Novel Approach to Rainfall Rate Estimation based on Fusing Measurements from Terrestrial Microwave and Satellite Links

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Introdution

The work deals with a novel technique able to generate a map of rainfall phenomenon from the measures of rain attenuation on a set of N_{CML} commercial microwave links (CML) and a set of N_{BSL} broadcast satellite links (BSL).

The work is based on the well known *power-law formula*, which connects the rainfall rate R_ℓ experienced by a wireless link ℓ with its attenuation A_ℓ relates

$$A_\ell = aR_\ell^b.$$

Combining CML and BSL 1/2

Motivations

- **Efficiency**: by exploiting already existing wireless infrastructure, at no extra costs for the required equipments;
- **Flexibility**: including satellite terminals already installed for TV reception or purposely installed in areas not adequately covered;
- **Diversity**: the signal levels coming from different links provide a diversity gain which can improve the accuracy and reliability of the overall joint system;
- **Accuracy**: the numerical results obtained by simulations corroborate the effectiveness of the proposed mixed strategy.

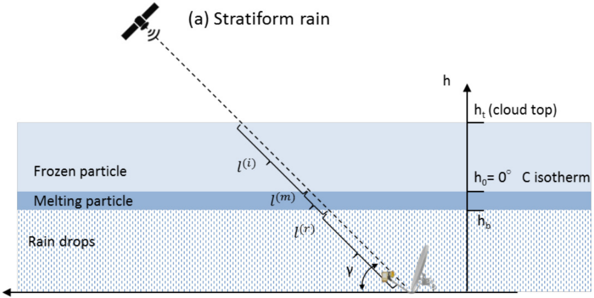
Combining CML and BSL 2/2

- We started from the the state-of-the-art *GMZ* algorithm[1].
- We extended the model in the z dimension in order to exploit also satellite links.
- Numerical results quantify the competitive performance in some practical scenarios.

[1] Oren Goldshtein, Hagit Messer, and Artem Zinevich. "Rain Rate Estimation Using Measurements From Commercial Telecommunications Links". In: *Signal Processing, IEEE Transactions on* 57 (May 2009), pp. 1616–1625.

The scenario is a box with square base of area B and height limited by the 0°C isotherm height h_0.

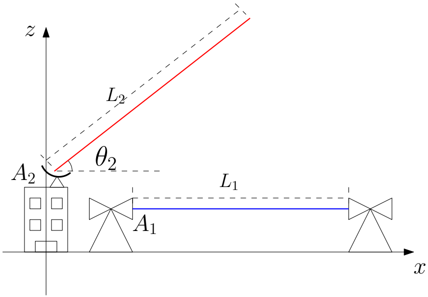
$$
\begin{aligned}
-\sqrt{B} \leq &x \leq \sqrt{B}, \quad \forall x,\\
-\sqrt{B} \leq &y \leq \sqrt{B}, \quad \forall y,\\
0 \leq &z \leq h_0, \quad \forall z.
\end{aligned}
$$



Preprocessing is needed to consider all the $N = N_{\text{CML}} + N_{\text{BSL}}$ links as an uniform data structure, regardless the geometrical differences.

For each active communication link ($\ell = 1, \ldots, N$) **do**:

1. the rainfall-induced attenuation A_ℓ is estimated;



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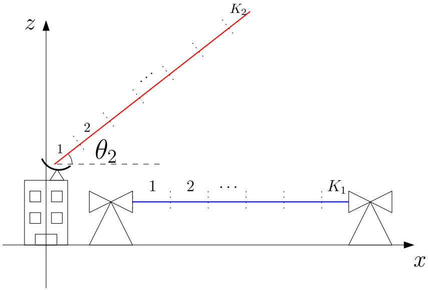
For each active communication link ($\ell = 1, \ldots, N$) **do**:

1. the rainfall-induced attenuation A_ℓ is estimated;
2. ℓ is divided in $K_\ell = \lceil L_\ell \cos(\theta_\ell)/D \rceil$ segments, where D is the distance in which rain can be assumed constant on plane (x, y);



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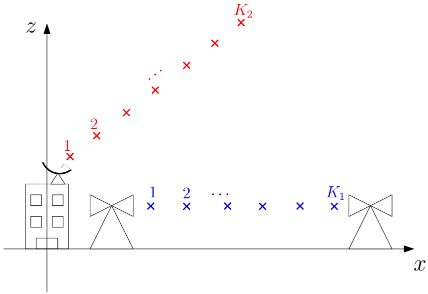
3. so-called **data points** are then located in the middle of each segment.



