Power maximization for a multiple - input and multiple - output wireless power transfer system described by the admittance matrix

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• Conclusion
In this work, the optimal loads to maximize the power transfer for a wireless power transfer (WPT) system with any number of transmitters and receivers are determined.

This was already done for WPT systems characterized by their impedance matrix, but for certain applications (e.g. capacitive WPT), an admittance matrix approach is much more straightforward.
Describing the WPT system as a multiport

A multiport network $\mathcal{N}$ with $M$ transmitters and $N$ receivers.

The $M$ input ports of the network are connected to $M$ current sources.

At the $N$ output ports $N$ load admittances $Y_{L,i}$ are present.
The relation between the voltages and the currents of the multiport can be described by an admittance matrix $\mathbf{Y}$ which can be partitioned in four submatrices:

$$
\begin{bmatrix}
\mathbf{i}_M \\
\mathbf{i}_N
\end{bmatrix} =
\begin{bmatrix}
\mathbf{Y}_{MM} & \mathbf{Y}_{MN} \\
\mathbf{Y}_{NM} & \mathbf{Y}_{NN}
\end{bmatrix}
\begin{bmatrix}
\mathbf{v}_M \\
\mathbf{v}_N
\end{bmatrix}
$$

$$
\mathbf{i}_M = 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_M
\end{bmatrix},
\mathbf{v}_M = 
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_M
\end{bmatrix}
$$

$$
\mathbf{i}_N = 
\begin{bmatrix}
I_{L,1} \\
I_{L,2} \\
\vdots \\
I_{L,N}
\end{bmatrix},
\mathbf{v}_N = 
\begin{bmatrix}
V_{L,1} \\
V_{L,2} \\
\vdots \\
V_{L,N}
\end{bmatrix}
$$
Norton equivalent circuit of the multiport

\[ \mathcal{N}' \text{ network} \ [Y] \]

\[ \mathcal{N}_0 \text{ network} \ [Y_0] \]

\[ \mathcal{N}_L \text{ network} \ [Y_L] \]

Norton’s theorem

M input ports are replaced by open circuits.

The \( N \) loads of the receiver are represented by the network \( \mathcal{N}_L \) described by the admittance matrix \( Y_L \).
$I^{(no)}_i$ are the **Norton currents**, given by:

$$i_N = Y_{NM} Y_{MM}^{-1} \cdot i_M \equiv i^{(no)}_N = \begin{bmatrix} I^{(no)}_1 \\ I^{(no)}_2 \\ \vdots \\ I^{(no)}_N \end{bmatrix}$$
The Norton admittance matrix $Y_0$ which characterizes network $\mathcal{N}_0$ is defined by

$$i_N = Y_0 \cdot v_N$$

As function of the original admittance matrix $Y$, $Y_0$ is given by:

$$Y_0 = Y_{NN} - Y_{NM} \cdot Y_{MM}^{-1} \cdot Y_{MN}$$
Optimal loads for power maximization

The goal of this work is to determine the loads that realize maximum power transfer from the $M$ transmitters to the $N$ receivers, i.e. that maximize the total output power $P_{out}$ defined as

$$P_{out} = \sum_{i=1}^{N} P_i$$

with $P_i$ the output power delivered to load $Y_{L,i}$. 

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Voltage condition for achieving maximum power transfer to loads [*]:

\[ v_N = (Y_0 + Y_0^+)^{-1} \cdot i^{(no)} \]

with \( Y_0^+ \) the conjugate transpose of \( Y_0 \).

This results in the current condition for the loads at the maximum power configuration:

\[ i_N = i^{(no)} - Y_0 \cdot (Y_0 + Y_0^+)^{-1} \cdot i^{(no)} \]

\[ \Rightarrow \text{The optimal loads are given by} \quad Y_{L,i} = \frac{I_{L,i}}{V_{L,i}} \]

with \( V_{L,i} \) and \( I_{L,i} \) the elements of \( v_N \) and \( i_N \).

Overview procedure

The general procedure to find the loads for power maximization for a WPT system with any number of transmitters and receivers:

1. Establish (e.g., by measurement or simulation) the admittance matrix $Y$ of the network.
2. Determine the Norton current sources $I_{i}^{(no)}$.
3. Set up the Norton admittance matrix $Y_0$.
4. Calculate the voltages $v_N$ and currents $i_N$ for the loads at the maximum power configuration.
5. Determine the optimal loads $Y_{L,i}$ from these voltages and currents.
Example: capacitive WPT with 2 transmitters and 3 receivers

Two current sources $I_1$ and $I_2$ power the system with operating angular frequency $\omega_0$. 

At the 3 output ports load admittances $Y_{L,1}$, $Y_{L,2}$ and $Y_{L,3}$ are connected.
The shunt conductances $g_{jj} (j=1,...,5)$ describe the losses in the circuit.

