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Phase-Only Nulling for Uniform Linear Array via Kronecker Decomposition

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Abstract

This paper presents a novel beam-forming algorithm and its application to phase-only nulling. The proposed algorithm aims to achieve the interference nulling only by modulate phases so that we can reduce cost when setting phase shifter. This algorithm is based on the Kronecker decomposition. We can decompose the steering vector and weight vector into the form of Kronecker product because of their Vandermonde structures. With Kronecker decomposition, the weight vector design can be transformed into the design for several sub-vectors. According to the theory of beam forming and interference nulling, we are able to get analytical solutions when designing the desired weight vector and its complexity is low. Simulation results are provided to demonstrate the performance of the proposed algorithm.

1 Introduction

Array antenna has been widely used in plenty of fields such as radar detection, navigation and wireless communication etc [1–3]. In radar system, it is quite necessary to design the array pattern, which is of significant importance to enhance system performance no matter in interference suppression or other aspects.

For the sake of setting nulls in the direction of interferences, the traditional method is usually realized by complex weighting. It is known that designing the weight vector for the elements is quite important to achieve an ideal array pattern [4]. It is necessary not only to form null in the interference direction, but also to set the mainlobe constraint. However, it is complex to modulate the amplitude and phase simultaneously. Besides, it will also lead to the loss of power, thus affecting the performance of detection. To simplify the beamforming network and reduce the cost, phase-only technique is also preferred [5–7]. Phase-only algorithm is mainly used in the control of nulls in beam pattern forming. In addition, the phase-only structure allows to use the single power-divider network, which is more efficient than those conventional structures that modulate the amplitude of excitation dynamically [8]. Comparing with other beamforming algorithm, phase-only algorithm does not need to modulate the amplitude. We can control the beam only by setting phase shifters.

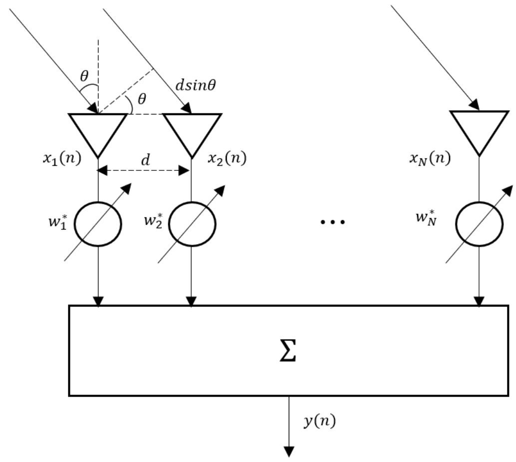


Figure 1. Uniform linear array.

In this paper, a phase-only nulling algorithm via Kronecker decomposition [9] is proposed. We design the weight vector by using the sub-vectors, then form the desired beam. The proposed algorithm has a low computational complexity with analytical solutions.

2 Preliminaries

2.1 Array Model

We consider a one-dimensional N-element uniform linear array as the array model, as shown in Fig.1. The corresponding steering vector can be written as

$$\mathbf{a}(\theta) = [1, e^{-j\phi}, \cdots, e^{-j(N-1)\phi}]^{\mathrm{T}} \in \mathbb{C}^N \tag{1}$$

where $\phi = 2\pi d \sin\theta/\lambda$ stands for the spatial phase, $(\cdot)^{\mathrm{T}}$ stands for the transpose operator, $j = \sqrt{-1}$ denotes the imaginary unit, and d denotes the element spacing.

Given the array weight vector $\mathbf{w} \in \mathbb{C}^N$, the beam pattern of the spatial filtering can be expressed as

$$f(\theta) = \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta) \tag{2}$$

where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose. Obviously, we can obtain an ideal beam pattern by designing an appropriate weight vector $\mathbf{w}$.

2.2 Phase-Only Nulling

In order to suppress the L interferences from some specific angles, interference nulling is a convenient method. Through some certain method of designing the weight vector $\mathbf{w}$ to make a null in the direction of the interference θ_l, where $l = 1, 2, \cdots, L$. The principle above can be expressed as

$$\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_l) = 0 \tag{3}$$

where $l = 1, 2, \cdots, L$. In this way, those interference signals are not able to pass the spatial filter except of the observed signal θ_0.

