Offline and online reconstruction for radio interferometric imaging

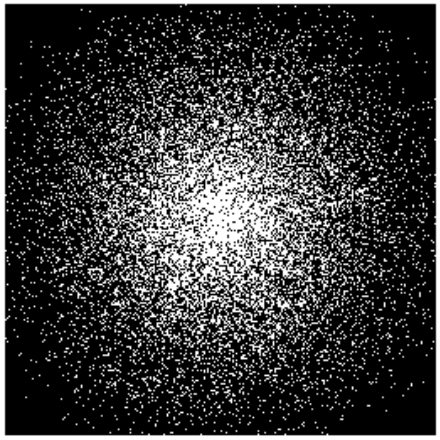
Xiaohao Cai

Mullard Space Science Laboratory (MSSL)
University College London
&
ECS, University of Southampton

Joint work with *Jason D. McEwen and Luke Pratley*

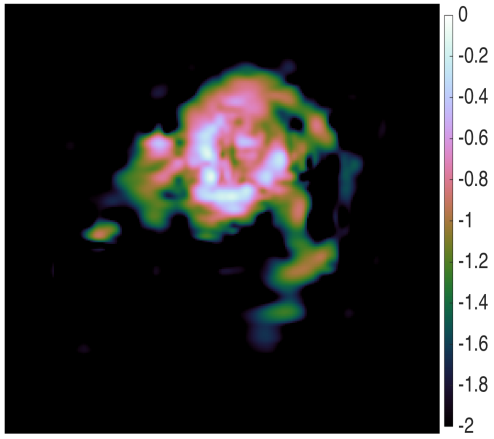
URSI GASS 2020

A high dimensional inverse problem



$y = \Phi x$

Big-data reconstruction



x: image we want to recover

y: big data acquired (noisy, incomplete)

A reconstruction model using Bayesian reference

$$p(\boldsymbol{x} \mid \boldsymbol{y}) \propto \exp\Big\{ -\underbrace{\|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2^2/2\sigma^2}_{\text{Likelihood}} - \underbrace{\lambda\|\Psi^\dagger \boldsymbol{x}\|_1}_{\text{Prior}} \Big\}$$

$$\max_{\boldsymbol{x}} p(\boldsymbol{x} \mid \boldsymbol{y}) = \arg\min_{\boldsymbol{x}} \Big\{ \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2^2/2\sigma^2 + \lambda\|\Psi^\dagger \boldsymbol{x}\|_1 \Big\}$$

ill-posed

derivation

Bayes' theorem

$$p(\boldsymbol{x} \mid \boldsymbol{y}) \propto p(\boldsymbol{y} \mid \boldsymbol{x}) p(\boldsymbol{x})$$

Reconstruction method: maximum a posteriori

$$\max_{\boldsymbol{x}} p(\boldsymbol{x} \mid \boldsymbol{y}) = \arg\min_{\boldsymbol{x}} \Big\{ \underbrace{\|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2^2 / 2\sigma^2}_{f_1(\boldsymbol{x})} + \underbrace{\lambda \|\Psi^\dagger \boldsymbol{x}\|_1}_{f_2(\boldsymbol{x})} \Big\}$$

- **Forward-backward splitting algorithm**

$$\boldsymbol{x}^{(i+1)} = \mathrm{prox}_{\gamma^{(i)} f_2}\big(\boldsymbol{x}^{(i)} - \gamma^{(i)} \nabla f_1(\boldsymbol{x}^{(i)})\big)$$

  (e.g. Combettes & Pesquet, '10)

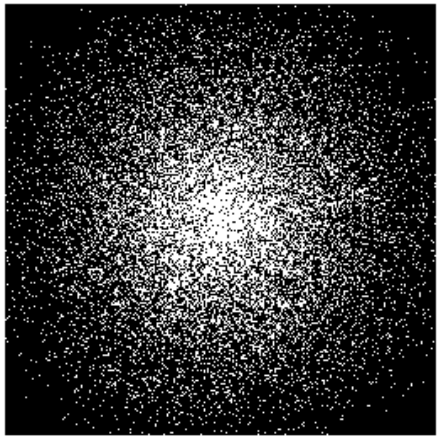
- **Parallelism and distributed calculation**
- **Online method**
- **Other efficient methods:** deep learning, etc.

