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Auto-Specifying Mainlobe Width for Beampattern Synthesis: An Iterative Reweighted Approach

Kangyu Tang, Xuejing Zhang*
University of Electronic Science and Technology of China, Chengdu 611731, China
Email: ky2mail@163.com, xjzhang7@163.com

Abstract

An iterative reweighted approach based on convex optimization for the array pattern synthesis is presented. The algorithm consists in solving the problem of auto-specifying the mainlobe width. In the optimization procedure, a slack variable is introduced in the region near the mainlobe region of a pattern. The value of the slack variable will change as well as the width of the region will be narrower based on iterations. The result is that the width of the region is equal to the width of mainlobe region. Representative simulation is provided to demonstrate the effectiveness of the proposed method in beampattern synthesis.

1 Introduction

Array antenna has been extensively applied in many fields, such as, radar, navigation and wireless communications [1–4]. Methods for optimal array antenna design play a critical role in improving system performance and reducing cost.

Over the past several decades, quite a number of approaches to pattern synthesis have been developed. The classical algorithms [5–7] have closed-form expressions but they are limited to some specific array geometries or array patterns. Global optimization-based methods like genetic algorithm (GA) [8], particle swarm optimization (PSO) method [9] and simulated annealing (SA) method [10] search the optimal solutions via stochastic approaches to design array patterns. Nevertheless, these methods take a lot of time for computation.

With recent advances in convex optimization [11], another class of algorithms for the array synthesis problem has been devised. For instance, Lebret and Boyd proposed convex programming (CP) method in [12]. The drawback while utilizing this method is that the width of mainlobe region is required to be set empirically. In such way, it may lead to two situations that the sidelobe exceeds the desired pattern or there is no solution to the problem in mathematics. This motivates us to develop a new approach, which is able to auto-specify the width of mainlobe region and make it as narrow as possible simultaneously.

More precisely, in this paper, an iterative reweighted approach for auto-specifying mainlobe region width is developed. In the proposed approach, a region which center lines in the direction angle of the antenna array is defined. It includes the mainlobe region as well as a little part of sidelobe. Then a vector is introduced in this region. Its value increases first and decreases later. Next, the value of the vector is added to the upper bound of the pattern in slack region to ascertain the width of mainlobe region step by step. To ensure the change of the value of the vector satisfies the requirement, a matrix is introduced. Besides, another vector is needed to assist the process of the previous vector.

Afterwards, the pattern synthesis problem can be expressed into a convex optimization problem. When the iterative process is terminated, the width of mainlobe region is confirmed and the weight vector is computed as well. The detail of the approach will be discussed in the following sections.

2 Problem Formulation

Let us consider an N-element antenna array with arbitrary geometry. For the sake of clarity, the problem is described for a one-dimensional pattern synthesis. Then the steering vector associated with the direction θ is given by

$$\mathbf{a}(\theta) = \left[g_1(\theta)e^{j\phi_1(\theta)}, \cdots, g_N(\theta)e^{j\phi_N(\theta)}\right]^{\mathrm{T}} \tag{1}$$

where $g_n(\theta)$ represents the radiation pattern of the nth element (we have $g_n(\theta) = 1$ when the antenna is isotropic), $\phi_n(\theta)$ stands for the phase delay of the nth element, $n = 1, \cdots, N$. Then the magnitude of the array is denoted as

$$f(\theta) = \mathbf{w}^H \mathbf{a}(\theta) \tag{2}$$

where $\mathbf{w} = [w_1, w_2, \cdots, w_N]^{\mathrm{T}}$ is the weight vector, $(\cdot)^{\mathrm{H}}$ denotes conjugate transpose operator, θ_0 stands for the direction of beam axis, which is also the incidence angle of the desired signal.

