# Analysis of polarization transformation in electromagnetic wave reflection from surfaces with complex boundary conditions 

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It is well known that the reflection of a circularly (or elliptically) polarized wave from a planar perfect electric conductor (PEC) surface changes the handedness of the polarization of the incident wave. Likewise happens for the reflection from a perfect magnetic conductor (PMC) boundary. For a linearly polarized incident wave, the reflected wave retains its linear character (in other words, it is the eigenpolarization) but the character of the reflection itself is dual in these two cases: electric field reflection coefficient from PEC is the same as the magnetic field reflection coefficient from PMC. As has been generalized [1], the so-called perfect electromagnetic conductor (PEMC) is a medium and surface which contains PEC and PMC as special cases. It has the very peculiar property that the reflection of a linearly polarized incident field results in a rotated linear polarization, and hence the linear polarization is no longer an eigenpolarization. And even more, the boundary is non-reciprocal. For a certain PEMC parameter [1], the reflection is totally cross-polarized.

Electromagnetic boundary conditions are relations of the tangential and/or normal components of the electric and magnetic fields $(\mathbf{E}, \mathbf{H})$ or flux densities $(\mathbf{D}, \mathbf{B})$ at the boundary of the domain of interest. Combining both the normal and tangential components leads to a rather general class on boundary conditions [2]; in this talk, we will focus on a particular subclass of boundaries; so-called impedance boundaries for which the boundary condition involves only tangential electric and magnetic fields

$$
\begin{equation*}
\mathbf{E}_{t}=\overline{\bar{Z}}_{s} \cdot\left(\mathbf{n} \times \mathbf{H}_{t}\right) \tag{1}
\end{equation*}
$$

This is a dyadic relation between the tangential electric and magnetic fields at the boundary: $\mathbf{E}_{t}=-\mathbf{n} \times(\mathbf{n} \times \mathbf{E})$ and $\mathbf{H}_{t}=-\mathbf{n} \times(\mathbf{n} \times \mathbf{H})$, with $\mathbf{n}$ as the unit normal of the boundary. (Note that despite the apparent simplicity of this impedance equation, due to the dyadic nature of $\overline{\bar{Z}}_{s}$, it spans still a rich domain.)

To appreciate the power of anisotropic metaboundaries to transform the polarization of wave in reflection, consider the perfect co-polarization reflector presented in [3]. With its metamaterial realization, it shows its performance over a wide angular range. For this co-polarization-boundary case, the surface impedance dyadic in (1) is anisotropic:

$$
\begin{equation*}
\overline{\bar{Z}}_{s}=-\mathrm{j} \eta_{0} \sinh (u) \overline{\overline{\mathrm{I}}}_{t}+\mathrm{j} \eta_{0} \cosh (u) \overline{\overline{\mathrm{L}}} \tag{2}
\end{equation*}
$$

where $u$ is a real parameter, and the two-dimensional dyadics are $\overline{\bar{I}}_{t}=\mathbf{v v}+\mathbf{w w}$ and $\overline{\overline{\mathrm{L}}}=\mathbf{v w}+\mathbf{w v}$. (The free-space impedance $\eta_{0}=\sqrt{\mu_{0} / \varepsilon_{0}}$ gives the units for the surface impedance.) The vectors $\mathbf{v}$ and $\mathbf{w}$ are tangential unit vectors such that $(\mathbf{v}, \mathbf{w}, \mathbf{n})$ form a right-handed base. Contrasting to PEMC (which is both isotropic and non-reciprocal), this boundary (2) is an-isotropic and reciprocal. In the talk, the particular properties of this boundary and others will be treated from the matched waves [2] point of view.

## References

[1] I. V. Lindell and A. H. Sihvola, "Perfect electromagnetic conductor," Journal of Electromagnetic Waves and Applications, 19, 7, pp. 861-869, 2005.
[2] I. V. Lindell and A. Sihvola: Boundary Conditions in Electromagnetics. IEEE Press, Wiley, Hoboken, NJ, USA, 2020.
[3] F. Liu, S. Xiao, A. Sihvola, and J. Li, "Perfect co-circular polarization reflector: A class of reciprocal perfect conductors with total co-circular polarization reflector," IEEE Transactions on Antennas and Propagation, 62, 12, pp. 6274-6281, 2014.

