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Exceptional Guided Waves

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Abstract

The planar interface of two dissimilar partnering mediums, at least one of which is anisotropic, can guide an exceptional surface wave that propagates in an isolated direction. In order for this to be achieved, the constitutive parameters of the partnering mediums must satisfy certain constraints. Exceptional surface waves have localization characteristics that distinguish them from unexceptional surface waves: the decay of fields of an exceptional surface wave in an anisotropic partnering medium exhibits a combined linear-exponential dependency on distance from the interface, whereas the decay is purely exponential for an unexceptional surface wave. The notion of exceptional surface waves can be extended to compound waves that are guided by a pair of parallel planar interfaces.

1 Preliminaries: Voigt waves

We consider a monochromatic electromagnetic field, oscillating with angular frequency ω, in a linear homogeneous medium. Without loss of generality, the electric and magnetic field phasors are expressed as

$$\left.\begin{array}{l}\underline{E}(\underline{r}) = \underline{e}(z)\,\exp\left[iq\left(x\cos\psi + y\sin\psi\right)\right]\\ \underline{H}(\underline{r}) = \underline{h}(z)\,\exp\left[iq\left(x\cos\psi + y\sin\psi\right)\right]\end{array}\right\}, \tag{1}$$

where q is the wavenumber in the xy plane and the propagation angle $\psi \in [0, 2\pi)$. Herein, an $\exp(-i\omega t)$ dependence on time t is implicit.

By substituting the phasor representations (1) into the source-free Maxwell curl equations, we arrive at the 4×4 matrix ordinary differential equation [1, 2]

$$\frac{d}{dz}[\underline{f}(z)] = i[\underline{\underline{P}}]\cdot[\underline{f}(z)], \tag{2}$$

where the 4×4 matrix $[\underline{\underline{P}}]$ depends on q, ψ, and the constitutive parameters of the medium; and the column 4-vector*

$$[\underline{f}(z)] = [\ \underline{\hat{u}}_x\cdot\underline{e}(z),\quad \underline{\hat{u}}_y\cdot\underline{e}(z),\quad \underline{\hat{u}}_x\cdot\underline{h}(z),\quad \underline{\hat{u}}_y\cdot\underline{h}(z)\ ]^T, \tag{3}$$

with the superscript T signaling the transpose. The components $\underline{\hat{u}}_z\cdot\underline{e}(z)$ and $\underline{\hat{u}}_z\cdot\underline{h}(z)$ are algebraically related to $[\underline{f}(z)]$ [2].

Consider possible degeneracies of the matrix $[\underline{\underline{P}}]$:

(i) In non-degenerate cases [3], the matrix $[\underline{\underline{P}}]$ has four distinct eigenvalues, each with algebraic multiplicity 1 and geometric multiplicity 1. Non-degenerate $[\underline{\underline{P}}]$ is the norm for planewave propagation in anistropic and bianisotropic materials [4].

(ii) In cases of semisimple degeneracy [3], the matrix $[\underline{\underline{P}}]$ has two distinct eigenvalues, each with algebraic multiplicity 2 and geometric multiplicity 2. Semisimple degeneracy is exhibited for every $\psi \in [0, 2\pi)$ by the matrix $[\underline{\underline{P}}]$ formulated for free space as well as for any isotropic dielectric-magnetic material [4].

(iii) In cases of non-semisimple degeneracy [3], the matrix $[\underline{\underline{P}}]$ has two distinct eigenvalues, each with algebraic multiplicity 2 and geometric multiplicity 1. A plane wave arising from a non-semisimple degeneracy of $[\underline{\underline{P}}]$ is called a Voigt wave; such waves were experimentally observed by Voigt in 1902 [5] and theoretically explained by Pancharatnam in 1958 [6]. Certain biaxial absorbing dielectric mediums, for example, support the propagation of Voigt waves for isolated values of ψ, depending on the orientation of the x and y axes [7, 8, 9]. Voigt waves are represented in band diagrams as exceptional points [10, 11]. These exceptional points can arise only if the medium of propagation is either dissipative or active [12, 13].

