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Direct Position Determination in the Presence of Direction-Dependent Mutual Coupling Using a Moving Array

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Abstract

In this paper, a new approach without iteration is proposed for direct position determination (DPD) with Doppler shifts in the presence of unknown direction-dependent mutual coupling (DDMC). The special banded symmetric Toeplitz structure of mutual coupling matrix and Doppler shift containing location information can be exploited for constructing a space-time transformation matrix, and the spectral function which is utilized to obtain the locations of sources is established via the orthogonality between space-time transformation matrix and noise subspace. Simulation results show that our solution can accurately determine the emitter position in a single-step without *a priori* information of the DDMC.

1 Introduction

Plenty of key techniques applied to wireless positioning play an important role in civilian use, such as wireless communications, navigation, and geophysical exploration. At present, in addition to further research on high-precision direct position determination (DPD) algorithm, it is also necessary to quantitatively analyze the theoretical performance of the DPD algorithm under the effect of limited sampling, model error or system error, which is conducive to quantitatively know the actual impact of various errors on the positioning performance, so as to be applied in the actual scene. One models the random disturbance of array manifold as model error [1]. In [2], for the maximum likelihood DPD algorithm with known waveforms, one analyzes the performance of DPD in the presence of model errors caused by multipath, calibration errors, mutual coupling, etc. And the simulation shows that in many cases of interest DPD is superior to the traditional two-step method. Reference [3] theoretically analyzes Doppler shifts based DPD method under the influence of model error. In addition, impact of different base station geometries on performance of DPD is analyzed [4]. The analysis of the performance of DPD is insufficient, especially for the case of direction-dependent mutual coupling (DDMC).

Recently, calibrating and compensating direction-independent mutual coupling have been intensively studied. However, considering the practical situation, the effect of mutual coupling makes a difference in signals incoming from different directions due to the directional beampattern of antenna array. In [5], a novel method estimates the DOA in the presence of direction-dependent mutual coupling (DDMC) in an iterative way, and the DDMC estimation can be regard as a convex minimization problem after unifying transformation. However, this iterative algorithm has high computational load, and then a low complexity algorithm is proposed to improve the efficiency of DOA estimation with DDMC [6].

All the above studies only focus on direction finding, and DPD in the presence of DDMC needs further. In this paper, a Doppler shifts based DPD method is proposed in the presence of DDMC. For sensor calibration DDMC, the transformation matrix is constructed using a special structure of DDMC matrix, and then we construct space-time transformation matrix by utilizing the Doppler shifts. Next, the orthogonality between the space-time transformation matrix and the noise subspace is exploited to construct a new spectral functions which can obtain position of target by searching the spectrum peaks. Finally, we derive the stochastic Cramér-Rao bound (CRB) and examine the performance of the proposed algorithm through simulations. Simulation results validate validity of the proposed DPD algorithm and also demonstrate that the proposed algorithm achieves high precision and robust direct position determine.

2 Problem Formulation

Consider Q stationary radio emitters and a moving receivers which equips an M-elements uniform linear array (ULA). The qth emitters is located in $\mathbf{p}_q = [x_q, y_q]^T$, and $[\cdot]^T$ denotes the operation of transpose. $\tilde{\mathbf{p}}_k = [\tilde{x}_k, \tilde{y}_k]^T$ and $\mathbf{v}_k$ are the known position and velocity of the moving array in the kth time slot. The transmitted signal is assumed to be narrowband with frequency centered at f_c in the far field. The observed signal time interval $[0, T]$ can be partitioned into N sections, each of length $T_s = T/N$. Assume the array is not well calibrated, and the DDMC effects cannot be ignored. Therefore, the observed signals by the kth time slot in the n-th section is well approximated by the time-dependent $M \times 1$ vector

$$\mathbf{r}_k(n) = \mathbf{A}_k \mathbf{G}_k \mathbf{s}_k(n) + \mathbf{n}_k(n) \tag{1}$$

