A Microwave Diagnostic Technique for Early-Stage Brain Stroke Characterization

A. Fedeli, A. Randazzo, A. Sciarrone, I. Bisio, F. Lavagetto, and M. Pastorino

Department of Electrical, Electronic, Telecommunications Engineering and Naval Architecture (DITEN) – University of Genoa, Italy
Introduction

- **Brain stroke** is well known as one of the leading causes of death and disability worldwide.

- Within **electromagnetic diagnostic techniques** [1, 2] an increasing interest is attracted by **brain stroke detection** [3 – 6].

- A novel **tomographic multistatic system** where the acquired data are processed by an **inexact Newton scheme** in **variable-exponent** $L^p$ spaces is presented.

- **Simulated and experimental results** are shown.


Inverse problem configuration & assumptions

- A simplified 2D scalar model with dielectric properties independent from the axial coordinate $z$ has been assumed [1]

- The head, located in a known investigation domain $D$, is illuminated by a known time-harmonic TM$_z$ incident electromagnetic field $e_{\text{inc}}$

- Head is surrounded by a lossy coupling medium with complex dielectric permittivity $\epsilon_b$

- Estimation of the reference dielectric profile of the head characterized by a contrast function
  \[
  \tilde{c} = \left( \frac{\tilde{\epsilon} - \epsilon_b}{\epsilon_b} \right)
  \]
  ($\tilde{\epsilon}$ being the complex dielectric permittivity of the reference profile)

The scattering problem is formulated as

\[
\Delta e(\mathbf{r}) = -k_b^2 \int_{D_{inv}} x(\mathbf{r}') \, e_{tot}(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') \, d\mathbf{r}', \quad \mathbf{r} \in \mathcal{M}
\]

\[
e_{tot}(\mathbf{r}) = e_{ref}(\mathbf{r}) - k_b^2 \int_{D_{inv}} x(\mathbf{r}') \, e_{tot}(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') \, d\mathbf{r}', \quad \mathbf{r} \in \mathcal{D}
\]

where

- \( c = (\varepsilon - \varepsilon_b)/\varepsilon_b \) is the contrast function of the actual configuration (which gives rise to the field \( e_{tot} \))
- \( c_{ref} = (\varepsilon_{ref} - \varepsilon_b)/\varepsilon_b \) represent the contrast function of the reference configuration (related to the field \( e_{ref} \))

By combining the two equations, we obtain the scattering model

\[
\Delta e(\mathbf{r}) = F(x)(\mathbf{r}) = G_{data} x (I - G_{state} x)^{-1} e_{ref}(\mathbf{r})
\]
To solve this nonlinear equation, an inexact-Newton iterative method is applied to minimize the residual functional $\Psi: X \to \mathbb{R}$

$$\Psi(x) = \frac{1}{2} \| F(x) - \Delta e \|^2_Y ,$$

where $x \in X$, $\Delta e \in Y$, $F: X \to Y$, and $\| \cdot \|^2_Y$ denotes the square of the norm of the functional space $Y$.

In particular, variable exponent Lebesgue spaces $L^{p(\cdot)}$ [3] are considered, in which the power $p$ used in the norm is not constant, but it is a function $p(\cdot)$.

Inversion procedure

- The exponent function for the space of the unknowns $X$ depends upon the position inside the investigation domain, allowing to set different values of the parameter $p$ to each point.

- The function $p(r)$ is updated at each step as

  $$p_k(\cdot) = p_{\text{min}} + (p_{\text{max}} - p_{\text{min}}) \left| x_k \right|/\max_{D} \left| x_k \right|$$

- Two possible initializations are considered:
  - A fixed value is used, i.e., $p_0(r) = p_{\text{start}}$
  - A delay-and-sum qualitative scheme is used to build the initial map, i.e.,

  $$I(r) = \int \int_{M} \int_{\Omega} E_s(r', \omega) e^{j2\omega v \|r-r'\|} d\omega dr' \rightarrow p_0(r) = p_{\text{min}} + (p_{\text{max}} - p_{\text{min}}) \frac{|I(r)|}{\max_{r \in D} |I(r)|}$$
Inversion procedure

**Initialization**

\[ k = 0, x_0 = 0 \]

The problem is linearized around the currently estimated solution \( x_k \)

\[ \Delta e - F(x_k) - F_{x_k} \delta_k = 0 \]

Update of the unknown with the regularized linear problem solution

\[ x_{k+1} = x_k + \tilde{\delta}_k \]

Update of the adaptive map of \( p(\cdot) \) inside the investigation domain \( \mathcal{D} \)

\[ p_{k+1}(\cdot) = p_{\text{min}} + (p_{\text{max}} - p_{\text{min}}) \frac{|x_{k+1}|_D}{\max |x_{k+1}|} \]

**Variable-exponent Landweber method initialization**

\[ \delta = 0, \tilde{\delta}_{k,0} = 0 \]

Iterative update of the linearized problem solution

\[ \delta_{k,i+1} = J_{X_k}^{p_k(\cdot)} (\delta_{k,i}) + \tau F_{x_k}^* J_{Y}^{p_{m,k}} (\Delta e - F(x_k) - F_{x_k} \delta_{k,i}) \]

where

- \( F_{x_k}^* \) is the adjoint of the linear operator \( F_{x_k} \)
- \( J_{X}^{p_k(\cdot)}, J_{X^*}^{p_k(\cdot)}, J_{Y}^{p_{m,k}} \) are duality maps of spaces \( X, X^* \) and \( Y \)
- \( \tau = 1/\|F_{x_k}\|^2 \) is the step size
- \( p_k^*(\cdot) \) is the point-wise Hölder conjugate of \( p_k(\cdot) \)

