Novel Approach to Rainfall Rate Estimation based on Fusing Measurements from Terrestrial Microwave and Satellite Links

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The work deals with a novel technique able to generate a map of rainfall phenomenon from the measures of rain attenuation on a set of $N_{\text{CML}}$ commercial microwave links (CML) and a set of $N_{\text{BSL}}$ broadcast satellite links (BSL).

The work is based on the well known *power-law formula*, which connects the rainfall rate $R_\ell$ experienced by a wireless link $\ell$ with its attenuation $A_\ell$ relates

$$A_\ell = a R_\ell^b.$$
COMBINING CML AND BSL 1/2

Motivations

- **Efficiency**: by exploiting already existing wireless infrastructure, at no extra costs for the required equipments;
- **Flexibility**: including satellite terminals already installed for TV reception or purposely installed in areas not adequately covered;
- **Diversity**: the signal levels coming from different links provide a diversity gain which can improve the accuracy and reliability of the overall joint system;
- **Accuracy**: the numerical results obtained by simulations corroborate the effectiveness of the proposed mixed strategy.
We started from the state-of-the-art GMZ algorithm\textsuperscript{1}.

We extended the model in the $z$ dimension in order to exploit also satellite links.

Numerical results quantify the competitive performance in some practical scenarios.

Scenario

The scenario is a box with square base of area $B$ and height limited by the $0^\circ$C isotherm height $h_0$. 

\[-\sqrt{B} \leq x \leq \sqrt{B}, \; \forall x,\]
\[-\sqrt{B} \leq y \leq \sqrt{B}, \; \forall y,\]
\[0 \leq z \leq h_0, \; \forall z.\]
Pre-processing is needed to consider all the $N = N_{\text{CML}} + N_{\text{BSL}}$ links as an uniform data structure, regardless the geometrical differences.

**For** each active communication link ($\ell = 1, \ldots, N$) **do**:

1. the rainfall-induced attenuation $A_\ell$ is estimated;
Preprocessing is needed to consider all the \( N = N_{\text{CML}} + N_{\text{BSL}} \) links as an uniform data structure, regardless the geometrical differences.

For each active communication link \((\ell = 1, \ldots, N)\) do:

1. the rainfall-induced attenuation \( A_\ell \) is estimated;
2. \( \ell \) is divided in \( K_\ell = \lceil L_\ell \cos(\theta_\ell)/D \rceil \) segments, where \( D \) is the distance in which rain can be assumed constant on plane \((x, y)\);
Pre-processing is needed to consider all the $N = N_{CML} + N_{BSL}$ links as an uniform data structure, regardless the geometrical differences.

For each active communication link $(\ell = 1, \ldots, N)$ do:

1. the rainfall-induced attenuation $A_{\ell}$ is estimated;
2. $\ell$ is divided in $K_{\ell} = \lceil L_{\ell} \cos(\theta_{\ell})/D \rceil$ segments, where $D$ is the distance in which rain can be assumed constant on plane $(x, y)$;
3. so-called data points are then located in the middle of each segment.
We want to estimate the rainfall rate on point $i$ assuming known the rainfall rate of other $M$ points.

**Assumption I: on different heights**
Two points having same $(x, y)$ coordinates and different $z$ coordinate are assumed to have the same rainfall rate. Any observable streak or shaft of precipitation falling from a cloud that evaporates or sublimes before reaching the ground, is considered negligible$^2$. $^3$

**Assumption II: on same plane $x − y$**
Rainfall rate $r_i$ of a generic point can be estimated through inverse distance weighting (IDW) knowing other local rainfall rate values on the same plane.

$^2$The authors are currently studying the impact of this phenomenon on the algorithm and the appropriate countermeasures.

We want to estimate the rainfall rate on point $i$ assuming known the rainfall rate of other $M$ points.