A Voigt wave is an exceptional plane wave. The notion of exceptional waves can be extended to guided waves, such as surface waves and compound waves, as we describe next.

2 Surface waves

A surface wave is guided by the planar interface of two dissimilar mediums [14]. To be specific, suppose that medium $\mathscr{A}$ occupies the half-space $z > 0$ while medium $\mathscr{B}$ occupies the half-space $z < 0$. The phasor representation (1) holds for

*The triad of Cartesian unit vectors is written as $\{\underline{\hat{u}}_x, \underline{\hat{u}}_y, \underline{\hat{u}}_z\}$.

all $z \in (-\infty, \infty)$ with q being the surface wavenumber and the direction of surface-wave propagation relative to the x axis in the xy plane being prescribed by ψ. The source-free Maxwell curl equations now deliver the 4×4 matrix ordinary differential equations [1, 2]

$$\frac{d}{dz}[\underline{f}(z)] = \begin{cases} i[\underline{\underline{P}}_{\mathscr{A}}] \cdot [\underline{f}(z)], & z > 0 \\ i[\underline{\underline{P}}_{\mathscr{B}}] \cdot [\underline{f}(z)], & z < 0 \end{cases} . \tag{4}$$

The electric and magnetic fields of surface-wave solutions to Eqs. (4) must decay as $z \to \pm\infty$ and satisfy the standard boundary condition

$$[\underline{f}(0^-)] = [\underline{f}(0^+)]. \tag{5}$$

For a certain value of ψ, suppose that a surface wave is excited. Let us consider possible degeneracies of the matrixes $[\underline{\underline{P}}_{\mathscr{A}}]$ and $[\underline{\underline{P}}_{\mathscr{B}}]$:

(i) $[\underline{\underline{P}}_{\mathscr{A}}]$ does not have a non-semisimply degenerate eigenvalue for decay as $z \to \infty$ nor does $[\underline{\underline{P}}_{\mathscr{B}}]$ have a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the surface wave is unexceptional [14].

(ii) $[\underline{\underline{P}}_{\mathscr{A}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to \infty$, but $[\underline{\underline{P}}_{\mathscr{B}}]$ has no non-semisimply degenerate eigenvalues for decay as $z \to -\infty$; in this case the surface wave is exceptional [15].

(iii) $[\underline{\underline{P}}_{\mathscr{A}}]$ has no non-semisimply degenerate eigenvalues for decay as $z \to \infty$, but $[\underline{\underline{P}}_{\mathscr{B}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the surface wave is exceptional [15].

(iv) $[\underline{\underline{P}}_{\mathscr{A}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to \infty$ and $[\underline{\underline{P}}_{\mathscr{B}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the surface wave is doubly exceptional [16].

Exceptional surface waves may be distinguished from unexceptional surface waves by their localization characteristics: the decay of fields of an exceptional surface wave in an anisotropic partnering medium exhibits a combined linear-exponential dependency on distance from the interface, whereas the decay is purely exponential for an unexceptional surface wave.

A variety of different types of exceptional surface wave [15] have been reported on recently:

(a) If both partnering mediums are dielectric materials, with one being anisotropic, then exceptional surface waves known as Dyakonov–Voigt surface waves can exist. These exceptional waves can arise if the partnering materials are nondissipative [17] or dissipative [18]. For example, while the planar interface of an isotropic dielectric material and a uniaxial dielectric material supports one exceptional surface wave for each quadrant of the interface plane, the planar interface of an isotropic dielectric material and a biaxial dielectric material supports two exceptional surface waves for each quadrant of the interface plane [19]. Doubly exceptional Dyakonov–Voigt surface waves have been reported on for the planar interface of a biaxial dielectric material and a uniaxial dielectric material [16].

(b) If one of the partnering mediums is a metal and the other is a dielectric material, with at least one of them being anisotropic, then exceptional surface waves known as surface–plasmon–polariton–Voigt waves can exist [20].