Apply to the synthesis model is straightforward!

$$\mathrm{prox}_h^\beta(\boldsymbol{z}) \equiv \arg\min_{\boldsymbol{u} \in \mathbb{R}^N} \big\{ h(\boldsymbol{u}) + \|\boldsymbol{u} - \boldsymbol{z}\|^2 / 2\beta \big\}$$

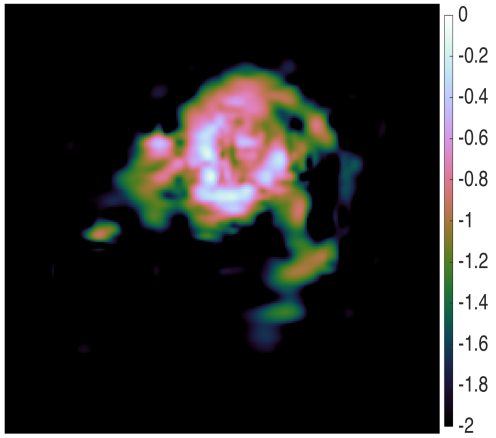
A high dimensional inverse problem

long time



$y = \Phi x$

Big-data reconstruction

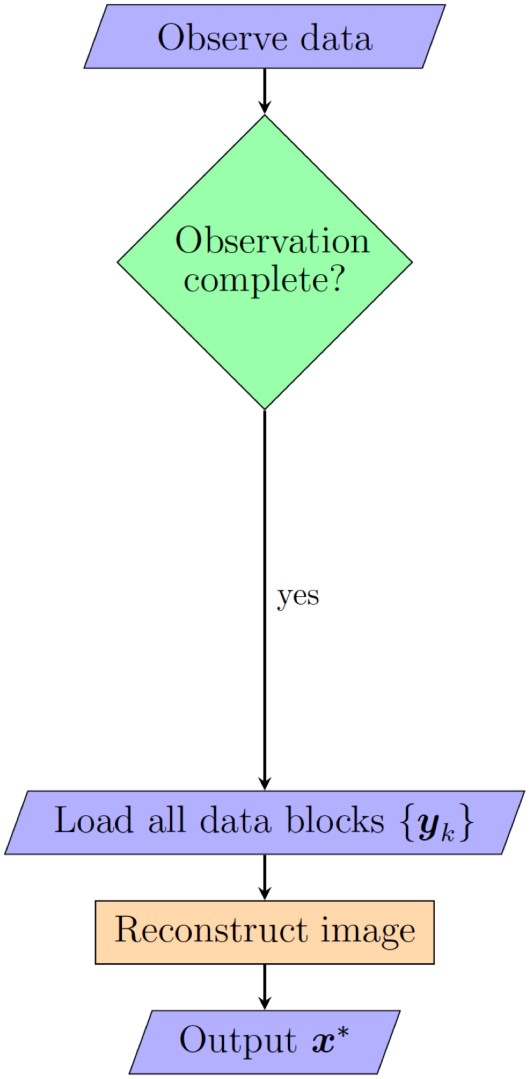


x: image we want to recover

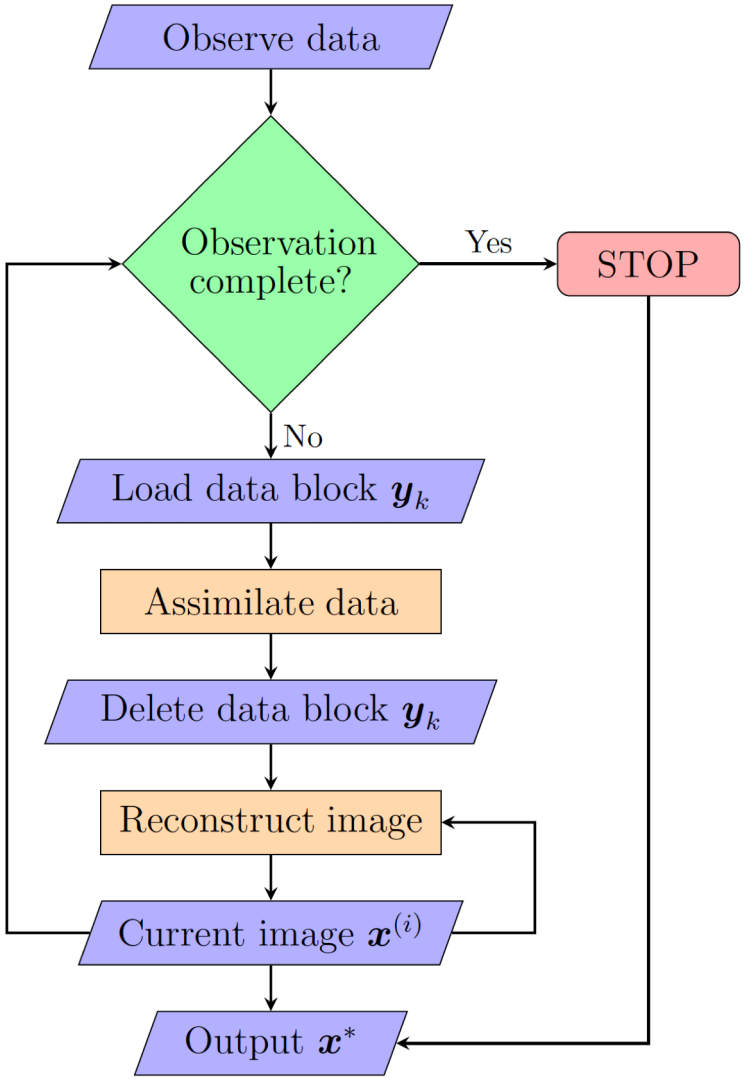
huge storage

y: big data acquired (noisy, incomplete)