In order to achieve the function of phase-only control, we still need to set

$$|w_n| = 1 \tag{4}$$

where $n = 1, 2, \cdots, N$ and w_n stands for the nth entry of the weight vector of an element.

3 Proposed Algorithm

Before presenting the algorithm, we introduce an important lemma firstly.

Lemma **1** *(Kronecker Decomposition)If a vector* $\mathbf{a}$ *is an* $N \times 1$ *vector having the Vandermonde structure* $\mathbf{a} = [1, e^{-j\phi}, \cdots, e^{-j(N-1)\phi}]^{\mathrm{T}}$ *with* ϕ *is a fixed. Assuming the element number* $N = 2^M$*, the vector* $\mathbf{a}$ *can be decomposed as*[1]

$$\mathbf{a} = \mathbf{a}_M \otimes \mathbf{a}_{M-1} \otimes \cdots \otimes \mathbf{a}_1 \tag{5}$$

where $\otimes$ *represents the Kronecker product* [10]. *And the factor is*

$$\mathbf{a}_m = [1, \exp(-j2^{(m-1)}\phi)]^{\mathrm{T}} \tag{6}$$

where $m = 1, 2, \cdots, M$.

It can be seen that the vector $\mathbf{a}$ and its Kronecker factor are all phase-shift vectors with uni-modulus elements. This also gives us the theoretical basis of the phase-only nulling.

According to *Lemma* **1**, for an one-dimensional N-element uniform linear array, we can use the formula (1) as its steering vector and the structure of the steering vector accords is Vandermonde structure. Then we can utilize the Kronecker decomposition to decompose the steering vector into the form like formula (5). In the same way, the steering vector of the interference signal $\mathbf{a}(\theta_l)$ can also be decomposed into the same form. If element number $N = 2^M$, the steering vector of the interference signal and observed signal can also be expressed as

$$\mathbf{a}(\theta_l) = \mathbf{u}_M^{(l)} \otimes \mathbf{u}_{M-1}^{(l)} \otimes \cdots \otimes \mathbf{u}_1^{(l)} \tag{7}$$

$$\mathbf{a}(\theta_0) = \mathbf{v}_M \otimes \mathbf{v}_{M-1} \otimes \cdots \otimes \mathbf{v}_1 \tag{8}$$

where $\mathbf{u}_m^{(l)} = [1, \exp(-j2^{(m-1)}\phi_l)]^{\mathrm{T}}$, $\mathbf{v}_m = [1, \exp(-j2^{(m-1)}\phi_0)]^{\mathrm{T}}$ stand for the Kronecker factors of the steering vectors and $\phi_l = 2\pi d \sin\theta_l / \lambda$, $\phi_0 = 2\pi d \sin\theta_0 / \lambda$ separatly stands for the spatial phase of their corresponding directions, and $l = 1, 2, \cdots, L$, $m = 1, 2, \cdots, M$.

To find a weight vector $\mathbf{w}$ satisfying equation (3) and (4), we propose to construct $\mathbf{w}$ as

$$\mathbf{w} = \mathbf{w}_M \otimes \mathbf{w}_{M-1} \otimes \cdots \otimes \mathbf{w}_1 \tag{9}$$

where $\mathbf{w}_m$ stands for the sub-vector needs to be designed and $m = 1, 2, \cdots, M$. Then the beam pattern in formula (2) can be rewritten as

$$\begin{aligned} f(\theta) &= \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta) \\ &= (\mathbf{w}_M^{\mathrm{H}} \otimes \mathbf{w}_{M-1}^{\mathrm{H}} \otimes \cdots \otimes \mathbf{w}_1^{\mathrm{H}})(\mathbf{a}_M \otimes \mathbf{a}_{M-1} \otimes \cdots \otimes \mathbf{a}_1) \\ &= (\mathbf{w}_M^{\mathrm{H}}\mathbf{a}_M) \otimes (\mathbf{w}_{M-1}^{\mathrm{H}}\mathbf{a}_{M-1}) \otimes \cdots \otimes (\mathbf{w}_1^{\mathrm{H}}\mathbf{a}_1) \\ &= (\mathbf{w}_M^{\mathrm{H}}\mathbf{a}_M)(\mathbf{w}_{M-1}^{\mathrm{H}}\mathbf{a}_{M-1}) \cdots (\mathbf{w}_1^{\mathrm{H}}\mathbf{a}_1) \end{aligned} \tag{10}$$

