URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

Scattering of a pulsed beam by cylindrical targets in a tomographic layout

C. Ponti[1], M. Santarsiero[1], and G. Schettini[1]
[1] Department of Engineering, Roma Tre University, Roma, Italy, {cristina.ponti; massimi.scantarsiero; giuseppe.schettini}@uniroma3.it

Abstract

An approach to study the scattering of a pulsed and spatially nonuniform field, by a set of dielectric cylinders, is presented. Cylinders are embedded in a semi-infinite medium and source field is impinging from an outside region. Source field is expressed through its space and time spectra, and solution is developed with the frequency-domain Cylindrical Wave Approach, applied to a discretized spectrum of monochromatic plane-wave sources, with amplitude belonging to a Gaussian waveform. Time-domain dependence of the scattered field is reconstructed through an inverse Fast Fourier Transform. The method has interesting applications in the modelling of the scattering by inclusions, like small vessels or fibers, in biological tissues, for the diagnostic at THz and optical frequencies. Simulated radargrams provided useful information on the physical properties of the targets, and data useful for the understanding the image formation.

1 Introduction

The modelling of time-domain scattering by buried targets is a topic that received a lot of attention in the microwave frequency range, due to its applications in radar techniques for the analysis of subsurface targets [1], or in applications of breast cancer imaging [2]. Other interesting fields may be in in the terahertz or optical domain, where pulsed sources are used for the diagnostics of the internal structures of biological tissues through Optical Coherence Tomography (OCT) techniques [3]. Time-of-arrivals information, relevant to the scattered pulses, allows reconstructing the internal structures of the targets, in terms of size and permittivity. Electromagnetic modelling of the scattered field has an important role in studying the image formation of the biological samples. Finite-Difference Time-Domain method is mainly employed as full-wave approach for the scattering by a pulsed source, being developed directly in the time domain [4]. The scattering solution is given in the form of an A-scan radargram, i.e., a one-dimensional plot of the scattered field intensity versus time, or scanning the source and the probe along a line, in a two-dimensional map, i.e., the so-called B-scan radargram. However, in the modelling of biological inclusions, the use of canonical shapes is a good approximation and allows a deeper insight to understand the mechanisms of interaction with a biological target. An example of analytical approach is proposed in [5] for an infinitely long circular cross-section cylinder in free-space. In the presence of the background medium hosting the cylinder, the development of an analytical solution is more complicated, as the interaction of the scattered field, that is described through cylindrical functions, and the interface should be suitably dealt with.

This paper presents an analytical-numerical approach for the scattering of a pulsed beam by circular cross-section targets, buried in a semi-infinite medium. The presence of the interface is considered and solved analytically through suitable reflected and transmitted cylindrical functions. The source of the scattering problem is a pulsed field non-uniform in time and space, and cylindrical targets buried in a biological background are modelled [6]-[7]. Scattered fields are evaluated in the form of A-scan, or polar B-scans obtained probing the scattered field along a circumference surrounding the target.

2 Theoretical approach

A 2D scattering problem is considered (see Figure 1), where the source is a pulsed and spatially non-uniform field with origin in the *x'z'*-plane in medium 0 (vacuum permittivity ε_{r0}), impinging in TM polarization. Its general expression, in a reference frame *xz*, including time- and space-dependence, is evaluated at the interface with medium 1 (refractive index n_{r1}) and given by:

$$v_{in}(0,z;t) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} b'(k_\perp sin\varphi + k_\parallel cos\varphi)s(\omega - \omega_0)e^{-i(k_\parallel x_0 - k_\perp z_0)}e^{i(k_\parallel z-\omega t)}\left(\frac{k_\parallel}{k_\perp}sin\varphi + cos\varphi\right)d\omega dk_\parallel. \quad (1)$$

In (1), $s(\omega)$ is the spectrum of the source, around the central frequency ω_0, that is obtained from:

$$s(\omega) = \int_{-\infty}^{+\infty} s(t)e^{i\omega t}dt. \quad (2)$$

and $b'(k_\parallel)$ is the spatial spectrum, given by:

$$b'(k_\parallel) = \int_{-\infty}^{+\infty} B'(z)e^{-ik_\parallel t}dt. \quad (3)$$