where $\mathbf{s}_k(n)$ is the unknown signal waveform, $\mathbf{w}(n)$ is white, zero mean, and complex Gaussian noise with common variance σ_w^2, $\mathbf{A}_k = [\mathbf{C}_{k1}\mathbf{a}_k(\mathbf{p}_1), \cdots, \mathbf{C}_{kQ}\mathbf{a}_k(\mathbf{p}_Q)]$ is the moving array response from the Q emitters in the kth time slot, and the steering vector corresponding to the qth emitter is $\mathbf{a}_k(\mathbf{p}_q) = [1, \beta(\mathbf{p}_q), \cdots, \beta(\mathbf{p}_q)^{M-1}]^T$ with $\beta(\mathbf{p}_q) = \exp(-j2\pi d(x_q - \tilde{x}_k)/(\lambda |(x_q - \tilde{x}_k, y_q - \tilde{y}_k)|))$, where d is the inter-sensor spacing, and λ is the carrier wavelength, where $\mathbf{C}_{kq} \in \mathbb{C}^{M \times M}$ denote the DDMC matrix at the qth emitter. For the ULA, $\mathbf{C}_{kq}$ can be approximated by a banded symmetric Toeplitz matrix, i.e., $\mathbf{C}_{kq} = Toeplitz([(\mathbf{c}_k^q)^T, \mathbf{0}_{M-P}^T])$, where $\mathbf{c}_k^q = [c_{k,1}^q, c_{k,2}^q, \cdots, c_{k,P}^q]^T$ with length P is the DDMC vector of qth signal, satisfying $\left|c_{k,1}^q\right| = 1 > \left|c_{k,2}^q\right| > \cdots > \left|c_{k,P}^q\right|$, $\mathbf{0}_{M-P} \in \mathbb{R}^{M-P}$ is a zero vector, the operators $(\cdot)^T$, $(\cdot)^H$ denote the operation of transpose and conjugate transpose, respectively. And $\mathbf{G}_k = diag(e^{j2\pi f_k(\mathbf{p}_1)nT_s}, \cdots, e^{j2\pi f_k(\mathbf{p}_Q)nT_s})$, where $f_k(\mathbf{p}_q)$ is the Doppler frequency shift of the qth source, namely

$$f_k(\mathbf{p}_q) = f_c \frac{\mathbf{v}_k^T(\mathbf{p}_q - \mathbf{u}_k)}{\dot{c}\left\|\mathbf{p}_q - \mathbf{u}_k\right\|_2} \tag{2}$$

where $\dot{c}$ denotes the propagation speed, the symbols $diag\{\cdot\}$ stands for diagonal matrix, and $\|\cdot\|_2$ denotes the operation of ℓ_2 (Euclidean) norm. .

Next, we extract the Doppler frequency shift by forming an $L-$factor temporally stacked data vector in kth time slot

$$\mathbf{x}_k(n) = [\mathbf{r}_k^T(nL), \mathbf{r}_k^T(nL+1), \cdots, \mathbf{r}_k^T(nL+L-1)]^T \tag{3}$$

Further, $\mathbf{x}_k(n)$ can be expressed as

$$\mathbf{x}_k(n) = \mathbf{B}_k\mathbf{s}_k(n) + \mathbf{w}_k(n) \tag{4}$$

where $\mathbf{s}_k(n) = [\mathbf{s}_{k1}(n), \cdots, \mathbf{s}_{kQ}(n)]^T$, $\mathbf{w}_k(n) = [\mathbf{n}_k^T(nL), \mathbf{n}_k^T(nL+1), \cdots, \mathbf{n}_k^T(nL+L-1)]^T$, and $\mathbf{B}_k$ can be written as

$$\mathbf{B}_k = [\mathbf{b}_k(\mathbf{p}_1), \mathbf{b}_k(\mathbf{p}_2), \cdots, \mathbf{b}_k(\mathbf{p}_Q)] \in \mathbb{C}^{LM \times Q} \tag{5}$$

where $\mathbf{B}_k$ stands for the extended array response matrix, $\mathbf{b}_k(\mathbf{p}_q) = \mathbf{g}_k(\mathbf{p}_q) \otimes (\mathbf{C}_k(\mathbf{p}_q)\mathbf{a}_k(\mathbf{p}_q))$ denotes the space-time steering vector, and $\mathbf{g}_k(\mathbf{p}_q) = [1, e^{j2\pi f_k(\mathbf{p}_q)T_s}, \cdots, e^{j2\pi f_k(\mathbf{p}_q)(L-1)T_s}]^{\mathrm{T}}$.