where we use the fact that $(\mathbf{w}_p^{\mathrm{H}} \otimes \mathbf{w}_q^{\mathrm{H}})(\mathbf{a}_p \otimes \mathbf{a}_q) = (\mathbf{w}_p^{\mathrm{H}}\mathbf{a}_p) \otimes (\mathbf{w}_q^{\mathrm{H}}\mathbf{a}_q)$.

For the interference signal and the observed signal, their beam pattern can be written as

$$\begin{aligned} f(\theta_l) &= \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_l) \\ &= (\mathbf{w}_M^{\mathrm{H}}\mathbf{u}_M^{(l)})(\mathbf{w}_{M-1}^{\mathrm{H}}\mathbf{u}_{M-1}^{(l)}) \cdots (\mathbf{w}_1^{\mathrm{H}}\mathbf{u}_1^{(l)}) \end{aligned} \tag{11}$$

$$\begin{aligned} f(\theta_0) &= \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0) \\ &= (\mathbf{w}_M^{\mathrm{H}}\mathbf{v}_M)(\mathbf{w}_{M-1}^{\mathrm{H}}\mathbf{v}_{M-1}) \cdots (\mathbf{w}_1^{\mathrm{H}}\mathbf{v}_1) \end{aligned} \tag{12}$$

where $l = 1, 2, \cdots, L$.

According to the theory of phase-only nulling, we need to choose one factor in the equation (11) to set it to zero, just like the form in equation (3). Then we are able to get the weight of the corresponding part by solving the equation $\mathbf{w}_m^{\mathrm{H}}\mathbf{u}_m^{(l)} = 0$ where $l = 1, 2, \cdots, L$, $m = 1, 2, \cdots, M$. Then according to the solution of the equation above, we will get the desired sub-vector for the corresponding interference

$$\mathbf{w}_l = [-\exp(j2^{(l-1)}\phi_l, 1)]^{\mathrm{T}} \tag{13}$$

where $l = 1, 2, \cdots, L$.

For the remaining factors, we can utilize then to constrain the mainlobe of the beam pattern. In other words, for the remaining factors, we are supposed to maximize $f(\theta_0)$ by designing these weight components. Because of the orthogonality, it is easy to get

$$\mathbf{w}_i = \mathbf{v}_i = [1, \exp(-j2^{(i-1)}\phi_0)]^{\mathrm{T}} \tag{14}$$

where $i = L+1, L+2, \cdots, M$.

[1]Actually, for any value of N, we can decompose the vector into the form of Kronecker product. However, 2 is the smallest prime number. In the case of the same number of interferences, the decomposition by 2 can save radar resources to the maximum extent and it is also convenient to decompose. Therefore, we set the element number to the power of 2 in this paper.

Algorithm 1 Phase-Only Nulling Algorithm for Uniform Linear Array via Kronecker Decomposition