Generally, the objective of the CP method is to find a weight vector $\mathbf{w}$ that makes the magnitude of $|f(\theta_i)|$ to be below a given envelope $\rho(\theta)$ specified in the sidelobe region Φ_s. Afterwards, the mainlobe region is specified as follow

$$\Phi_m = [\theta_0 - \alpha, \theta_0 + \alpha] \tag{3}$$

where α is a real variable empirically chosen. Once Φ_m is specified, Φ_s is specified as well. Then we can express pattern synthesis problems as convex optimization problems, which can be written as

$$\begin{aligned} &\textbf{find} \quad \mathbf{w} && (4a)\\ &\text{s.t.} \quad \mathbf{w}^H\mathbf{a}(\theta_0)=1 && (4b)\\ &\qquad |\mathbf{w}^H\mathbf{a}(\theta_i)| \le \rho(\theta_i), \quad \theta_i \in \Phi_s && (4c)\end{aligned}$$

Finding a weight vector $\mathbf{w}$ is the objective of the CP method as shown in (4a). (4b) shows a main beam radiated in the direction θ_0. The major part of the CP method is (4c). In the optimal process, every θ in Φ_s is required to satisfy (4c) so as to obtain a beampattern with mainlobe region given in (3). Based on this analysis, the performance of the CP method depends on Φ_s, in other word, it depends on the width of Φ_m. If the empirically chosen variable α is large, the Φ_m will be wide, then the signal in sidelobe region will exceed the given envelop $\rho(\theta)$ near the mainlobe region , in contrast, a small α will make the synthesis problem unsolvable.

3 The Proposed Method

To overcome the drawback while using the CP method in (4), we want to redefine the mainlobe region and named it as slack region, which is denoted as

$$\Omega = [\theta_0-\beta, \theta_0+\beta] \tag{5}$$

where β is a real variable chosen optionally. The difference between Ω and Φ_m is that β is an initial value and it will change during the optimal process, which results in the reduction of the width of Ω. Therefore, the region Ω can be divided into two subregions, one is the mainlobe region Ω_m at the central and the other is sidelobe region Ω_s. Then we define a vector denoted as $\mathbf{s}=[s_1,s_2,\cdots,s_L]^{\mathrm{T}}$ in this region. The subscript L here stands for the length of vector $\mathbf{s}$ and the discrete Ω.

Next, we will explain the effect of Ω and the vector $\mathbf{s}$. As we have mentioned in Section II, the empirically set mainlobe region directly affect the performance of the CP method. The empirical method cannot suit to every situation. The function of vector $\mathbf{s}$ is a value added on the envelop $\rho(\theta)$ in Ω to determine the boundary of Ω_m. The vector $\mathbf{s}$ means the constraint intensity of $\rho(\theta)$ in Ω. In the left side of Ω, the constraint we want here should be strong, so the value of vector $\mathbf{s}$ in this part should be small. It's the same way in the right side of Ω. The ideal situation is that the constraint in both side of Ω is strong enough to make the value of vector $\mathbf{s}$ become zero and these part of Ω will turn into sidelobe region from mainlobe region. And in the central part of Ω, the constraint here should be weak because the mainlobe exceeds $\rho(\theta)$ a lot normally. Hence the value of vector $\mathbf{s}$ should be larger.

As for the vector $\mathbf{s}$, the several numbers at the beginning are zero and then the value of vector $\mathbf{s}$ gradually increases, once it reaches the middle of vector $\mathbf{s}$, the value gradually decreases and several numbers at the end of vector $\mathbf{s}$ should also be zero. To formalize this problem, let us consider that L is odd. Thus, we can acquire the following inequations

$$s_l - s_{l+1} \le 0, \;\; l=1,\cdots,\frac{L-1}{2} \tag{6a}$$

$$s_{l+1} - s_l \le 0, \;\; l=\frac{L+1}{2},\cdots,L-1 \tag{6b}$$

If we use matrix to represent the inequations above, the following expression is obtained

$$\mathbf{G}\mathbf{s} \preceq 0 \tag{7}$$

where the matrix $\mathbf{G}$ has the following form

$$\mathbf{G} = \begin{bmatrix} \mathbf{V} & 0 \\ 0 & -\mathbf{V} \end{bmatrix} \in \mathbb{R}^{(L-1)\times L} \tag{8}$$

and the matrix $\mathbf{V}$ is showed as follow

$$\mathbf{V} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{bmatrix} \in \mathbb{R}^{\frac{L-1}{2}\times\frac{L+1}{2}} \tag{9}$$

It's easy to get the following inequations when L is even in the same way.

$$s_l - s_{l+1} \le 0, \;\; l=1,\cdots,\frac{L}{2}-1 \tag{10a}$$

$$s_{l+1} - s_l \le 0, \;\; l=\frac{L}{2}+1,\cdots,L-1 \tag{10b}$$

In this way, the matrix $\mathbf{G}$ is attained in both two situations.