In real systems, the covariance matrix can be estimated from a finite set of snapshots by

$$\hat{\mathbf{R}}_k = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}_k(n)\mathbf{x}_k^{\mathrm{H}}(n) \tag{6}$$

Assuming the Q signals are uncorrelated to each other, then the rank of $\hat{\mathbf{R}}_k$ is Q. Consequently, the Eigen-decomposition (EVD) of $\hat{\mathbf{R}}_k$ can be expressed as

$$\hat{\mathbf{R}}_k = \mathbf{U}_{k,s}\mathbf{\Sigma}_{k,s}\mathbf{U}_{k,s}^H + \mathbf{U}_{k,w}\mathbf{\Sigma}_{k,w}\mathbf{U}_{k,w}^H \tag{7}$$

where $\mathbf{U}_{k,s}$ and $\mathbf{U}_{k,w}$ are the signal subspace and noise subspace, respectively, and $\mathbf{\Sigma}_{k,s}$ and $\mathbf{\Sigma}_{k,w}$ are two diagonal matrices containing the large and small eigenvalues, respectively.

3 Proposed Algorithm

However, as the array moves, it is difficult to obtain the DDMC matrix in advance. The positioning accuracy will be severely affected by DDMC. Therefore, it is necessary for DPD to calibrate DDMC. Fortunately, according to symmetric banded Toeplitz structure of DDMC matrix and the *Lemma* in [5], $\mathbf{C}_{kq}$ can be written as

$$\mathbf{C}_{kq} = \sum_{i=1}^{l}\mathbf{E}_{k,i}\mathbf{c}_{k,i}^q \tag{8}$$

where the (p,m)th entry of M×M matrix $\mathbf{E}_{k,i}$ can be written as

$$\mathbf{C}_{kq} = \sum_{i=1}^{l}\mathbf{E}_{k,i}c_{k,i}^q \tag{9}$$

and the transformation matrix can be defined as

$$\mathbf{T}_k(\mathbf{p}_q) = [\mathbf{E}_{k,1}\mathbf{a}_k(\mathbf{p}_q), \mathbf{E}_{k,2}\mathbf{a}_k(\mathbf{p}_q), \cdots, \mathbf{E}_{k,l}\mathbf{a}_k(\mathbf{p}_q)] \tag{10}$$

then we have

$$\mathbf{C}_{kq}\mathbf{a}_k(\mathbf{p}_q) = \mathbf{T}_k(\mathbf{p}_q)\mathbf{c}_k^q \tag{11}$$

Therefore, Based on the orthogonality between the noise subspace $\mathbf{U}_{k,w}$ and the space-time steering vector $\mathbf{b}_k(\mathbf{p}_q)$, we have

$$(\mathbf{c}_k^q)^H\mathbf{Q}(\mathbf{p}_q)\mathbf{c}_k^q = 0 \tag{12}$$

For brevity of (14), we defined

$$\mathbf{Q}(\mathbf{p}) = (\mathbf{g}_k(\mathbf{p}) \otimes \mathbf{T}_k(\mathbf{p}))^H\mathbf{U}_{k,w}\mathbf{U}_{k,w}^H(\mathbf{g}_k(\mathbf{p}) \otimes \mathbf{T}_k(\mathbf{p})) \tag{13}$$

if it satisfies $M - P > Q$, $\mathbf{Q}(\mathbf{p})$ will be a full rank matrix. Nevertheless, when $\mathbf{p}$ contains any one of the Q location of target, i.e., $\mathbf{p} = \mathbf{p}_q$, $\mathbf{Q}(\mathbf{p})$ will be rank deficiency, so it determinant is equal to zero. Therefore, DPDs of target can be estimated by searching the $\hat{\mathbf{p}}$ to make $\mathbf{Q}(\mathbf{p})$ become a non-full rank matrix. a new spectral function can be constructed as

$$P_{\det}(\mathbf{p}) = \left\{\sum_{k=1}^{K}\det[\mathbf{Q}_k(\mathbf{p})]\right\}^{-1} \tag{14}$$

where $\det[\cdot]$ denotes the determinant of a matrix.