We want to estimate the rainfall rate on point i assuming known the rainfall rate of other M points.

Assumption I: on different heights

Two points having same (x, y) coordinates and different z coordinate are assumed to have the same rainfall rate. Any observable streak or shaft of precipitation falling from a cloud that evaporates or sublimates before reaching the ground, is considered negligible[2].[3]

Assumption II: on same plane $x - y$

Rainfall rate r_i of a generic point can be estimated through inverse distance weighting (IDW) knowing other local rainfall rate values on the same plane.

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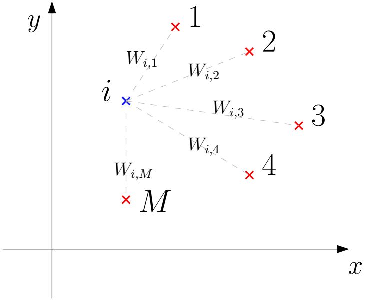
[2] The authors are currently studying the impact of this phenomenon on the algorithm and the appropriate countermeasures.

[3] Simone Lolli et al. "Vertically Resolved Precipitation Intensity Retrieved Through a Synergy Between the Ground-Based NASA MPLNET Lidar Network Measurements, Surface Disdrometer Datasets and an Analytical Model Solution". In: *Remote Sens.* 10 (May 2018).

We want to estimate the rainfall rate on point i assuming known the rainfall rate of other M points.

Mixing together the previous assumption, and defining $W_{i,m}$ as the weight between i and m, we obtain the *rainfall estimation* (RE) formula as

$$\text{RE: } \hat{r}_i = \frac{\sum_{m=1}^{M} W_{i,m}\, r_m}{\sum_{m=1}^{M} W_{i,m}}$$



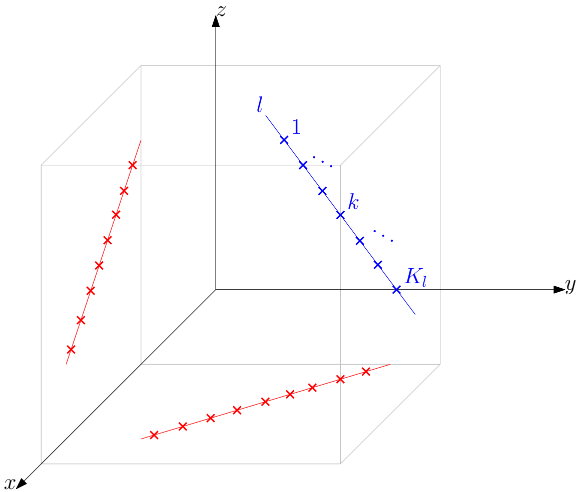
Estimation of data point rain rate 1/3

The procedure able to estimate the data points rain rate is inspired from the well-known *GMZ* algorithm. For each link, we perform:

1. the estimation of the data point rain seen by the other links;
2. the solution of the optimization problem to match the attenuation estimated by the link.

FIRST STEP:

We can obtain the rain rate of each data point $\hat{r}_{\ell,k}$ seen by all the other links using RE



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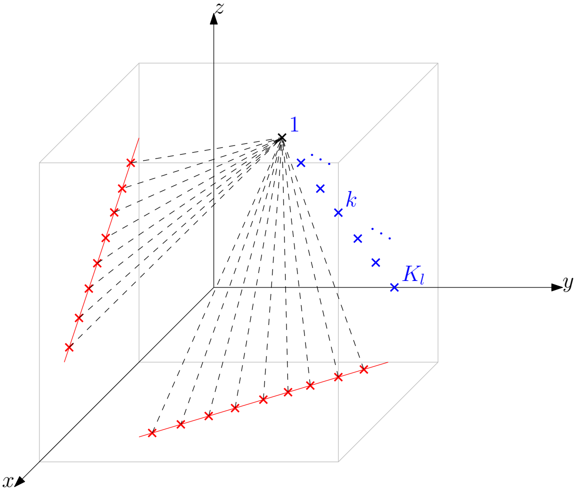

