Statistical model for MIMO propagation channel in cavities and random media

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Introduction and motivation

• Determine the channel capacity of a MIMO interconnect in a cavity
• Consider noisy fields as a proxy for information transfer
• Statistical theories necessary to obtain noisy transfer functions.
• Impedance based formalism used: random coupling model
• Channel capacity distribution generated numerically
• Dependence on average loss parameter critical.
Example, small package antennas communicate within a resonant environment. How do we model the channel transfer functions?

\[ H = ? \]

- PCB scattering
- Multiple resonances
- Losses
- Antenna efficiency
Linear relation between transmitted \((x)\) and received \((y)\) signals \(y = Hx + n\)

Channel transfer matrix \(H\)

\[
\begin{bmatrix}
  h_{11} & \cdots & h_{1T} \\
  \vdots & \ddots & \vdots \\
  h_{R1} & \cdots & h_{RT}
\end{bmatrix}, \quad h_{ij} \in C
\]
Multi-antenna communications in a box

Transfer function defined between port quantities at the antenna terminals

\[ y = Hx \]

\[ V_T = H V_T \]

\[ Z_{cav} = \begin{bmatrix} Z_{TT} & Z_{TR} \\ Z_{RT} & Z_{RR} \end{bmatrix} \]
Transfer function defined between port quantities at the antenna terminals

\[ y = Hx \]

\[ V_R = HV_T \]

- Transfer matrix involves *sums* and *products* of impedance matrices.
- Information theoretic \( H \) takes a simplified form for uncoupled MIMO channels.
- Leverage on RMT model of impedance matrix in **irregular cavities** – the Random Coupling Model.
MIMO channel transfer matrix in resonant cavities

For a linear time invariant multi-port system

\[ H = \frac{e^{-j\phi}}{R_r} C_T^{-1/2} Z_{TR} C_R^{-1/2} \]

\[ C_T = \frac{\Re\{Z_{TT}\}}{R_r} \]

\[ C_R = \frac{\Re\{Z_{RR}\}}{R_r} \]

Impedance matrix of antennas radiating inside the cavity (cav):

\[ Z_{cav} \]

Impedance matrix of antennas radiating in free-space (rad):

\[ Z_{rad} \]
Random Coupling Model (RCM): a Random Matrix Theory model for cavity impedance matrix

\[ Z^{cav} = i \text{Im}(Z^{rad}) + \left[ \text{Re}(Z^{rad}) \right]^{1/2} \cdot \xi_\alpha \cdot \left[ \text{Re}(Z^{rad}) \right]^{1/2} \]

Coupling coefficients are GRVs

\[ \left[ \xi_\alpha \right]_{ps} \approx -\frac{i}{\pi} \sum_{m=1}^{M} \frac{\varphi_{pm} \cdot \varphi_{sm}}{k_0^2 - k_m^2} + i\alpha \]

RMT random spectrum:
Generated by eigenvalues of matrices from the Gaussian Orthogonal Ensemble (GOE).

Average loss parameter:
Measures the average Q-width of a resonant mode relative to average mode spacing \( \Delta k^2 \).

\[ \alpha = \frac{k^2}{Q \Delta k^2} = \frac{B_Q}{\Delta k^2} \]

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PDF of the normalized cavity impedance

An example of Monte Carlo computation...

\[ z \approx [\xi_\alpha]_{11}; \quad \xi_\alpha = \left[ \text{Re} \left( Z^{\text{rad}} \right) \right]^{-1/2} \cdot \left[ Z^{\text{cav}} - i \text{Im} \left( Z^{\text{rad}} \right) \right] \cdot \left[ \text{Re} \left( Z^{\text{rad}} \right) \right]^{-1/2} \]

High loss: Gaussian; zero-loss: Lorenzian

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Each constitutive impedance of the channel transfer function can be treated with the RCM

\[ H = \frac{e^{-j\phi}}{R_r} C_T^{-1/2} Z_{TR} C_R^{-1/2} \]

**RCM**

\[
Z_{TT}^{cav} = \text{Im} \left( Z_{TT}^{rad} \right) + \left[ \text{Re} \left( Z_{TT}^{rad} \right) \right]^{1/2} \xi_\alpha \left[ \text{Re} \left( Z_{TT}^{rad} \right) \right]^{1/2} \\
Z_{TR}^{cav} = \left[ \text{Re} \left( Z_{TT}^{rad} \right) \right]^{1/2} \xi_\alpha \left[ \text{Re} \left( Z_{TR}^{rad} \right) \right]^{1/2} \\
Z_{RT}^{cav} = \left[ \text{Re} \left( Z_{RR}^{rad} \right) \right]^{1/2} \xi_\alpha \left[ \text{Re} \left( Z_{TT}^{rad} \right) \right]^{1/2} \\
Z_{RR}^{cav} = \text{Im} \left( Z_{RR}^{rad} \right) + \left[ \text{Re} \left( Z_{RR}^{rad} \right) \right]^{1/2} \xi_\alpha \left[ \text{Re} \left( Z_{RR}^{rad} \right) \right]^{1/2}
\]
High average losses \( \alpha > 1 \) yield

\[
Z_{TT} \approx Z_{TT}^{\text{rad}}, \quad Z_{RR} \approx Z_{RR}^{\text{rad}}.
\]

and

\[
\mathbb{R}\{\xi_{\alpha,(c)}\}, \mathbb{I}\{\xi_{\alpha,(c)}\} \sim \mathcal{N}_N \left( 0, \frac{1}{\sqrt{2\pi \alpha}} \mathbf{I} \right)
\]

whence the Gramian matrix \( \mathbf{H} \mathbf{H}^\dagger \approx \xi_\alpha \xi_\alpha^\dagger \)

takes the complex Wishart matrix form

\[
\mathbf{H} \mathbf{H}^\dagger \sim \mathcal{CW}_{NT} \left( N_R, \sqrt{\frac{2}{\pi \alpha}} \mathbf{I} \right)
\]
Eigenvalue distribution of the channel transfer matrix

Predicted by Marchenko-Pastur (MP) law

\[ f_\beta(\lambda) = \left(1 - \frac{\alpha}{\beta}\right)^+ \delta(\lambda) + \frac{\sqrt{(\lambda - a)^+} \sqrt{(b - \lambda)^+}}{2\pi \lambda \left(\frac{b}{\alpha}\right)} \]

with

\[ \beta = \frac{N_T}{N_R}, \quad (z)^+ = \max(0, z) \]

\[ \langle \lambda \rangle \sim \frac{\beta}{\alpha}, \quad \alpha > 1. \]
Channel capacity distribution function at variable losses

\[ C = \log_2 \left( \det \left( \mathbf{I} + \frac{\rho}{N_T} \mathbf{H} \mathbf{H}^\dagger \right) \right) \]

\[ C = \sum_{n=1}^{N_C} \log_2 \left( 1 + \frac{\rho}{N_T} \lambda_n \right) \]
Conclusion

• Physics based model of information theoretic transfer function found

• Model for MIMO systems operating in complex resonators/environments

• Radiation resistance becomes a random matrix – coupling with ergodic cavity eigenmodes

• Channel transfer matrix universally depends on a loss factor

• Channel capacity distribution calculated from Gramian eigenvalue distribution

• Results relevant in calculations of outage probability within real life electromagnetic environments
Thank you for your attention

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