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Unphysical Oscillations in Numerical Solutions to Electromagnetics Problems: Analytical Studies

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We summarize recent work showing that certain straightforward numerical methods applied to simple electromagnetics problems can yield unphysical and oscillatory solutions. We obtain analytical results for the aforementioned simple problems, and discuss extensions to more complicated ones.

We first consider a certain integral equation satisfied exactly by the electrostatic charge distribution on a conducting cylindrical wire [1]. Many antenna and electromagnetics textbooks discuss an approximate form of this equation and solve it using Galerkin methods. The first work to follow this approach was probably a 1978 article by Tsai and Smith, published in the *IEEE Transactions on Education.* We show that the aforementioned approximate electrostatics equation is not as innocent as it might appear because it admits Galerkin solutions that are unphysical. We explain why such solutions occur and propose several remedies. We include a theorem pertinent to the solvability of the equation in the Lebesgue space of integral functions. Our approach has parallels in recent works on Hallén's equation; this is a well-known first-kind Fredholm integral equation satisfied by the steady-state current distribution on a center-fed linear antenna. In particular, we use a detailed analytical study of the wire of infinite length as a means of deducing information about the actual (finite-length) wire.

A very simple scattering electromagnetic (or acoustic) scattering problem is considered next, together with its solution obtained via the Method of Auxiliary Sources (MAS) [2]. We show that it is possible to concurrently have divergence of the auxiliary currents together with convergence of the scattered field generated by these divergent currents. The divergence manifests itself as rapid, unphysical oscillations in the auxiliary currents. It is stressed that the oscillations are not due to effects such as roundoff, matrix ill-conditioning, or to the well-studied phenomenon of internal resonances. We arrive at our conclusions using a number of means including asymptotic methods and a thorough discussion of the singularities of the analytic continuation of the scattered field. We discuss connections of our results to well-known phenomena pertaining to Fredholm integral equations of the first kind. We make a detailed comparison to corresponding discretizations of the Extended Integral Equation (EIE), in which similar phenomena do not occur. Finally, we point out connections to the well-known phenomenon of antenna superdirectivity.

References

[1] G. Fikioris, I. Tastsoglou, G. D. Kolezas, and T. Hatziafratis, "Unphysical moment-method solutions of an approximate integral equation of electrostatics," *IEEE Antennas and Propagation Magazine*, **11**, June 2017, pp. 142-153.

[2] G. Fikioris and N. L. Tsitsas, "Convergent fields generated by divergent currents in the Method of Auxiliary Sources," chapter 5 in T. Wriedt and Y. Eremin, Eds., *The Generalized Multipole Technique for light scattering: recent developments.* (Springer Series on Atomic, Optical, and Plasma Physics, vol. 99). Springer International Publishing, 2018.