No

Convergence

Yes

End

No

Convergence

Yes
Brain stroke detection – Numerical results

Simulation parameters

- Head of the AustinWoman 3D model [1] with 2-mm voxel size
- Time-domain forward simulation by using gprMax FDTD [2]
- Dispersive tissue properties [3, 4]
- Domain size: 28.4 × 32 × 30 cm (3.4 × 10^6 cells of 2 mm side)
- PML boundary (10 cells)
- Time step: 3.85 × 10^{-12} s
- Time window: 3 × 10^{-8} s
- S = 21 antennas (Hertzian dipoles) on an ellipse of semi-axes 9.2 cm and 11 cm.
- Excitation signal: Gaussian derivative centered at 1 GHz
- Background coupling medium: glycerin/water mixture 70%
- Scattered field data corrupted by additive white Gaussian noise with SNR = 25 dB.

Brain stroke detection – Numerical results

Measurement configuration

- Ellipsoidal inclusion: hemorrhagic brain stroke at (11.7, 17.2, 17.5) cm
- Healthy head profile used as reference model
- Investigation domain composed by 1485 cells with 4 mm side
- Frequency hopping started from 500 Mhz and with step 50 MHz.
- Range of values of the exponent function: [1.4,2].
- Initial exponent map: constant value equal to 1.4.


Reconstructed dielectric properties at 0.5 GHz

Variable-exponent approach

Fixed-exponent approach ($p_{opt} = 1.2$)

Reconstructed dielectric properties at 0.8 GHz

Variable-exponent approach

Fixed-exponent approach ($p_{opt} = 1.4$)
Brain stroke detection – Numerical results

Measurement configuration

- Ellipsoidal inclusion: hemorrhagic brain stroke at (11.7, 17.2, 17.5) cm

- Partially homogeneous configuration (except skull) used as reference model

- Investigation domain composed by 1485 cells with 4 mm side

- Frequency hopping started from 500 Mhz and with step 50 MHz.

- Range of values of the exponent function: [1.4, 2].

- Initial exponent map: constant value equal to 1.4.

Reconstructed dielectric properties (variable exponent approach)


© 2020 IEEE. Reprinted, with permission, from [1].
Brain stroke detection – Numerical results

Measurement configuration

- Ellipsoidal inclusion: hemorrhagic brain stroke at (11.7, 17.2, 17.5) cm
- Healthy head profile used as reference model
- Investigation domain composed by 1485 cells with 4 mm side
- Frequency hopping started from 500 MHz and with step 50 MHz.
- Range of values of the exponent function: [1.4, 2].
- Initial exponent map: obtained by applying the DAS scheme.

Reconstructed dielectric properties at 0.7 GHz
Brain stroke detection – Experimental results

Configuration of the system prototype

Body under test

Coupling layer

Antennas

Coaxial cables

Vector Network Analyzer
Keysight 8753E

Port 1

Port 2

RF switch matrix

In 2

In 1

Coaxial cables

GPIB

USB

Personal Computer

LAN

Cylindrical test target

16 antennas

Bowtie-like antenna structure [1]


© 2018 IEEE. Reprinted, with permission, from [1].
Preliminary experimental results

- **Target properties**
  - **Outer structure** (filled with 70% glycerin/water mixture)
    - 5 L PP beaker (external diameter of 180 mm, 4 mm thickness)
      filled with 70% glycerin/water mixture
  - **Cylindrical inclusions** (filled with 0.9% saline solution)
    - 100 mL PP circular cylinder, 20 mm diameter
    - 500 mL PP circular cylinder, 52 mm diameter

- **Configuration parameters**
  - Coupling medium (70% glycerin/water mixture) in PE bags
    (40 x 80 mm, 100 µm thick, 20 ml volume) around the outer cylinder
  - Investigation domain partitioned into
    $N_i = 1264$ square cells of side $d_i = 4.5$ mm

- **Parameters of the inverse solver**
  - $p_{\text{start}} = p_{\text{min}} = 1.4$, $p_{\text{max}} = 2.0$
  - Number of maximum allowed inner and outer iterations
    to $N_{IN} = N_{LW} = 100$, minimum residual variation
    $r_{IN} = r_{LW} = 0.35$

---

Preliminary experimental results

Target configuration – Single circular inclusion
(Ø 52 mm cylindrical inclusion)

Target configuration – Two circular inclusions
(Ø 20 mm and Ø 52 mm cylindrical inclusions)

Reconstructed relative dielectric properties (variable exponent)

\[ f = 600 \text{ MHz} \]

\[ f = 900 \text{ MHz} \]

Conclusions

- A novel **tomographic multistatic microwave imaging system** for **brain stroke detection** has been designed.

- A **variable-exponent Lebesgue-space inversion** scheme is adopted for processing the acquired data.

- Two initialization strategies have been considered:
  - Constant exponent function
  - Variable exponent function obtained by a delay-and-sum scheme.

- **Numerical simulations and preliminary experimental results** have been carried out.

- **Further activities will be devoted to**
  - Improve the measurement system
  - Test the method in more realistic configurations, also with clinical data.