Target is a dielectric circular cross-section cylinder of radius *a*, refraction index n_{rc}, and center in (*h, d*) in medium 1. The scattered field is probed around a

circumference of radius ρ centered on the cylinder, and it is expressed through two contributions: a scattered field $v_s(x,z;t)$ and a scattered-reflected field $v_{sr}(x,z;t)$. Scattered fields are expressed through their spatial spectra, and the temporal waveform is obtained through an inverse Fourier Transform, as in (1). The plane-wave spectrum of a cylindrical wave is employed to the space dependence [6]:

$$F_m(x,k_{||}) = \frac{2e^{i|x|\sqrt{1-(k_{||})^2}}}{\sqrt{1-(k_{||})^2}} \begin{cases} e^{-im\cos^{-1}\theta}, & x \geq 0 \\ e^{-im\cos^{-1}\theta}, & x \geq 0 \end{cases} \quad (4)$$

From (4), the basis functions of the scattered field, i.e., cylindrical waves of m-th order, are defined:

$$CW_m\,(u,v) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_m(u,k_{||})e^{ik_{||}v}dk_{||} \quad (5)$$

Using (5), the space-dependence of the scattered field is expressed as

$$E_s(x,z) = V_0 \sum_{m=-\infty}^{+\infty} c_{\mathrm{qm}} CW_m\,[k_1(x-h), k_1(z-d)] \quad (6)$$

where c_{qm} are the expansion coefficients.
Reflected-cylindrical functions are also introduced, employing the spectrum (4):

$$RW_m\,(u,v) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Gamma_{10}(k_{||})\,F_m(-n_1u,k_{||})e^{ik_{||}v}dk_{||} \quad (7)$$

and are used as bass functions of the scattered-reflected field

$$E_{sr}(x,z) = V_0 \sum_{m=-\infty}^{+\infty} c_{\mathrm{qm}} RW_m\,[-n_1(x+h), n_1(z-d)] \quad (8)$$

3 Numerical results

The approach presented in Section 2 can be applied to the modelling of cylindrical inclusions in biological tissues. The source field in (1) is modelled as a Gaussian beam with waist size w_0 and center in (x_0, z_0), having a spatial spectrum:

$$b'(k'_{||}) = B_0\sqrt{\pi}w_0 \mathrm{e}^{-(w_0k'_{||}/2)^2} \quad (9)$$

A Gaussian frequency spectrum is also assigned to the source field

$$s(\omega) = \sqrt{\pi}\sigma \mathrm{e}^{-(\omega\sigma/2)^2} \quad (10)$$

where σ is the duration of the Gaussian pulse, which is related to the full-width half-maximum of the frequency spectrum $\Delta\omega$ through $\sigma = 4\sqrt{ln2}/\Delta\omega$.
In the numerical implementation, the CWA solution on the expression (1) is applied to a finite set of monochromatic plane waves, sampled in the frequency spectrum [7]. The temporal waveforms of the scattered fields are reconstructed through an inverse Fast Fourier Transform.
Numerical results are presented in a tomographic geometry. The beam has waist size w_0 = 5 μm and is in normal incidence. A broadband light source is modelled, with σ = 7.5 × 10^{-3} ps, around the central wavelength λ_0 = 1300 nm, The target is a cylinder of refraction index n_c = 1.5, radius a = 10 μm, buried in (30 μm, 0) in a hosting medium of refraction index n_1 = 1.4.

