Generalized Force Approach to Point-to-Point Ray Tracing in the Anisotropic Ionosphere

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In this work, we present an extension of the generalized force approach [1] to point-to-point ray tracing in a magneto-active ionosphere. We start with the equation for the phase path functional:

$$S = \int_{t_A}^{t_B} n(\vec{r},\vec{u})(\dot{\vec{r}}\cdot\vec{u})dt = \int_{A}^{B} \vec{n}(\vec{r},\vec{u})\cdot d\vec{l}. \tag{1}$$

Here, the endpoints A and B are associated with the transmitter and receiver of the ray, $\vec{r}$ and $\dot{\vec{r}}$ are the position vector and the unit tangent to the ray, respectively, $\vec{u}$ is the normal to the wavefront, dl is the length element along the ray, $n(\vec{r},\vec{u})$ is the refractive index defined by the Appleton–Hartree equation, $\vec{n}(\vec{r},\vec{u}) = n(\vec{r},\vec{u})\vec{u}$, and $d\vec{l} = dl\dot{\vec{r}}$. The approach makes use of Fermat's variational principle directly, without solving the eikonal equation, and involves iterative transformation of some initially defined trajectory to the optimal configuration satisfying a stationary condition of the phase path functional. By considering only the paths with their endpoints fixed at the transmitter and receiver sites, the boundary conditions are satisfied automatically, which is an advantage of the method. Following the generalized force approach [1], we discretize the functional using the midpoint rule:

$$S(\mathbf{r},\mathbf{u}) = \sum_{i=0}^{P-1} n_{i+\frac{1}{2}}\vec{u}_{i+\frac{1}{2}}(\vec{r}_{i+1}-\vec{r}_i), \tag{2}$$

where $\vec{r}_i = (x_i, y_i, z_i)$ is the position of the ith vertex, $n_{i+\frac{1}{2}}$ and $\vec{u}_{i+\frac{1}{2}}$ are refractive index and normal to the wavefront at the middle vertex $\vec{r}_{i+\frac{1}{2}} = (\vec{r}_{i+1}+\vec{r}_i)/2$, respectively. The initial trajectory is then approximated by a polygonal line connecting $P+2$ vertices, where the end points are fixed, but P intermediate vertices need to be adjusted to an optimal configuration. According to Fermat's variational principle, radio paths correspond to extrema of general functional $S(\mathbf{r},\mathbf{u})$ so that point-to-point ionospheric ray tracing essentially becomes a problem of locating stationary points.

In our approach, the stationary points of $S(\mathbf{r},\mathbf{u})$ are found using an iterative optimization procedure guided by the force $\mathbf{F}$. The definition of the generalized force depends on the character of extrema: minima for high rays and saddle points for low rays. For high rays, the force is simply a negative gradient of S, and the rays are found by a minimization of the phase path functional. For low rays, the transformation of the gradient is applied, converting the neighborhood of a saddle point to that of a local minimum. The anisotropic formulation of $\mathbf{F}$ for both high and low rays includes additional components defining the rotation of the vector normal to the wavefront which also considered in the optimization process. The final vertex and normal vector configuration corresponding to zero of $\mathbf{F}$ gives a discrete representation of the ray path.

The generalized force approach applied to the magneto-active ionosphere makes it possible to calculate both ordinary and extraordinary rays. Discrimination of the ray types makes it possible to organize a global ray-tracing technique where all relevant rays are identified one after another so that there is no need to provide an accurate initial estimate for each ray. Development of the variational approach (P. F. Bessarab) was supported by the Russian Science Foundation (Grant No. 19-72-10138). Ray tracing simulation was performed (I. A. Nosikov, M. V. Klimenko) as a part of the SPbU "Ozone Layer and Upper Atmosphere Research laboratory" activity supported by the Government of the Russian Federation (project No. 075-15-2021-583).

References

[1] I. A. Nosikov, M. V. Klimenko, G. A. Zhbankov, A. V. Podlesnyi, V. A. Ivanova, and P. F. Bessarab. "Generalized Force Approach to Point-to-Point Ionospheric Ray Tracing and Systematic Identification of High and Low Rays," *IEEE Transactions on Antennas and Propagation*, **68**, 1, January 2020, pp. 455-467, doi:10.1109/TAP.2019.2938817.