The proposed method now can be transformed into the following convex optimization problem

$$\begin{aligned} &\textbf{find} \quad \mathbf{w} && (11a)\\ &\text{s.t.} \quad \mathbf{w}^H\mathbf{a}(\theta_0)=1 && (11b)\\ &\qquad \mathbf{s} \succeq 0 && (11c)\\ &\qquad \mathbf{G}\mathbf{s} \preceq 0 && (11d)\\ &\qquad |\mathbf{w}^H\mathbf{a}(\theta_l)| \le \rho(\theta_l)+s_l, \quad \theta_l \in \Omega && (11e)\\ &\qquad |\mathbf{w}^H\mathbf{a}(\theta_i)| \le \rho(\theta_i), \quad \theta_i \in \Phi_s && (11f)\end{aligned}$$

In the above formulation, the constraints (11a), (11b) and (11f) is the same as (4a), (4b) and (4c). (11c) means that each value of vector $\mathbf{s}$ should be larger than zero because the upper bound of the pattern is $\rho(\theta)$. (11d) shows the result of the interaction of $\mathbf{G}$ and $\mathbf{s}$, which is applied to ensure the trend of $\mathbf{s}$. The crucial part of the proposed method is (11e). The upper bound of the pattern in Ω is the $\rho(\theta)$ plus a value from vector $\mathbf{s}$. It will assist to specify the width of Ω_m.

In the proposed method, the fewer nonzero elements of vector $\mathbf{s}$ will help to specify the narrower Ω_m. Thus, a new vector $\mathbf{p}=[p_1,p_2,\cdots,p_L]^{\mathrm{T}}$ is needed to assist the process of vector $\mathbf{s}$. After every iteration, the value of vector $\mathbf{p}$ is determined as follow

$$\mathbf{p} = \mathbf{1} \oslash \mathbf{s} \tag{12}$$

where the operator $\oslash$ denotes the element division operation.

After every itetation, the outcome of $\mathbf{s}$ is reflected on $\mathbf{p}$, in the place where the value of $\mathbf{s}$ is larger, the value of $\mathbf{p}$ will be smaller; it will be reverse when the value of $\mathbf{s}$ is small. The large values of $\mathbf{s}$ are penalized more heavily than the small values. The vector $\mathbf{s}$ will be reweighted in this way in each iteration. Such operation will reach the target of reducing the number of nonzero elements of $\mathbf{s}$.

As we want to reduce the number of nonzero elements of vector $\mathbf{s}$, finding the minimal value of $\mathbf{p}^H\mathbf{s}$ can be our target in optimal process. So the expression (11) can be rewritten as follow

$$\begin{aligned}
\min_{\mathbf{w},\mathbf{s}} \quad & \mathbf{p}^H\mathbf{s} && \text{(13a)}\\
\text{s.t.} \quad & \mathbf{w}^H\mathbf{a}(\theta_0) = 1 && \text{(13b)}\\
& \mathbf{s} \succeq 0 && \text{(13c)}\\
& \mathbf{G}\mathbf{s} \preceq 0 && \text{(13d)}\\
& |\mathbf{w}^H\mathbf{a}(\theta_l)| \le \rho(\theta_l) + s_l, \quad \theta_l \in \Omega && \text{(13e)}\\
& |\mathbf{w}^H\mathbf{a}(\theta_i)| \le \rho(\theta_i), \quad \theta_i \in \Phi_s && \text{(13f)}
\end{aligned}$$

Besides, the proposed method for pattern synthesis is summarized in Table I.

Table 1.
Summary of the proposed method for pattern synthesis.

Input	θ_0, $\mathbf{a}(\theta)$, β, Ω, $\rho(\theta)$, ε
Step 1.	(1). Calculate the length of Ω, L.
	(2). Initialize $\mathbf{G}$ using (8) and (9) with L.
	(3). Initialize $\mathbf{p}$ with L.
Step 2.	Solve the convex optimization problem (13).
Step 3.	(1). Remove the zero value of $\mathbf{s}$.
	(2). Computing the new β with new $\mathbf{s}$ and update Ω.
	(3). Compute the new L and update $\mathbf{G}$.
	(4). Update $\mathbf{p}$ using (12).
Step 4.	(1). Using $\mathbf{w}$ to obtain the beampattern.
	(2). Compute the PEI η.
Step 5.	Go to **Step 1.** unless η is less than or equal to ε twice.
Output	$\mathbf{w}$, $\mathbf{s}$, η.