For the proposed method, we take advantage of the Doppler shifts of moving array to expand the array manifold. Therefore, the proposed method can identify $M - P - 1$ targets. The computational complexity of the proposed method is approximately $O((\mathrm{L}M)^3) + S \times O(P^3)$, where S is the number of the spectral peak searches, $K \times O((\mathrm{L}M)^3)$ comes from EVD, and $S \times O(P^3)$ is due to finding peaks of the spectrum $P_{\det}(\mathbf{p})$.

4 Stochastic CRB

The CRB provides a benchmark for the highest positioning accuracy that can be obtained by any unbiased estimator. Consider the array output vector, the covariance matrix $\mathbf{R}_k$ can be calculated by

$$\mathbf{R}_k = E\{\mathbf{x}_k(t)\mathbf{x}_k^H(t)\} = \mathbf{B}_k\mathbf{R}_s\mathbf{B}_k^H + \sigma_w^2\mathbf{I}_{LM} \tag{15}$$

In the problem of DPD in the presence of DDMC, the unknown parameters of interest are defined as

$$\boldsymbol{\eta} = [\mathbf{x}^T, \mathbf{y}^T, \bar{\mathbf{c}}^T, \tilde{\mathbf{c}}^T]^T \tag{16}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_Q]^T$, $\mathbf{y} = [y_1, y_2, \cdots, y_Q]^T$, $\bar{\mathbf{c}} = \mathrm{Re}\{\mathbf{c}_k^1, \cdots, \mathbf{c}_k^Q\}$, and $\tilde{\mathbf{c}} = \mathrm{Im}\{\mathbf{c}_k^1, \cdots, \mathbf{c}_k^Q\}$ are unknown targets positions, real and imaginary parts of direction-dependent mutual coupling coefficients, respectively. The Fisher information matrix (FIM) of the parameter of $\boldsymbol{\eta}$ can be expressed as

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\mathbf{xx}} & \mathbf{J}_{\mathbf{xy}} & \mathbf{J}_{\mathbf{x}\bar{\mathbf{c}}} & \mathbf{J}_{\mathbf{x}\tilde{\mathbf{c}}} \\ \mathbf{J}_{\mathbf{xy}}^T & \mathbf{J}_{\mathbf{yy}} & \mathbf{J}_{\mathbf{y}\bar{\mathbf{c}}} & \mathbf{J}_{\mathbf{y}\tilde{\mathbf{c}}} \\ \mathbf{J}_{\mathbf{x}\bar{\mathbf{c}}}^T & \mathbf{J}_{\mathbf{y}\bar{\mathbf{c}}}^T & \mathbf{J}_{\bar{\mathbf{c}}\bar{\mathbf{c}}} & \mathbf{J}_{\bar{\mathbf{c}}\tilde{\mathbf{c}}} \\ \mathbf{J}_{\mathbf{x}\tilde{\mathbf{c}}}^T & \mathbf{J}_{\mathbf{y}\tilde{\mathbf{c}}}^T & \mathbf{J}_{\bar{\mathbf{c}}\tilde{\mathbf{c}}}^T & \mathbf{J}_{\tilde{\mathbf{c}}\tilde{\mathbf{c}}} \end{bmatrix} \tag{17}$$

where $\mathbf{J}_{\mathbf{xx}}$ is the FIM associated with the target positions, $\mathbf{J}_{\mathbf{x}\bar{\mathbf{c}}}$ is the FIM associated with the target positions and direction-dependent mutual coupling coefficients, and all other blocks are similarly defined. According to [7] the (m,n)th element of $\mathbf{J}$ can be written as

$$\mathbf{J}_{mn} = \sum_{k=1}^{K} L \cdot \mathrm{tr}\left\{\mathbf{R}_k^{-1}\frac{\partial \mathbf{R}_k}{\partial m}\mathbf{R}_k^{-1}\frac{\partial \mathbf{R}_k}{\partial n}\right\} \tag{18}$$