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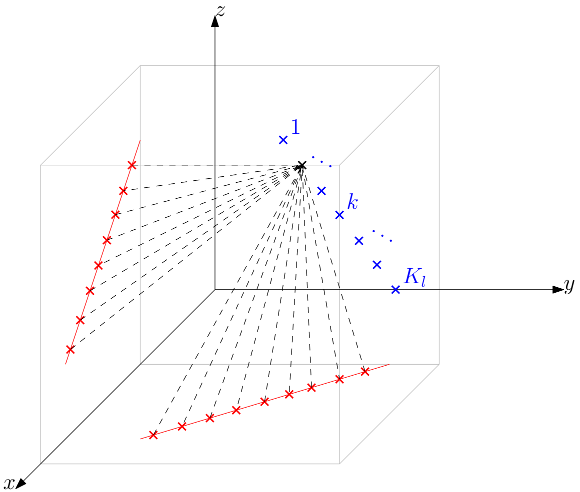


Second step:

In order to match the global attenuation seen by the link with A_ℓ, the following *optimization problem* (OP) is solved

$$\text{OP:}\ \arg\min_{\mathbf{r}_\ell} \left\{ ||\mathbf{r}_\ell - \hat{\mathbf{r}}_\ell||^2 \,\middle|\, K_\ell \frac{A_\ell}{a} - \sum_{k=1}^{K_\ell} r_{\ell,k}^b = 0 \right\},$$

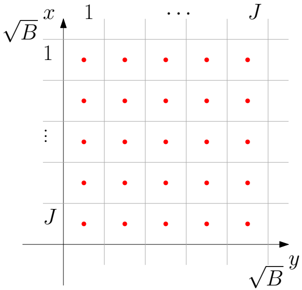
with

$$\mathbf{r}_\ell = [r_{\ell,1}, \ldots, r_{\ell,K_\ell}]^T$$
$$\hat{\mathbf{r}}_\ell = [\hat{r}_{\ell,1}, \ldots, \hat{r}_{\ell,K_\ell}]^T$$

$$\underset{\text{dB/km}}{A} = a\underset{\text{mm/h}}{r}^b$$

In order to obtain the graphical rain map from the numerical one obtained above:

1. **sampling**: the area B is sampled into a grid of $J \times J$ points.
2. **regression**: map is computed through RE formula from data points rain rate to all possible points of the grid.



Evaluation

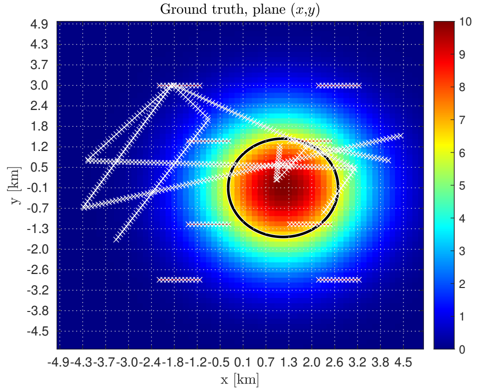
Results are given in terms of RMSE [mm/h] and correlation factor ρ between the real map of rainfall rate and the estimated one. Furthermore, an example of estimated map with different number of links is presented.

B	100 km^2	f_0	18 GHz
J	64	polariz.	vertical
D	50 m	a	0.0601
h_0	1 km	b	1.1154
θ_ℓ	40°		

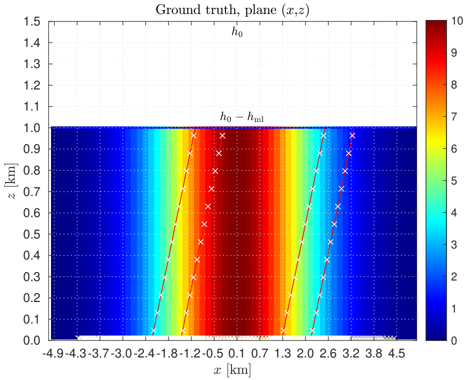
Table: Simulation parameters

Rain

The rain is generated following a two dimensional Gaussian shape with maximum value $r_t = 10$ mm/h and standard deviation $\sigma = 1$ km.