4 Numerical Results

In this section, two simulation results are provided to demonstrate the efficacy and convenience of the proposed method. First, an uniform linear array with uniform sidelobe is considered. We intend to introduce the comparisons of the results between adjacent iterations. Second, there is the result of a nonuniformly spaced linear array, comparing to the CP method.

4.1 Uniform Linear Array

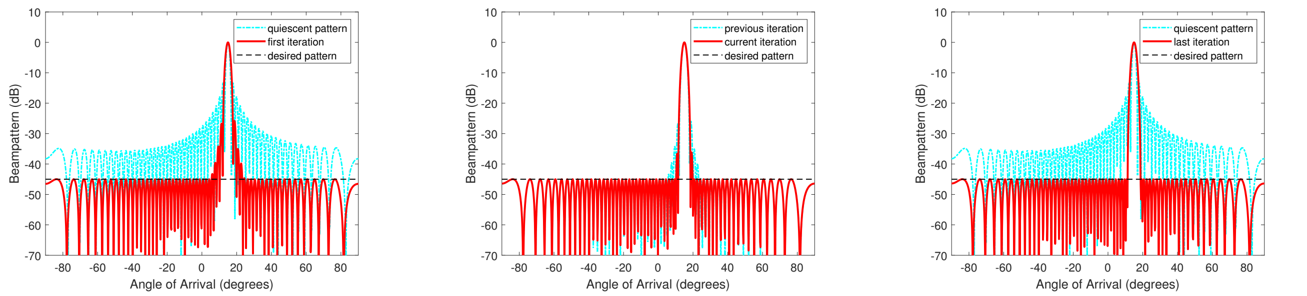
In this example, an ULA consisted of 60 elements with uniform sidelobe set to -45 dB is considered. The interval between each element is half of the wave length and the beam center is fixed at $\theta_0 = 15°$. The width of the slack region Ω can be configured freely but here we are going to set its width to 30°, which means that the value of β is 15°. As a result, Ω in this example is $[0°, 30°]$.

Through five iterations by the proposed method, the beampattern for the ULA is attained. Fig.1 displays the patterns of some iterations and the comparisons between them. Fig.1(a) shows the quiescent pattern and first iteration of the proposed method. It's intuitive to find out the mainlobe region of the first iteration of the proposed method is the same as the quiescent pattern. Besides, the sidelobe outside of the slack region perfectly meet the requirement of the desired pattern. However, in the slack region especially in $[5°, 25°]$ the sidelobe exceeds the desired pattern a lot. In Fig.1(b), the first and the second iteration of the proposed method are drawn together. Compared to the pattern of the first iteration, the sidelobe of the second one has a conspicuous decrease in slack region. The pattern of the fifth iteration is shown in Fig.1(c), now the sidelobe is completely less than the desired pattern.

4.2 Nonuniformly Spaced Linear Array

In this section, a 20-element nonuniform spaced linear array is considered. We set the beam axis at $\theta_0 = -30°$. The sidelobe level is expected to be lower than -45 dB if $\theta \in [-90°, -25°]$, -40 dB if $\theta \in [-25°, -5°]$, -50 dB if $\theta \in [5°, 25°]$ and -35 dB for θ in the interval $[25°, 90°]$.

Under such prerequisite, the proposed method still have a great performance in ascertaining the width of mainlobe region. Making a comparison between our method and the CP method can assist us to see this point clearly. The simulation result is shown in Fig.2. The width of mainlobe region in the CP method is set to 24°, 26° and 36°, respectively. For the CP method, the first width of mainlobe region as 24° is too narrow, it is unable to attain a pattern under this configuration. The second width of mainlobe region as 26° is appropriate for the CP method, a pattern that meets the requirement is obtained. The width of mainlobe region as 36° is too wide for the CP method, though there is a gained pattern, the sidelobes near the left side and right side of mainlobe region both exceed the desired pattern. On the contrary, the proposed algorithm acquires an eximious pattern without setting the width of mainlobe region manually. Besides, the pattern gained by utilizing the CP method with