A representative numerical example is illustrated in Fig. 1, wherein the real and imaginary parts of the surface wavenumber q (relative to the free-space wavenumber k_0) is plotted against ψ for the instance where material $\mathscr{A}$ is a uniaxial dielectric material with relative permittivity dyadic $(6.4962 + 0.09830i)\,\hat{\underline{u}}_x\hat{\underline{u}}_x + (2.2 + 0.5i)\left(\hat{\underline{u}}_y\hat{\underline{u}}_y + \hat{\underline{u}}_z\hat{\underline{u}}_z\right)$; material $\mathscr{B}$ is an isotropic metal with relative permittivity $-16.07 + 0.44i$; and the free-space wavelength is 633 nm. The unexceptional surface waves (i.e., surface–plasmon–polariton waves) are represented by the two curves in Fig. 1; and the solitary exceptional surface wave (i.e., a surface–plasmon–polariton–Voigt wave) is identified by the black star at $\psi = 35°$ (with $q = (1.8222 + 0.2045i)k_0$).

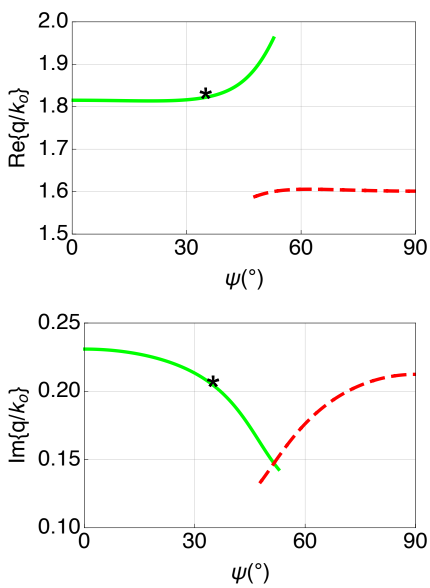

Figure 1. $\mathrm{Re}\{q/k_0\}$ and $\mathrm{Im}\{q/k_0\}$ plotted versus $\psi \in (0, 90)°$ for unexceptional (curves) and exceptional (black stars) surface–plasmon–polariton waves. See text for parameter values.

(c) The notion of exceptional surface waves extends to planar interfaces involving nonhomogeneous materials. For example, if one of the partnering mediums is a homogeneous uniaxial dielectric material and the other is an isotropic dielectric material that is periodically nonhomogeneous in the direction normal to interface, then exceptional surface waves known as Dyakonov–Tamm–Voigt surface waves can exist [21]. Indeed, multiple such exceptional waves can exist for each quadrant of the interface plane.

3 Compound waves

Let us now turn from the two-medium structure that guides surface waves to the three-medium structure that guides compound waves. For example, suppose that medium $\mathscr{A}$ occupies the space $z > D$, medium $\mathscr{B}$ occupies space $0 < z < D$, and medium $\mathscr{C}$ occupies the space $z < 0$. For definiteness, let medium $\mathscr{B}$ be an isotropic metal film of thickness D, while mediums $\mathscr{A}$ and $\mathscr{C}$ are dielectric materials with at least one them being anisotropic. If D is sufficiently large (i.e., large compared to the skin depth of the metal), the two metal/dielectric interfaces will not interact and each could guide an surface–plasmon–polariton wave on its own. But, when D is sufficiently small, the two metal/dielectric interfaces can interact to engender compound plasmon-polariton (CPP) waves.

The phasor representation (1) again holds for all $z \in (-\infty, \infty)$ with q now being the compound wavenumber and the direction of compound-wave propagation relative to the x axis in the xy plane being prescribed by ψ. The source-free Maxwell curl equations now deliver the 4×4 matrix ordinary differential equations [1, 2]

$$\frac{d}{dz}[\underline{f}(z)] = \begin{cases} i[\underline{\underline{P}}_{\mathscr{A}}] \cdot [\underline{f}(z)], & z > D \\ i[\underline{\underline{P}}_{\mathscr{B}}] \cdot [\underline{f}(z)], & 0 < z < D \\ i[\underline{\underline{P}}_{\mathscr{C}}] \cdot [\underline{f}(z)], & z < 0 \end{cases} . \tag{6}$$

The electric and magnetic fields of compound-wave solutions to Eqs. (6) must decay as $z \to \pm\infty$ and satisfy the boundary conditions

$$\left. \begin{array}{l} [\underline{f}(0^-)] = [\underline{f}(0^+)] \\ [\underline{f}(D^-)] = [\underline{f}(D^+)] \end{array} \right\} . \tag{7}$$

