Quantification of the artificial mains network impedance contribution to the uncertainty of conducted emission measurements

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3.4.8 Although this *Guide* provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value.

**JCGM 100:2008**  
GUM 1995 with minor corrections  
**Evaluation of measurement data — Guide to the expression of uncertainty in measurement**
Scope

- The scope is to provide a case study of measurement uncertainty (MU) quantification showing that the result is not unique, but it essentially depends on subjective statistical modeling.

- The considerations reported here are relevant to both measurement and computational uncertainty since the common root of statistical modeling is addressed.
Case study

• Non-reproducibility of conducted emission measurement due to the deviation from the nominal value of the artificial mains network impedance

• Such non-reproducibility is the dominating contribution to measurement uncertainty of conducted disturbances in Electromagnetic Compatibility measurements (*)

Circuit description of the problem

$U_{d0}$ is the open-circuit disturbance voltage generated by the EUT
$Z_{d0}$ is the EUT impedance
$Z_{13}$ is the nominal AMN impedance, as defined by the standards specifications
$Z_{in}$ is the actual AMN impedance
Since $Z_{in}$ shall unavoidably differ from $Z_{13}$, also the voltage developed across these impedances by the same EUT, $U_m$ and $U_{mt}$, respectively, will be different
Circuit parameters: $Z_{d0}$

- The impedance of the EUT, $Z_{d0}$, is an unspecified parameter: its value and frequency behavior depend on the specific EUT and they are usually unknown.

- It is assumed that $Z_{d0}$ can be any complex value in the right half of the complex impedance plane, including the imaginary axis.
Circuit parameters: $Z_{in}$

- The impedance of the AMN, $Z_{in}$, is specified by the standard (*) through the concept of the “impedance tolerance circle”

The impedance tolerance circle

\[ \frac{Z_{in}}{Z_{13}} = 1 + z \text{ where } z = \rho \cdot \exp(j\theta) \text{ and } 0 \leq \rho \leq \rho_M \text{ and } 0 \leq \theta < 2\pi \]
$Z_{d0}$ to $k$ transformation

- Through the biunivocal transformation $k = \frac{Z_{13}}{Z_{d0} + Z_{13}}$ the infinite domain of the impedance $Z_{d0}$ (left) is transformed into the compact domain of the parameter $k$ (right).

- The domain of $k$, $D_k$, is a disc centered at $k_0 = \frac{1}{2} \left(1 + j \frac{X_{13}}{R_{13}}\right)$ of radius $|k_0|$, contour $k_C$.

- Random sampling over a compact domain is more practical than random sampling over an infinite domain (see next slides).
Non-reproducibility

• Measurement non-reproducibility is quantified through the relative deviation $\delta$ between $U_m$ and $U_{mt}$

$$\delta = \frac{U_m - U_{mt}}{U_{mt}}$$

• $\delta$ is a complex quantity

• A more practical measure of non reproducibility that will be used here is

$$\Delta = 20\log_{10}|1 + \delta| = 20\log_{10}\left|\frac{U_m}{U_{mt}}\right|$$
Random sampling

In order to evaluate measurement non-reproducibility (uniform), random sampling is carried out over the domains of $z$ and $k$, thus generating random samples of the non-reproducibility, as quantified by

$$\Delta = 20\log_{10} \left| \frac{1 + z}{1 + kz} \right|$$
Preliminary considerations

• We consider the case of the 50 Ω | | (50 μH + 5 Ω) AMN, the network designed to cover the broad frequency range from 9 kHz to 30 MHz and adopted by many product standards.

• The magnitude of $1 + k \rho_M$ is greater than 0 if $\rho_M |k_M| < 1$ and this inequality is met by the impedance of the 50 Ω | | (50 μH + 5 Ω) AMN over the whole frequency range of operation.

• Therefore, $\Delta$ is an analytic function of $z$ and $k$ over the compact regions $D_z$ and $D_k$.

• Due to the properties of analytic functions the limited maximum and minimum values of $\Delta$ exist and they are achieved on the borders of such regions.
Impact of different random sampling strategies

- Limits of the 95% probabilistically symmetric coverage interval of $\Delta$
- Different colors correspond to different sampling strategies:
  - Black lines: uniform sampling the interior of $D_z$ and $D_k$
  - Blue lines: uniform sampling of over the border of $D_z$ (imaginary EUT impedance), uniform sampling over the interior of $D_k$
  - Red lines: uniform sampling of the border of $D_k$ (AMN impedance at the tolerance limit) and of the interior of $D_z$
  - Green lines: uniform sampling of both the borders of the domains $D_z$ and $D_k$
  - Magenta lines: uniform sampling over the border of the disc $D_z$ and $k = \hat{k}$
Worst case EUT impedance

• $\hat{k}$ is a value on the border of $D_k$ such that the root-mean-square value of $\Delta$, for $z$ varying over the border of $D_z$, is maximum

• $\hat{k}$ corresponds to the “worst-case” EUT impedance

\[
Z_{d0} = -j \frac{|Z_{13}|^2 (1 - \rho_M^2)}{X_{13}}
\]
Range

The range (difference *maximum* – *minimum* of the 95 % coverage interval) of ∆ is maximum at the frequency of 52.8 kHz, where the phase angle of the impedance of the 50 Ω || (50 μH + 5 Ω) AMN is maximum, whatever is the adopted sampling strategy.
Mean

The mean of $\Delta$ is zero for all the sampling strategies and over the full frequency range of interest.
Range over standard deviation

- In the case of the triangular distribution the value of this ratio is $2\sqrt{6 \left(1 - \sqrt{0.05}\right)}$ and it is represented through a dashed line.
- The triangular distribution is, in this sense, suitable to describe the spread of the possible values in the case represented by the green curve (i.e. when both $z$ and $k$ uniformly sample the borders of the corresponding domains).
Conclusion

• The non-reproducibility of the disturbance voltage due to the imperfect realization of the AMN impedance is a knowledge-based source of uncertainty

• Experimental observation would require experiments involving EUTs characterized by all the possible disturbance source impedances and AMNs characterized by all the possible deviations from the nominal impedance, which is clearly impossible

• AMN impedance is close to the tolerance limits at the lowest and highest frequencies of operation

• The impedance of the disturbance source is dominated by the output impedance of the filter used to attenuate disturbances at the power port, which is essentially reactive

• It is safe but also reasonable to estimate the non-reproducibility assuming that $z$ and $k$ are at the borders of their respective domains (results corresponding to the green curves in the plots)

• Similarly to the case of the mismatch correction, the expected value of the non-reproducibility (when expressed in decibel) is zero
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