POLARIMETRIC TWO-SCALE MODEL FOR
ROUGH SURFACE BISTATIC SCATTERING EVALUATION

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Introduction and motivation

- Microwave sea observations are of fundamental importance, since they allow retrieving parameters of the sea and of objects on the sea surface.

- In recent years the interest in GNSS reflectrometry (GNSS-R) is increasing, due to the promise of low revisit times and low costs.

- For similar reasons, low-orbit small-satellite constellations have become a hot research topic, too.

- Appropriate electromagnetic models are required to design the system, assess its performance through simulation tools, and to support the development of adequate inversion techniques.
Introduction and motivation

What surface model?
- Sea surface → Anisotropic roughness

What scattering model?
- GNSS or constellation → Bistatic scattering

What applications?
- Sea state → Specular or near specular acquisition geometry
- Object detection → Far from specular acquisition geometry
- Wide range of scattering angles
Models for scattering from natural (randomly rough) surfaces

- Approximate analytical, closed-form (SPM, GO, empirical)
- Approximate analytical/numerical (TSM, SPM2, SSA, SSA2)
- “Exact” fully numerical (MoM + Monte Carlo simulation)

TSM is widely used to model the scattering from the sea surface
Two-Scale Model (TSM)

Total NRCS = large scale roughness NRCS (computed via GO) + small scale roughness NRCS (computed via SPM)

GO dominates at low incidence angles
SPM dominates at intermediate/high incidence angles

Range of validity: union of GO and SPM ones.
no cross-pol and de-pol unless SPM term is averaged over random slopes of tilted mean plane
Average over slopes -> numerical integration!

PTSM

• Almost ten years ago the Polarimetric Two-Scale Model (PTSM) was introduced\(^1\), allowing for closed-form evaluation of the average integral, via a moderate slope approximation.

• PTSM allows accounting for cross- and de-polarisation effects actually present in measured data even when surface scattering is the only present mechanism.

• PTSM has been used to devise soil moisture retrieval schemes for bare soils\(^1\).

• Recently PTSM has been extended to the case of the anisotropic sea surface (A-PTSM)\(^2\), but in backscattering configuration only.

**Extension to the case of bistatic scattering configuration is considere in this work.**


Theory

Surface description

Small-scale roughness:

High-frequency part of the directional Elfouhaily spectrum

\[ W_{2D}(\kappa, \varphi) = W(\kappa) \Phi(\kappa, \varphi) \]

\[ W(\kappa) = \frac{\pi \alpha_m c_m}{c \kappa^4} \exp \left[ -\frac{1}{4} \left( \frac{\kappa}{\kappa_m} - 1 \right)^2 \right] \]

\[ \Phi(\kappa, \varphi) = 1 + \Delta(\kappa) \cos \left[ 2(\varphi_w - \varphi) \right] \]

\[ c = \sqrt{\frac{g}{\kappa}} \left[ 1 + \left( \frac{\kappa}{\kappa_m} \right)^2 \right] \]

\[ u^* = \sqrt{C_d} u_{10} \]

\[ \alpha_m = \begin{cases} 0.01 \left[ 1 + \ln \left( \frac{u^*}{c_m} \right) \right] & \text{for } u^* \leq c_m \\ 0.01 \left[ 1 + 3 \ln \left( \frac{u^*}{c_m} \right) \right] & \text{for } u^* > c_m \end{cases} \]

\[ \kappa_m = 363 \text{ m}^{-1} \]

\[ c_m = 0.23 \text{ m/s} \]

\[ \Delta(\kappa) = \tanh \left[ 0.173 + 4 \left( \frac{c}{c_p} \right)^{2.5} + a_m \left( \frac{c_m}{c} \right)^{2.5} \right] \]

\[ c_p \approx u_{10}/0.84 \text{ and } a_m = 0.13u^*/c_m \]

if \( 20 \text{ m}^{-1} < \kappa < 200 \text{ m}^{-1} \)

\[ c \approx \sqrt{\frac{g}{\kappa}} \]

\[ W(\kappa) \approx \frac{S_0}{\kappa^{3.5}} \]

\[ \Phi(\kappa, \varphi) \approx \Phi(\varphi) = 1 + \Delta(\kappa) \cos \left[ 2(\varphi_w - \varphi) \right] \]

\[ \Delta(\kappa) < \sim 0.2 \]

\[ u_{10} : \text{wind velocity} \]

\[ \varphi_w : \text{wind direction} \]
Theory

Surface description

Large-scale roughness:

Up-wind and cross-wind slopes $s_{up}$ and $s_{cross}$: independent zero-mean Gaussian variables with $\sigma_{up}$ and $\sigma_{cross}$ standard deviations.

Katzberg model for $f=1.5$ GHz (GNSS)

$$
\sigma_{up0}^2 = 0.45 \left[ 0.00316 \cdot 6 \ln (u_{10}) \right]
$$

$$
\sigma_{cross0}^2 = 0.45 \left[ 0.003 + 0.00192 \cdot 6 \ln (u_{10}) \right]
$$

Evaluation for generic $f$:

$$
\sigma_{up,cross}^2 \approx \sigma_{up0,cross0}^2 + \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{\kappa_{cut}} \kappa^2 \cos^2 (\varphi - \varphi_w - \psi_{up,cross}) W(\kappa) \Phi(\varphi) \kappa \, d\kappa \, d\varphi
$$

$$
= \sigma_{up0,cross0}^2 + \frac{S_0}{2\pi} \left( 1 \pm \frac{\Delta(\kappa_0)}{2} \right) \left( \sqrt{\kappa_{cut}} - \sqrt{\kappa_{cut0}} \right)
$$

$$
\psi_{up} = 0, \psi_{cross} = \pi/2
$$

Azimuth and range slopes $s_a$ and $s_r$: correlated zero-mean Gaussian variables with $\sigma_a$ and $\sigma_r$ standard deviations and $\rho$ correlation coefficient.

