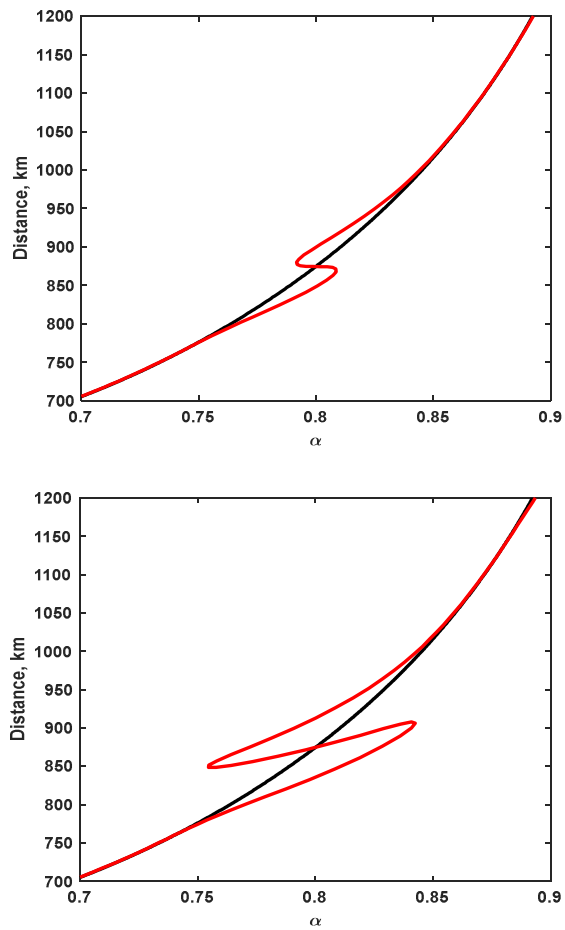

Figure 2. Fragments of the angle-distance curves for the disturbed (red) and undisturbed (black) ionospheric reflection channel of propagation. A=0.05 and 0.14 for upper and bottom panels, respectively.

According to Fig. 2, in the case of very weak local disturbances (upper panel), the disturbed angle-distance curve depicts the single-valued dependence of α on the distance, so that solely one stationary point corresponds to each distance, just as in the undisturbed case.

Alternatively, as the intensity of the disturbance gets higher, this unambiguity breaks down (bottom panel), and the additional stationary points appear. In this particular case, this occurs at the distances from 849 km to 908 km from the source, producing the multipath and local focusing.

5 Conclusions

The theory was presented for interpretation of the effects of a high-frequency wave field local focusing of different orders due to the mid-scale inhomogeneities of the electron density on the ionospheric reflection paths of propagation. In principle, it also includes the case of complex caustics, occurring in the case of non-zero values of the wave parameter of local mid-scale inhomogeneities. The effects of focusing of different orders can be well described in terms of the mathematical theory of catastrophes of different orders (see, e.g., in the review paper [13] and many other publications of these authors).

To conclude this investigation, it is also possible to further extend the developed here theory, when constructing the solution based on more general interference type (as in [14]), or oscillatory type (as in [15]) integrals, where the background medium is already not a layered medium. In this case, the component waves are of a more general type than in just mentioned references, which are constructed in the style as described in [16].

6 References

1. A. Koval, Y. Chen, A. Stanislavsky, A. Kashcheyev, and Q.-H. Zhang, (2018). "Simulation of Focusing Effect of Traveling Ionospheric Disturbances on Meter-Decameter Solar Dynamic Spectra," *Journal of Geophysical Research - Space Physics*, pp. 1-15, October 2018.

2. C.S. Carrano, J.M. Retterer, K.M. Groves, G. Crowley, T. M. Duly, and D. E. Hunton, (2020). "Wave-Optics Analysis of HF Propagation through Traveling Ionospheric Disturbances and Developing Plasma Bubbles". *2020 XXXIIIrd General Assembly and Scientific Symposium of the International Union of Radio Science*, Rome, Italy, 2020, pp. 1-4, DOI: 10.23919/URSIGASS49373.2020.9232348

3. N.N. Zernov, (1990). "Generalization of Rytov's method to the case of a point source field ln inhomogeneous media (in Russian)". *Radiotekhnika i Elektronika, **35**,* 1590-1595, 1990.

4. N.N. Zernov, V.E. Gherm, N.Yu. Zaalov, A.V. Nikitin, (1992). "The generalization of Rytov's method to the case of inhomogeneous media and HF propagation and scattering in the ionosphere", *Radio Science*, ***27***, 2, pp. 235-244. 1992. doi:10.1029/91RS02920

5. N.N. Zernov and B. Lundborg, (1996). "An integral representation of the wave field in inhomogeneous media in terms of diffracting component waves", *Radio Science, **31**,* 1, 1996, pp. 67-80, doi: 10.1029/95RS02452

6. N.N. Zernov and B. Lundborg, (1993). *The Statistical Theory of Wave Propagation and HF Propagation in the Ionosphere with Local Inhomogeneities.* ISSN 0284-1703. 138 pp. Swedish Institute of Space Physics, Uppsala Division, Sweden, (printed in Kiruna), 1993.

7. G.N. Watson, (1918). "The diffraction of electric waves by the Earth". *Proceedings Royal Society*. London A. vol. ***95***, pp. 83-99, 1918.

8. G.N. Watson, (1919). "The transmission of electric waves around the Earth". *Proceedings Royal Society.* London A. vol. ***95***, pp. 546-563, 1919.

9. J.R. Wait, (1981). "*Lectures on waves propagation theory"*, New-York: Pergamon press, 348 pp., 1981.

10. G.I. Makarov, V.V. Novikov, S.T. Rybachek, (1991). "*Electromagnetic waves propagation over the Earth's surface"* (in Russian). Moscow, Nauka, 195 pp. 1991.

11. G.I. Makarov, V.V. Novikov, S.T. Rybachek, (1993). "*Radiowaves propagation in the Earth-Ionosphere – Waveguide and the Ionosphere"* (in Russian). Moscow, Nauka, 147 pp. 1993.

12. Yu.A. Kravtsov, and Yu.I. Orlov, (1980). *Geometrical optics of inhomogeneous media* (in Russian), Nauka, Moscow, 304 pp. 1980.

13. A.S. Kryukovsky, D.S. Lukin, E.A. Palkin and D.V. Rastyagaev, (2019). "The Solution of the Fundamental Problems of the Propagation and Focusing of Electromagnetic Waves in Inhomogeneous, Anisotropic, Absorbing Media on the Basis of Bicharacteristics and Wave Catastrophe Special Functions," *2019 Russian Open Conference on Radio Wave Propagation (RWP)*, Kazan, Russia, 2019, pp. 12-18, DOI: 10.1109/RWP.2019.8810241

14. Yu.I. Orlov, (1972). "Asymptotic method for wave field description in arbitrary fluently inhomogeneous media (in Russian)". *Sov. Transactions MEI*, ***119***, pp. 82-113, 1972.

15. J.M. Arnold, (1982). "Oscillatory integral theory for uniform representation of wave functions". *Radio Science, **17**,* pp. 1181-1191, 1982.

16. V.E. Gherm, Yu.A. Gogin, N.N. Zernov, (2001). "Diffraction of the wave field on the weak inhomogeneities of the dielectric permittivity in the 3D smoothly inhomogeneous medium (in Russian)". *St. Petersburg University Bulletin*: series 4, ***2***, 12, pp. 32-38, 2001.