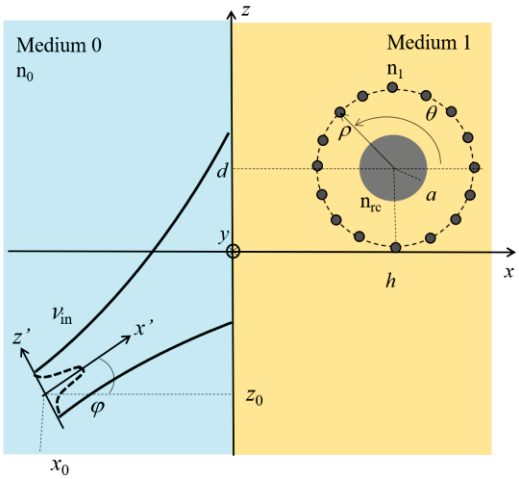

Figure 1. Geomety of the scattering problem.

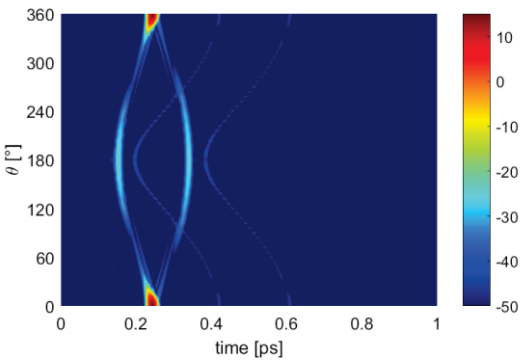

Figure 3. Polar B-scan radargram.

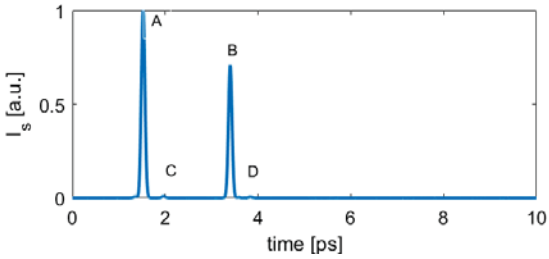

Figure 3. A-scan radargram at the receiver in θ= 180°.

The time-domain intensity of scattered field is probed at along a circumference of radius $\rho = 80$ μm centered on the cylinder and plotted in the form of a polar B-scan in Fig. 2, where different scattering contributions are visible. They are visible also in the A-scan of Fig. 3, evaluated at the receiving point in $\theta = 180°$, where the labels A-B assigned to the pulses with strongest intensity correspond to: A) scattering from the frontside of the cylinder, and B) from its backside. The two weaker contributions, labelled with C and D, are relevant to the reflection of the contributions A and B at the planar interface. Information of the arrival times allows to reconstruct the size of the target, and distance from the interface.

4 Acknowledgements

This research was partly supported by the Italian Ministry for Education, University, and Research under the project PRIN2017 "Quick, reliable, cost effective methodology for DIagnostics of Conformal Antennas (DI-CA)," grant number 20177C3WRM_003.

5 References

1. D. J. Daniels, Surface Penetrating Radar, 2nd edition, London: IEE, 2004.

2. E. Porter, M. Coates, and M. Popovic, "An Early Clinical Study of Time-Domain Microwave Radar for Breast Health Monitoring," *IEEE Trans. Biomedical Eng*. **63**, pp. 530 – 539, 2016.

3. D. Huang et al., "Optical Coherence Tomography," *Science*, **254**, pp. 1178-1181, 1991.

4. T. Brenner, D. Reitzle, and A. Kienle, "Optical coherence tomography images simulated with an analytical solution of Maxwell's equation for cylinder scattering," *J. Biomed. Opt.*, **21**, 045001, 2016.

5. P.R. Munro, A. Curatolo, and D.D. Sampson, "Full wave model of image formation in optical coherence tomography applicable to general samples," *Opt. Express*, **23**, pp. 2541-2556, 2015.

6. C. Ponti, M. Santarsiero, and G. Schettini, "Electromagnetic Scattering of a Pulsed Signal by Conducting Cylindrical Targets Embedded in a Half-Space Medium," *IEEE Trans. Ant. Propag*., **65**, pp. 3073-3083, 2017.

7. C. Ponti, M. Santarsiero, and G. Schettini, "Full-wave analysis of the scattering of a pulsed light beam by dielectric cylinders embedded in a homogeneous medium," *J. of Optics*, **65**, 045606, 2019.