$$
\sigma_a^2 = \sigma_{cross}^2 \cos^2 \varphi_w + \sigma_{up}^2 \sin^2 \varphi_w
$$

$$
\sigma_r^2 = \sigma_{up}^2 \cos^2 \varphi_w + \sigma_{cross}^2 \sin^2 \varphi_w
$$

$$
\rho = \frac{1}{2} \sin 2\varphi_w \frac{\sigma_{cross}^2 - \sigma_{up}^2}{\sigma_r \sigma_a}
$$
Theory

**Bistatic A-PTSM**

1) Compute tilted surface’s polarimetric covariance matrix via SPM in terms of the local incidence $\vartheta_{li}$ and scattering $\vartheta_{ls}$, $\varphi_{ls}$ angles, and of rotation angles $\beta_i$ and $\beta_s$ of incidence and scattering planes.

2) Express $\vartheta_{li}$, $\vartheta_{ls}$, $\varphi_{ls}$, $\beta_i$ and $\beta_s$ in terms of global incidence $\vartheta_i$ and scattering $\vartheta_s$, $\varphi_s$ angles and of local surface slopes $s_x$ and $s_y$.

3) Second order expansion of tilted surface’s covariance matrix around $s_x = 0$ and $s_y = 0$.

4) Averaging tilted surface’s NRCS and other entries of the covariance matrix over $s_x$ and $s_y$ by using:
   
   $< s_x > = < s_y > = 0, \quad < s_x^2 > = \sigma_x^2, \quad < s_y^2 > = \sigma_y^2, \quad \text{and} \quad < s_x s_y > = \rho \sigma_x \sigma_y$

Expressions for $\vartheta_{li}$ and $\beta_i$ are already available, while those for $\vartheta_{ls}$, $\varphi_{ls}$ and $\beta_s$ are an original contribution of this work.
Theory

Covariance matrix elements

\[
\langle R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s; s_x, s_y) \rangle_{s_x,s_y} \approx R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s; 0, 0) + \\
+ D_{2,0}^{pq,rs} \sigma_x^2 + D_{0,2}^{pq,rs} \sigma_y^2 + D_{1,1}^{pq,rs} \rho \sigma_x \sigma_y
\]

\[
\kappa_y = -k \sin \vartheta_s \sin \varphi_s
\]

\[
\kappa_x = -k \sin \vartheta_s \cos \varphi_s + k \sin \vartheta_i
\]

\[
\bar{\kappa} = \sqrt{\kappa_x^2 + \kappa_y^2}
\]

\[
\bar{\varphi} = \arctan(\kappa_y / \kappa_x)
\]

Expansion coefficients

\[
D_{k,n-k}^{pq,rs} = \frac{1}{n!} \binom{n}{k} \partial^n R_{pq,rs}^{SPM} |_{s_x=s_y=0}
\]

Bragg coefficients

\[
F_{hh} = \frac{\varepsilon_r - 1}{\varepsilon_r} \cos \varphi_s \\
\times \left( \cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \right) \left( \cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i} \right)
\]

\[
F_{lv} = \sin \varphi_s \left( \varepsilon_r - 1 \right) \left( \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \right) \left( \cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i} \right)
\]

\[
F_{vh} = \sin \varphi_s \left( \varepsilon_r - 1 \right) \left( \sqrt{\varepsilon_r - \sin^2 \vartheta_i} \right) \left( \cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \right)
\]

\[
F_{vv} = \left( \varepsilon_r - 1 \right) \left( \sqrt{\varepsilon_r - \sin^2 \vartheta_i} \right) \left( \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \right) \left( \cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i} \right)
\]

Standard SPM elements covariance matrix

\[
R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s, 0, 0)
= \frac{1}{4} k^4 \cos^2 \vartheta_i \cos^2 \vartheta_s F_{pq}(\vartheta_i, \vartheta_s, \varphi_s) F_{rs}^*(\vartheta_i, \vartheta_s, \varphi_s) W_{2D}(\bar{\kappa}, \bar{\varphi})
\]
Results

All experiments at L band, $\vartheta_i = 45^\circ$, and $u_{10} = 10$ m/s.

- $\varphi_s = 0^\circ$, $\varphi_w = 0^\circ$
- $\varphi_s = 30^\circ$, $\varphi_w = 0^\circ$
- $\varphi_s = 60^\circ$, $\varphi_w = 0^\circ$
- $\varphi_s = 90^\circ$, $\varphi_w = 0^\circ$

Bistatic A-PTSM results are closer to the SSA2 than to the SSA1 ones.

SSA2\(^1\) considers multiple scattering (up to second order), but it requires computationally intensive numerical evaluation of fourfold integrals.

Conclusions

- Closed-form PTSM extended to the anisotropic sea surface case (A-PTSM) and to the bistatic scattering configuration

- All elements of the linear polarization polarimetric covariance matrix analytically expressed in closed form

- Reasonable agreement with SSA2, which is more accurate but computationally intensive

- For applications in which computational efficiency is important, use A-PTSM! (for instance, wind speed and direction retrieval, or, more in general, surface parameter retrieval).

- Extendable to the case of agricultural anisotropic soil surfaces, upon appropriate modeling of the roughness
THANK YOU FOR YOUR ATTENTION!

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