Mixing together the previous assumption, and defining $W_{i,m}$ as the weight between $i$ and $m$, we obtain the *rainfall estimation* (RE) formula as

\[
\text{RE: } \hat{r}_i = \frac{\sum_{m=1}^{M} W_{i,m} r_m}{\sum_{m=1}^{M} W_{i,m}}
\]
The procedure able to estimate the data points rain rate is inspired from the well-known GMZ algorithm. For each link, we perform:

1. the estimation of the data point rain seen by the other links;
2. the solution of the optimization problem to match the attenuation estimated by the link.
**First step:**
We can obtain the rain rate of each data point $\hat{r}_{\ell,k}$ seen by all the other links using RE.
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SECOND step:
In order to match the global attenuation seen by the link with $A_\ell$, the following *optimization problem* (OP) is solved

$$
\text{OP: } \arg \min_{\mathbf{r}_\ell} \left\{ \| \mathbf{r}_\ell - \hat{\mathbf{r}}_\ell \|^2 \left| K_\ell \frac{A_\ell}{a} - \sum_{k=1}^{K_\ell} r_{\ell,k}^b = 0 \right. \right\},
$$

with

$$
\mathbf{r}_\ell = [r_{\ell,1}, \ldots, r_{\ell,K_\ell}]^T \quad \hat{\mathbf{r}}_\ell = [\hat{r}_{\ell,1}, \ldots, \hat{r}_{\ell,K_\ell}]^T
$$

$$
A \text{ dB/km} = ar^b \quad \text{mm/h}
$$
In order to obtain the graphical rain map from the numerical one obtained above:

1. **sampling**: the area $B$ is sampled into a grid of $J \times J$ points.

2. **regression**: map is computed through RE formula from data points rain rate to all possible points of the grid.
Numerical results

Evaluation
Results are given in terms of RMSE [mm/h] and correlation factor \( \rho \) between the real map of rainfall rate and the estimated one. Furthermore, an example of estimated map with different number of links is presented.

<table>
<thead>
<tr>
<th>( B )</th>
<th>100 km(^2)</th>
<th>( f_0 )</th>
<th>18 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
<td>64</td>
<td>polariz.</td>
<td>vertical</td>
</tr>
<tr>
<td>( D )</td>
<td>50 m</td>
<td>( a )</td>
<td>0.0601</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>1 km</td>
<td>( b )</td>
<td>1.1154</td>
</tr>
<tr>
<td>( \theta_\ell )</td>
<td>40(^\circ)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table: Simulation parameters

Rain
The rain is generated following a two dimensional Gaussian shape with maximum value \( r_t = 10 \) mm/h and standard deviation \( \sigma = 1 \) km.
True value of rain

Ground truth, plane $(x,y)$

Ground truth, plane $(x,z)$
Estimation

CML = 14, BSL = 0

RMSE = 2.416, $\rho = 0.658$

CML = 14, BSL = 8

RMSE = 1.772, $\rho = 0.790$
**Results: RMSE, ρ vs CML**

**Figure:** RMSE (solid lines) and ρ (dashed lines) vs the number of BSL
Conclusions & future works

Conclusions

◦ The problem concerning the estimation of the rain map based on both CML and BSL has been defined;
◦ The numerical results corroborate the effectiveness of our approach.

Future works

◦ Different models regarding different kinds of rain must be studied;
◦ Dependence on the environmental condition of the rain along the z-axis in order to refine both the model and the algorithm is currently investigated.
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Appendix: RMSE and ρ

\[
\text{RMSE} = \sqrt{\frac{||r_e - r_t||^2}{J^2}}
\]

\[
\rho = \frac{\text{cov}(r_e, r_t)}{\sigma_{r_e} \sigma_{r_t}}
\]

where:

- \(r_e\) is the estimated rain map;
- \(r_t\) is the real rain map;
- \(\sigma_{r_x} = \sqrt{\sum_{i=1}^{J^2} |r_x - \mu_x|^2 / J^2}\) is the standard deviation of the points of the rain map, \(x \in \{e, r\}\);
- \(\mu_x = \sum_{i=1}^{J^2} r_x / J^2\) is the mean value of the points of the rain map, \(x \in \{e, r\}\).
The process of estimation

\[ \hat{r}_i = \frac{\sum_{m=1}^{M} W_{i,m} v^{(z_i)}(r_m)}{\sum_{m=1}^{M} W_{i,m}} \]

is optimal when \( v^{(z)}(\cdot) \) is known and when the rain is Gaussian distributed on \((x, y)\) plane with correlation matrix \( \Lambda = \text{diag}(1/W_{i,m}) \).