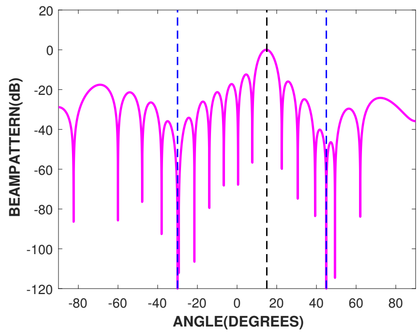
Input: $d, \lambda, N = 2^M, \theta_0, \theta_l, l = 1, 2, \cdots, L$
for $l = 1, 2, \cdots, L$ **do**
  $\phi_l = 2\pi d \sin\theta_l / \lambda$
  $\mathbf{a}(\theta_l) = \mathbf{u}_M^{(l)} \otimes \mathbf{u}_{M-1}^{(l)} \otimes \cdots \otimes \mathbf{u}_1^{(l)}$, where $\mathbf{u}_m^{(l)} = [1, \exp(-j2^{(m-1)}\phi_l)]^{\mathrm{T}}, m = 1, 2, \cdots, M$
  solve $\mathbf{w}_m^{\mathrm{H}} \mathbf{u}_m^{(l)} = 0$
  $\mathbf{w}_l = [-\exp(j2^{(l-1)}\phi_l, 1)]^{\mathrm{T}}$
end for
$\mathbf{a}(\theta_0) = \mathbf{v}_M \otimes \mathbf{v}_{M-1} \otimes \cdots \otimes \mathbf{v}_1$, where $\mathbf{v}_m = [1, \exp(-j2^{(m-1)}\phi_0)]^{\mathrm{T}}$
for $i = L+1, L+2, \cdots, M$ **do**
  $\phi_0 = 2\pi d \sin\theta_0 / \lambda$
  $\mathbf{w}_i = \mathbf{v}_i = [1, \exp(-j2^{(i-1)}\phi_0)]^{\mathrm{T}}$, where $i = L+1, L+2, \cdots, M$
end for
$\mathbf{w} = \mathbf{w}_M \otimes \mathbf{w}_{M-1} \otimes \cdots \otimes \mathbf{w}_L \otimes \cdots \otimes \mathbf{w}_1$

4 Simulation

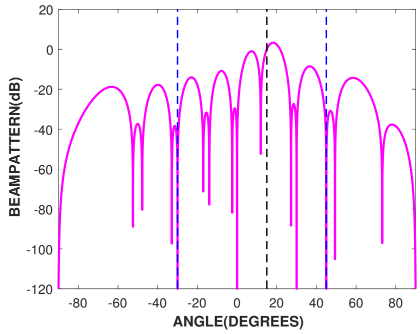
4.1 Simulation with different Kronecker factors

In the section above, we have already explained the proposed algorithm. The key part is the Kronecker decomposition. There will be a selection for us after decomposing. Considering about the Kronecker factors we got, their phases are different. In that case, assuming there are two interferences from -45 and 30 degree, and the observed signal is in the direction of 15 degree. Executing Algorithm 1, then for the interference suppression, we will give two different cases in which we select $\mathbf{w}_1^{\mathrm{H}}\mathbf{u}_1$, $\mathbf{w}_2^{\mathrm{H}}\mathbf{u}_2$ and $\mathbf{w}_M^{\mathrm{H}}\mathbf{u}_M$, $\mathbf{w}_{M-1}^{\mathrm{H}}\mathbf{u}_{M-1}$ to achieve it separately for contrast. In the simulation, we use the one-dimensional uniform linear array. The element number is the power of 2. Therefore we set the element number to $N = 2^6 = 64$ with the $\lambda/8$ as the element spacing. Then the beam pattern simulations will be given in Fig.2.

Fig.2(a) shows a desired beam pattern whose mainlobe points to 15 degree and formed two nulls on the required angles. Although the beam pattern in Fig.2(b) also form the required nulls successfully, there are some deviations in the orientation of the mainlobe, and the level of the side lobe is relatively high. Therefore, it is clear to see that the performance in Fig.2(a) is much better than the other one. To explain this phenomenon, we can compare the $\mathbf{w}_1$ and $\mathbf{w}_M$. In this case, we can assume $\mathbf{w}_1 = e^{j\Psi}$ and $\mathbf{w}_M = e^{j32\Psi}$. Obviously, $\mathbf{w}_1$ has a smaller period than $\mathbf{w}_M$, which can provide a better performance in beam pointing. Because of this, we will select the factors with the smallest footmarks to suppress interferences in the following simulations.



(a)



(b)

Figure 2. Patterns with different Kronecker factors. (a) Utilizing factors with the smallest footmarks. (b) Utilizing factors with the biggest footmarks.

