Wiener-Hopf Method for Arbitrary Linear Scattering Problems in Angular Regions

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The Wiener-Hopf technique in its generalized form has been applied effectively in electromagnetic wave scattering problem for angular regions (wedge problems), see the monographies [1-2] and references there in. Following the procedure first proposed in [3], in this work, we illustrate how the extension of the Wiener-Hopf (WH) technique in angular regions for arbitrary linear wave scattering problems [3-6]. This technique can be also extended to geometries containing angular regions and/or stratified planar regions, see for instance [7]. We start our formulation from electromagnetic applications [3-5] and we extend the procedure to elasticity [6]. The method is based on two steps: the deduction of the Generalized Wiener-Hopf Equations (GWHEs) for angular region problems [3-6] and the solution of the equations using the semi-analytical procedure of factorization known as Fredholm factorization method, see for instance [8-9].

In [10] a general procedure to obtain first order vector differential formulation for transverse components of the field is reported for rectangular shaped regions. Starting from the wave motion differential formulation

$$\Gamma \cdot \psi = \theta \tag{1}$$

where Γ, ψ, θ are respectively a matrix differential operator, and two fields related together by constitutive relations. By transversalization procedure we get

$$-\frac{\partial}{\partial y}\psi_t = M(\frac{\partial}{\partial z}, \frac{\partial}{\partial x}) \cdot \psi_t \tag{2}$$

where ψ_t are the field components that are continuous at interfaces at constant y. In this work, we extend the procedure to angular regions by defining suitable oblique Cartesian coordinate system. We obtain the first order vector differential formulation for transverse components of the field in oblique coordinates that are similar and concatenated to the ones of the rectangular regions (using Laplace transformation in the spectral domain we have related eigenvalues and eigenvectors and construction of the Green function). The solution of this system of equation and its projection to the reciprocal eigenvectors give functional equations in spectral domain. The application of boundary conditions allow getting the GWHEs of the angular regions. The procedure follows the paradigm of Jones' method where we obtain the WH formulation in spectral domain from the differential formulation in physical domain instead of using integral equation formulation in physical domain. We note that the GWHEs present plus and minus functions that are defined in different complex planes. In several cases, suitable mappings allow transforming GWHEs into CWHEs [1-4] and the solution is obtained by Fredholm factorization [8-9].

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