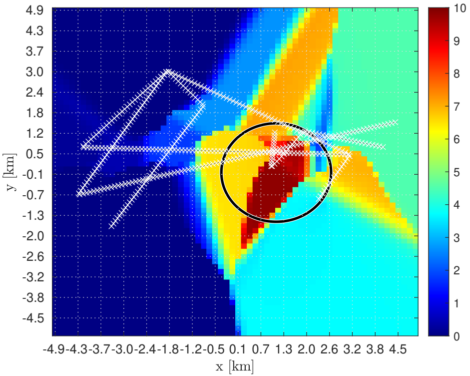


Ground truth, plane (x,y)
4.9
4.3
3.7
3.0
2.4
1.8
1.2
0.5
-0.1
-0.7
-1.3
-2.0
-2.6
-3.2
-3.8
-4.5
y [km]
-4.9 -4.3 -3.7 -3.0 -2.4 -1.8 -1.2 -0.5 0.1 0.7 1.3 2.0 2.6 3.2 3.8 4.5
x [km]
0
1
2
3
4
5
6
7
8
9
10



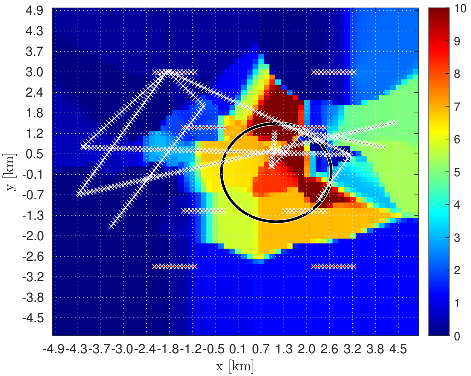
Ground truth, plane (x,z)
h_0
$h_0 - h_{\mathrm{ml}}$
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
z [km]
-4.9 -4.3 -3.7 -3.0 -2.4 -1.8 -1.2 -0.5 0.1 0.7 1.3 2.0 2.6 3.2 3.8 4.5
x [km]
0
1
2
3
4
5
6
7
8
9
10

CML = 14, BSL = 0



RMSE $= 2.416$, $\rho = 0.658$

CML = 14, BSL = 8



RMSE $= 1.772$, $\rho = 0.790$

Results: RMSE, ρ vs CML

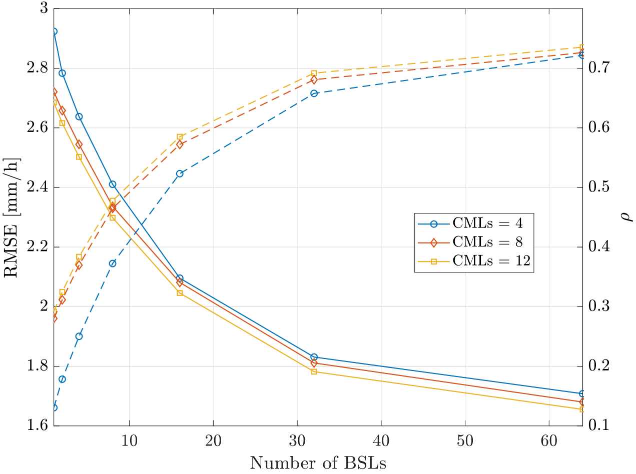


Figure: RMSE (solid lines) and ρ (dashed lines) vs the number of BSL

Conclusions

- The problem concerning the estimation of the rain map based on both CML and BSL has been defined;
- The numerical results corroborate the effectiveness of our approach.

Future works

- Different models regarding different kinds of rain must be studied;
- Dependence on the environmental condition of the rain along the z-axis in order to refine both the model and the algorithm is currently investigated.

Acknowledgment

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Thank you for the attention.

About Me

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Appendix: RMSE and ρ

$$\text{RMSE} = \sqrt{\frac{||\mathbf{r}_\text{e} - \mathbf{r}_\text{t}||^2}{J^2}}$$

$$\rho = \frac{\text{cov}(\mathbf{r}_\text{e}, \mathbf{r}_\text{t})}{\sigma_{r_\text{e}}\sigma_{r_\text{t}}}$$

where:

- r_e is the estimated rain map;
- r_t is the real rain map;
- $\sigma_{r_x} = \sqrt{\sum_{i=1}^{J^2} |r_x - \mu_x|^2 / J^2}$ is the standard deviation of the points of the rain map, $x \in \{\text{e}, \text{r}\}$;
- $\mu_x = \sum_{i=1}^{J^2} r_x / J^2$ is the mean value of the points of the rain map, $x \in \{\text{e}, \text{r}\}$.

The process of estimation

$$\hat{r}_i = \frac{\sum_{m=1}^{M} W_{i,m}\, v^{(z_i)}(r_m)}{\sum_{m=1}^{M} W_{i,m}}$$

is optimal when $v^{(z)}(\cdot)$ is known and when the rain is Gaussian distributed on (x, y) plane with correlation matrix $\Lambda = \mathrm{diag}(1/W_{i,m})$.