(a) Comparison between quiescent pattern and beampattern at the first iteration

(b) Comparison of beampatterns at the first and second iteration

(c) Comparison between quiescent pattern and beampattern at the fifth iteration

Figure 1. Comparisons of the beampatterns at different iterations

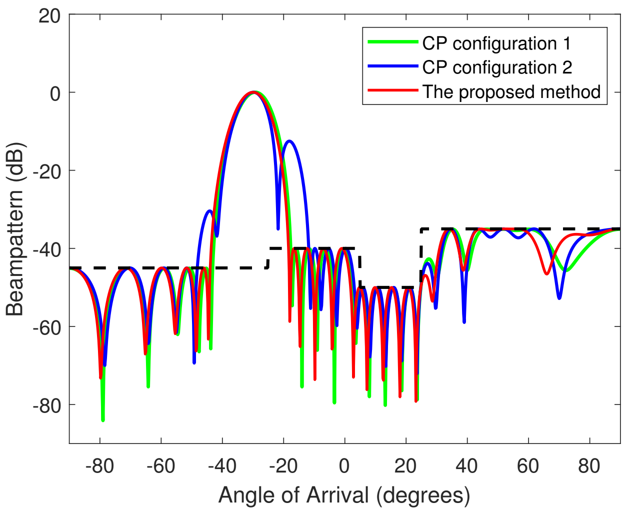


Figure 2. Comparison of the proposed method and CP method in nonuniformly spaced linear array

the width of mainlobe region as 26° is almost the same as the one obtained in the proposed algorithm.

5 Conclusion

In this paper, a novel method of auto-specifying the mainlobe width based on convex optimization is proposed. In this algorithm, the major portion is the process of the slack vector. Through reducing the nonzero elements of the slack vector, the slack region will narrow at the same time, which is equal to the mainlobe region eventually. Next, representative simulations have been carried out to verify the effectiveness of proposed approach under various scenarios.

References

[1] X. Zhang, Z. He, B. Liao, X. Zhang, Z. Cheng, and Y. Lu, “A^{2}RC: an accurate array response control algorithm for pattern synthesis,” *IEEE Trans. Signal Process.*, vol. 65, pp. 1810-1824, 2017.

[2] X. Zhang, Z. He, B. Liao, X. Zhang, and W. Peng, “Pattern synthesis for arbitrary arrays via weight vector orthogonal decomposition,” *IEEE Trans. Signal Process.*, vol. 66, pp. 1286-1299, 2018.

[3] X. Zhang, Z. He, B. Liao, X. Zhang, and W. Peng, “Pattern synthesis with multipoint accurate array response control,” *IEEE Trans. Antennas Propag.*, vol. 65, pp. 4075-4088, 2017.

[4] R. J. Mailous, *Phased Array Antenna Handbook.* Norwood, MA: Artech House, 1994.

[5] C. L. Dolph, “A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level,” *Proc. IRE*, vol. 34, pp. 335-348, 1946.

[6] H. Unz, “Linear arrays with arbitrarily distributed elements,” *IRE Trans. Antennas Propag.*, vol. 8, pp. 222-223, 1960.

[7] A. Koretz and B. Rafaely, “Dolph–Chebyshev beampattern design for spherical arrays,” *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2417-2420, Jun. 2009.

[8] K. K. Yan and Y. Lu, “Sidelobe reduction in array-pattern synthesis using genetic algorithm,” *IEEE Trans. Antennas Propag.*, vol. 45, no. 7, pp. 1117-1122, Jul. 1997.

[9] D. W. Boeringer and D. H. Werner, “Particle swarm optimization versus genetic algorithms for phased array synthesis,” *IEEE Trans. Antennas Propag.*, vol. 52, no. 3, pp. 771-779, Mar. 2004.

[10] V. Murino, A. Trucco, and C. S. Regazzoni, “Synthesis of unequally spaced arrays by simulated annealing,” *IEEE Trans. Signal Process.*, vol. 44, no. 1, pp. 119-122, Jan. 1996.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge, U.K.: Cambridge Univ. Press, 2004.

[12] H. Lebret and S. Boyd, “Antenna array pattern synthesis via convex optimization,” *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 526-532, Mar. 1997.