For a certain value of ψ, suppose that a CPP wave is excited. Let us consider possible degeneracies of the matrixes $[\underline{\underline{P}}_{\mathscr{A}}]$ and $[\underline{\underline{P}}_{\mathscr{C}}]$:

(i) $[\underline{\underline{P}}_{\mathscr{A}}]$ does not have a non-semisimply degenerate eigenvalue for decay as $z \to \infty$ nor does $[\underline{\underline{P}}_{\mathscr{C}}]$ have a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the CPP wave is unexceptional [14].

(ii) $[\underline{\underline{P}}_{\mathscr{A}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to \infty$, but $[\underline{\underline{P}}_{\mathscr{C}}]$ has no non-semisimply degenerate eigenvalues for decay as $z \to -\infty$; in this case CPP wave is exceptional [22].

(iii) $[\underline{\underline{P}}_{\mathscr{A}}]$ has no non-semisimply degenerate eigenvalues for decay as $z \to \infty$, but $[\underline{\underline{P}}_{\mathscr{C}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the CPP wave is exceptional [22].

(iv) $[\underline{\underline{P}}_{\mathscr{A}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to \infty$ and $[\underline{\underline{P}}_{\mathscr{C}}]$ has a non-semisimply degenerate eigenvalue for decay as $z \to -\infty$; in this case the CPP wave is doubly exceptional.

Exceptional CPP waves have been reported on recently for the case of a metal film embedded in a uniaxial dielectric material (i.e., in this case mediums $\mathscr{A}$ and $\mathscr{C}$ are the same) [23]: up to two exceptional CPP waves were found for each quadrant of the interface plane. And more general instances of exceptional CPP waves have been reported on too [22].

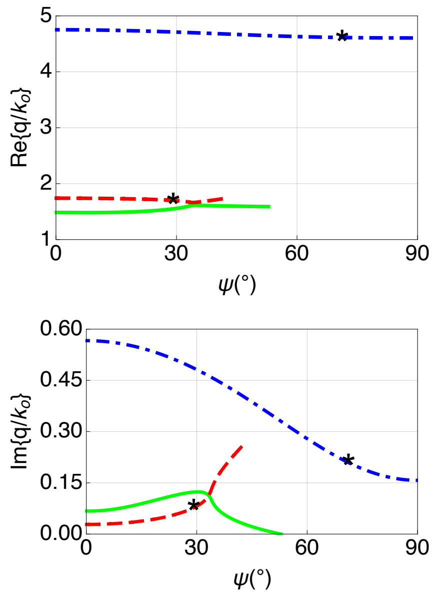

Figure 2. Re$\{q/k_0\}$ and Im$\{q/k_0\}$ plotted versus $\psi \in (0, 90)^\circ$ for unexceptional (curves) and exceptional (black stars) CPP waves. See text for parameter values.

A representative numerical example is illustrated in Fig. 2, wherein the real and imaginary parts of the compound wavenumber q (relative to the free-space wavenumber k_0) is plotted against ψ for the instance where material $\mathscr{A}$ is a uniaxial dielectric material with relative permittivity dyadic $(3.1635 + 3.5687i)\,\hat{\underline{u}}_x\hat{\underline{u}}_x + (2.2 + 0.2i)\left(\hat{\underline{u}}_y\hat{\underline{u}}_y + \hat{\underline{u}}_z\hat{\underline{u}}_z\right)$; material $\mathscr{B}$ is an isotropic metal with relative permittivity $-16.07 + 0.44i$; material $\mathscr{C}$ is an isotropic dielectric material with relative permittivity 6.26; $D = 15$ nm; and the free-space wavelength is 633 nm. The unexceptional CPP waves are represented by the three curves in Fig. 2; and the two exceptional CPP waves are identified by the black stars at $\psi = 29.1868^\circ$ (with $q = (1.7007 + 0.0771i)k_0$) and $\psi = 71.2253^\circ$ with ($q = (4.6133 + 0.2093i)k_0$).

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