Ulteriorly, we define the following notations

$$\mathbf{B}_{k\mathbf{x}} = \left[\left.\frac{\partial\left(\mathbf{b}_k(\mathbf{p}_1)\right)}{\partial x}\right|_{x=x_1}, \cdots, \left.\frac{\partial\left(\mathbf{b}_k(\mathbf{p}_Q)\right)}{\partial x}\right|_{x=x_Q}\right] \tag{19}$$

$$\mathbf{B}_{k\bar{\mathbf{c}}_i} = \left[\left.\frac{\partial\left(\mathbf{b}_k(\mathbf{p}_1)\right)}{\partial c_i}\right|_{c_i=c_{k,i}^1}, \cdots, \left.\frac{\partial\left(\mathbf{b}_k(\mathbf{p}_Q)\right)}{\partial c_i}\right|_{c_i=c_{k,i}^Q}\right] \tag{20}$$

$$\frac{\partial\left(\mathbf{C}_{kq}\right)}{\partial \bar{c}_i} = Toeplitz(e_i^T), \frac{\partial\left(\mathbf{C}_{kq}\right)}{\partial \tilde{c}_i} = jToeplitz(e_i^T) \tag{21}$$

where $\bar{\mathbf{c}}_i$ stands for the ith row of $\bar{\mathbf{c}}$.

$$\begin{aligned}\mathbf{J}_{\eta_m\eta_n} = 2L\,\mathrm{Re}\Big\{&\left(\mathbf{R}_s\mathbf{B}_k^H\mathbf{R}_k^{-1}\dot{\mathbf{B}}_{k\mathbf{n}_n}\right) \odot \left(\mathbf{R}_s\mathbf{B}_k^H\mathbf{R}_k^{-1}\dot{\mathbf{B}}_{k\eta_m}\right)^T \\ &+ \left(\mathbf{R}_s\mathbf{B}_k^H\mathbf{R}_k^{-1}\mathbf{B}_k\mathbf{R}_s\right) \odot \left(\dot{\mathbf{B}}_{k\eta_n}^H\mathbf{R}_k^{-1}\dot{\mathbf{B}}_{k\eta_m}\right)^T\Big\}\end{aligned} \tag{22}$$

Then the average CRB can be given as

$$CRB_{\mathbf{p}} = \sqrt{\frac{1}{2Q}\sum_{i=1}^{2Q}[\mathbf{J}^{-1}]_{ii}} \tag{23}$$

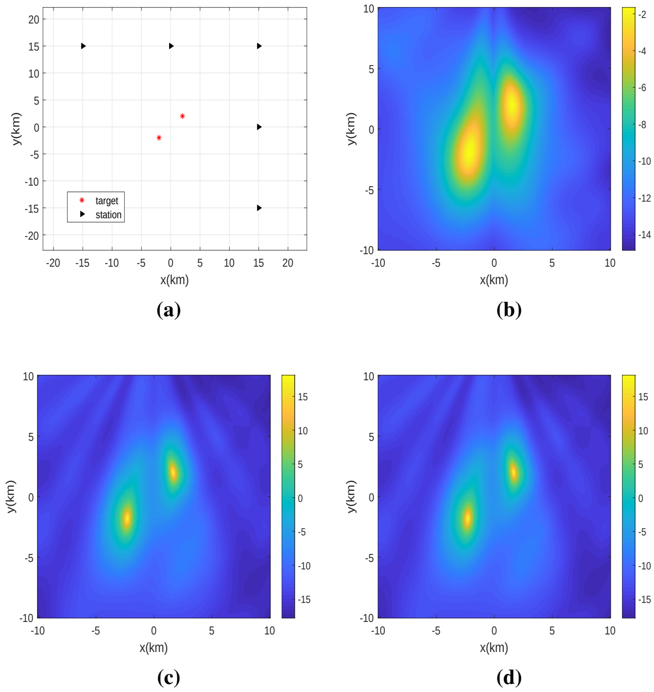

Figure 1. The performance of DPD comparison under different conditions. (a) the geometry of targets and the trajectory of the moving array, (b) DPD spectrum with unknown DDMC, (c) proposed DPD method spectrum with unknown DDMC, (d) DPD spectrum with known DDMC, where SNR = 5 dB, and the number of snapshots is 200.