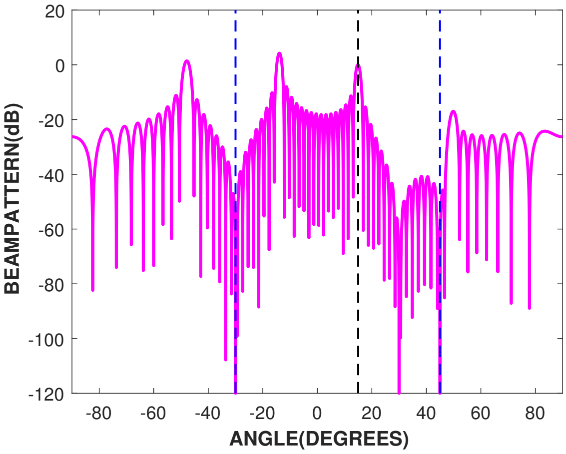
4.2 Simulation with different element spacing

In the proposed algorithm, the requirement for the element spacing is relatively high. In order to show the effect of element spacing on the performance of the algorithm, we will change the element spacing from $\lambda/2$, $\lambda/4$ to $\lambda/6$. The element number N is still $2^6 = 64$. The simulation result are shown in Fig.3.

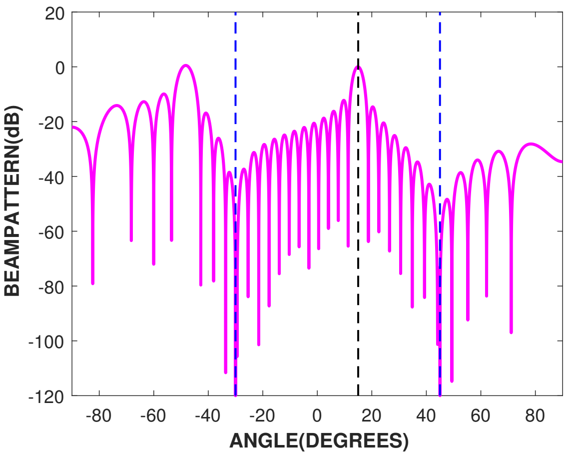
We can see that, the patterns in these three cases can meet our basic requirements for mainlobe and nulling. Moreover, with the decrease of the element spacing, the level of grating lobe also decreases. Although the mainlobe is widened to a certain extent, we can still say that the performance is getting better. We can get one of the limitations of the proposed algorithm, which is required to be in the case of small element spacing.

5 Conclusion

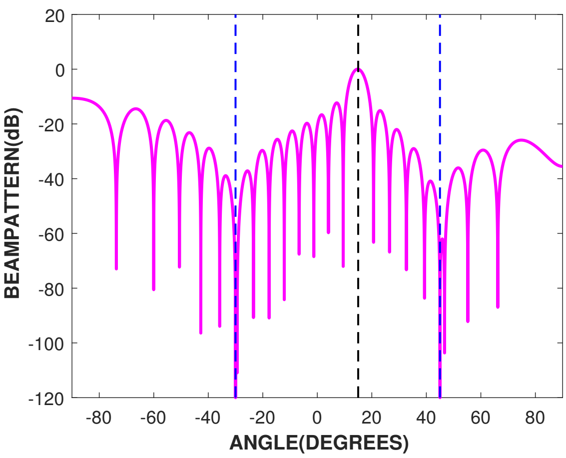
In this paper, we have presented a novel algorithm for phase-only nulling via Kronecker decomposition. In this algorithm, we transform the weight vector into the form of Kronecker product which is composed by several subvectors. In this way, we achieve the function of phase-only modulation. This algorithm not only has a low complexity but also gives us analytical solutions when design the



(a)



(b)



(c)

Figure 3. Patterns with different elment spacing. (a) $d = \lambda/2$. (b) $d = \lambda/4$. (c) $d = \lambda/6$.

weight vector. We also have simulated the algorithm in different situations. From the simulation result, the algorithm is feasible, obviously. The performance also have changed with the setting.

However, the proposed algorithm still have several limitations. Firstly, it is can only be used for uniform linear array. And then, it has a high requirement for the element spacing which needs to be small enough. On the basis of the current work, we will study how to increase the array spacing while maintaining the performance of this algorithm.

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