The mutual capacitances $C_{13}, C_{14}, C_{15}, C_{23}, C_{24}$ and $C_{25}$ represent the desired electric coupling between the transmitter capacitances $C_1, C_2$, and the receiver capacitances $C_3, C_4, C_5$.

**Undesired** electric coupling is present between both transmitters, indicated by the mutual capacitance $C_{12}$.

Also between the receivers, an undesired coupling is present: $C_{34}, C_{35}$ and $C_{45}$. 

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In order to obtain a resonant scheme, the inductors $L_j$ are added: 

$$L_j = \frac{1}{\omega_0^2 C_j}$$

The coupling factor $k_{ij}$ is defined as 

$$k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}}$$

$(i,j=1,...,5)$
The entire multiport system (indicated by the dashed rectangle) is fully determined by the admittance matrix $Y$ which is, at the resonance angular frequency $\omega_0$, equal to:

$$Y = \begin{bmatrix} Y_{MM} & Y_{MN} \\ Y_{NM} & Y_{NN} \end{bmatrix} = \begin{bmatrix} g_{11} & -j b_{12} & -j b_{13} & -j b_{14} & -j b_{15} \\ -j b_{12} & g_{22} & -j b_{23} & -j b_{24} & -j b_{25} \\ -j b_{13} & -j b_{23} & g_{33} & -j b_{34} & -j b_{35} \\ -j b_{14} & -j b_{24} & -j b_{34} & g_{44} & -j b_{45} \\ -j b_{15} & -j b_{25} & -j b_{35} & -j b_{45} & g_{55} \end{bmatrix}$$

with $b_{ij} = \omega_0 c_{ij}$.
In order to verify the analytical results, **circuital simulations** have been performed in SPICE with the following example values:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{11}$</td>
<td>1.00 mS</td>
<td>$C_1$</td>
<td>350 pF</td>
</tr>
<tr>
<td>$g_{22}$</td>
<td>1.25 mS</td>
<td>$C_2$</td>
<td>300 pF</td>
</tr>
<tr>
<td>$g_{33}$</td>
<td>1.50 mS</td>
<td>$C_3$</td>
<td>250 pF</td>
</tr>
<tr>
<td>$g_{44}$</td>
<td>1.75 mS</td>
<td>$C_4$</td>
<td>225 pF</td>
</tr>
<tr>
<td>$g_{55}$</td>
<td>2.00 mS</td>
<td>$C_5$</td>
<td>200 pF</td>
</tr>
<tr>
<td>$I_1$</td>
<td>100 mA</td>
<td>$f_0$</td>
<td>10 MHz</td>
</tr>
<tr>
<td>$I_2$</td>
<td>200 mA</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Desired coupling</th>
<th>Value</th>
<th>Undesired coupling</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{13}$</td>
<td>30 %</td>
<td>$k_{12}$</td>
<td>10 %</td>
</tr>
<tr>
<td>$k_{14}$</td>
<td>25 %</td>
<td>$k_{34}$</td>
<td>5 %</td>
</tr>
<tr>
<td>$k_{15}$</td>
<td>20 %</td>
<td>$k_{35}$</td>
<td>2 %</td>
</tr>
<tr>
<td>$k_{23}$</td>
<td>25 %</td>
<td>$k_{45}$</td>
<td>5 %</td>
</tr>
<tr>
<td>$k_{24}$</td>
<td>20 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{25}$</td>
<td>15 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optimal terminating admittances according to the developed theory are:

<table>
<thead>
<tr>
<th>$G_{L,1}$ (mS)</th>
<th>$L_{L1}$ (μH)</th>
<th>$G_{L,2}$ (mS)</th>
<th>$L_{L2}$ (μH)</th>
<th>$G_{L,3}$ (mS)</th>
<th>$L_{L3}$ (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4</td>
<td>524.0</td>
<td>23.0</td>
<td>443.0</td>
<td>22.6</td>
<td>370</td>
</tr>
</tbody>
</table>

First, a simulation with the network terminated on the optimal admittances returns an output power of 4.64 W.
Next, simulations were performed by varying one load conductance $G_{L,i}$ at a time while keeping all the others constant at their optimal value.

The results confirm the data provided by the theory for this example.
Next, simulations were performed by varying one load inductance $L_{L,i}$ at a time while keeping all the others constant at their optimal value.

The results confirm the data provided by the theory for this example.
Conclusion

A general procedure was shown to easily determine the terminating loads that maximize power transfer for a WPT system

• with any number of transmitters or receivers,

• characterized by its admittance matrix.
Thank you for reading

Any questions? Mail me at ben.minnaert@odisee.be