5 Experimental Result and Discussion

In this section, we examine the performance of the proposed method, by performing extensive Monte Carlo simulations and comparing the results with different conditions. We focus on the root mean square error (RMSE) of position estimates defined by

$$\mathrm{RMSE}_p = \sqrt{\frac{1}{K_M Q}\sum_{i=1}^{K_M}\sum_{q=1}^{Q}\left\|\widehat{\mathbf{p}}_q - \mathbf{p}_q\right\|^2} \tag{24}$$

where K_M signifies the number of Monte Carlo simulations.

In each of the experiments, consider two coherent source located at $(-2,-2)km$ and $(2,2)km$ while moving array with 7 sensors moves in an L-shaped trajectory and observes the source at $(-15,15)km$, $(0,15)km$, $(15,15)km$, $(15,0)km$, and $(15,-15)km$ in five time slots, and the geometry is shown in Fig. 1 (a). The mutual coupling coefficients of receiver is assumed to be $\mathbf{c}_k^q = \alpha_{k,q}[1/\alpha_{k,q}, -0.6545 + 0.4755\mathrm{i}, -0.4414 - 0.3414i]$, $\alpha_{k,q} \sim N(0,1)$.

Fig. 1 shows that the contour plots for DPD without compensating DDMC, proposed DPD algorithm, and DPD with known DDMC, respectively. The signal to noise (SNR) is 5

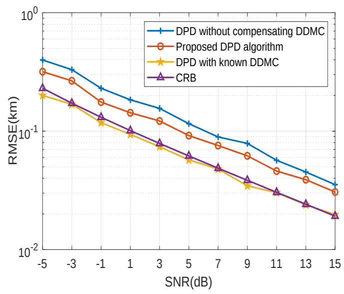

Figure 2. RMSE of DPD estimates versus SNR when N = 200.

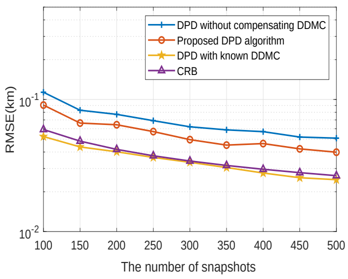

Figure 3. RMSE of DPD estimates versus the number of snapshots when SNR = 10 dB.

dB, the number of snapshots is 200, and $L = 3$. From Fig. 1 (b) and (d), it can be clearly seen that the positioning accuracy deteriorates for DDMC, and to compare Fig. 1 (c) and (d), the proposed method can significantly alleviate the effect of DDMC and approached to the performance of DPD with know DDMC.

Next, we consider the moving array capture $N = 200$ snapshots, the SNR varied between -5 dB and 15 dB with a step size of 2 dB. Fig. 2 shows that, the proposed DPD algorithms achieves better estimation performance with higher SNRs, and can effectively estimate the locations of targets at lower SNRs. The performance of proposed method asymptotically approaches the performance of DPD in the presence of known MC and CRB.

The final numerical experiment examines the performance of proposed DPD algorithm with unknown and known mutual coupling under the number of snapshots changed from 100 to 500 with a step size of 50 and SNR = 10 dB. Fig. 3 shows the RMSE of the proposed DPD algorithm compared with DPD without compensating DDMC, CRB and known MC as the snapshots change, respectively. As the number of snapshots increases, the estimation performance improves slightly in both conditions for the same array model.

6 Conclusion

This paper investigated the problem of DPD on a moving platform in the presence of DDMC. With a moving array, we propose a new Doppler shifts based DPD algorithm by taking advantage of the special structure of mutual coupling matrix for a uniform linear array, and the spatial spectrum function can be constructed via the orthogonality of the modified space-time transformation matrix and the noise subspace. Compared with previous DPD algorithms, the proposed one has two advantages: 1) positions of emitters can be estimated without compensating DDMC; 2) the resolution of the proposed estimator in a single-step manner is much better for multiple targets localization, while the computational complexity can be still preserved at a relatively low level. Besides, simulation results demonstrate the proposed algorithm can attain the corresponding CRB.

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