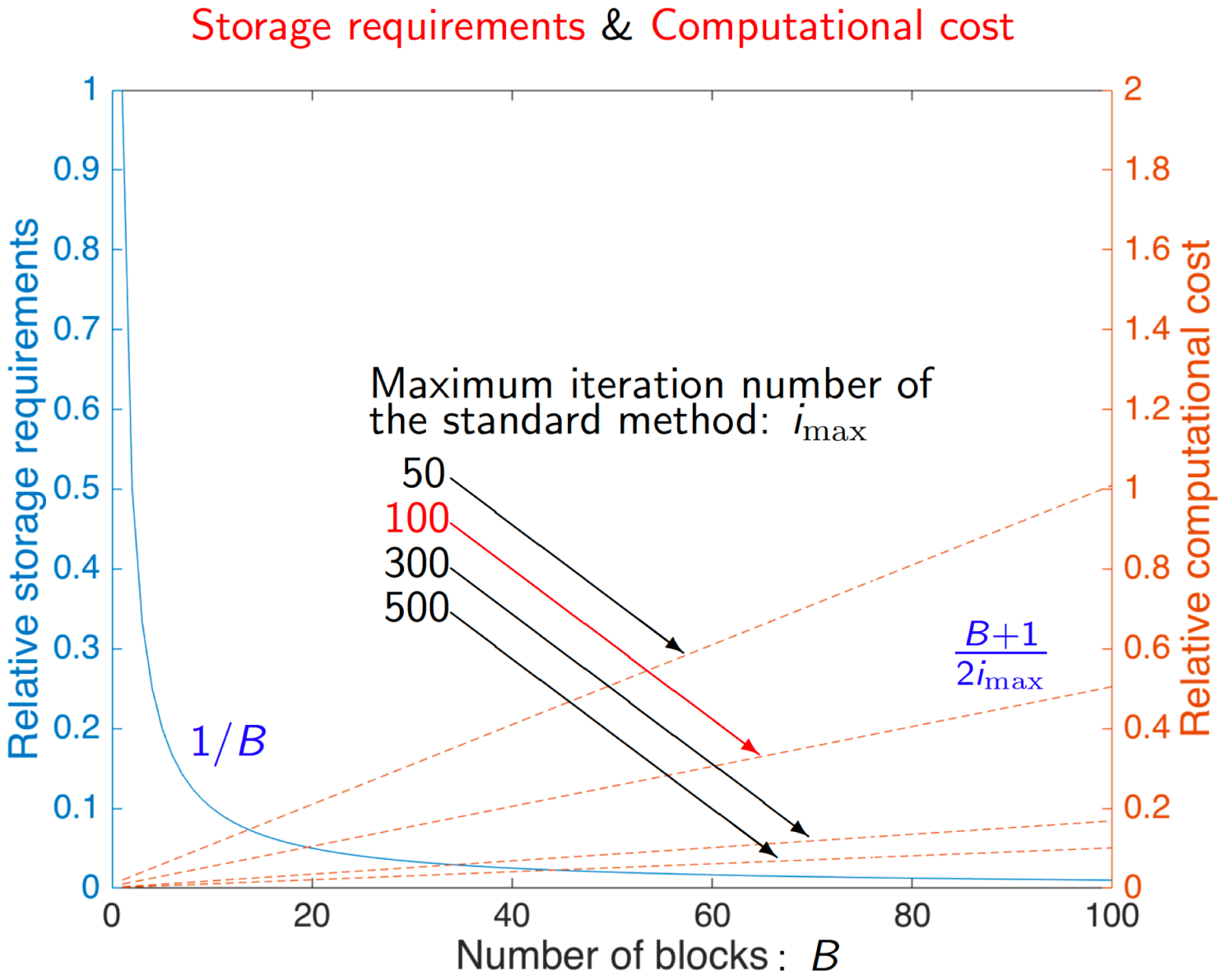
Offline reconstruction



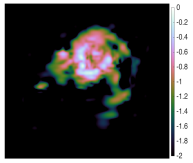
Online reconstruction

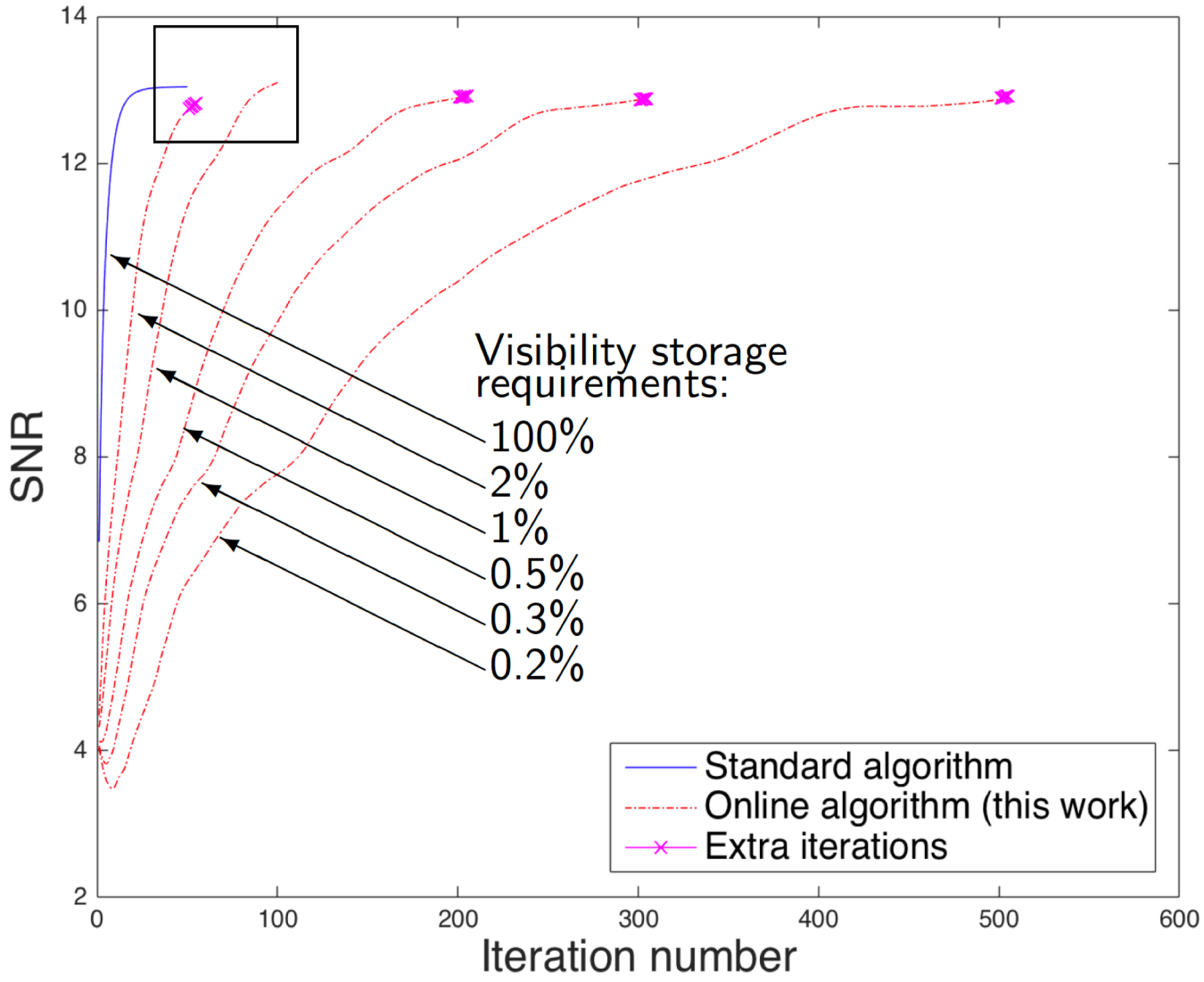


Offline VS online reconstruction



Offline VS online reconstruction





Offline VS online reconstruction

- Data storage requirements
  - Offline: store the whole data
  - Online: store one block of the whole data
- Time
  - Offline: data acquisition + reconstruction
  - Online: data acquisition + reconstruction
- Reconstruction quality
  - Same quality: Offline = Online

References

- X. Cai, L. Pratley and J. McEwen. Offline and online reconstruction for radio interferometric imaging. URSI GASS 2020.

- X. Cai, L. Pratley and J. McEwen. Online radio interferometric imaging: assimilating and discarding visibilities on arrival. MNRAS, 485(4):4559-4572, 2019.

- L. Pratleya, J. McEwen, M. D'Avezac, X. Cai, et al. Distributed and parallel sparse convex optimization for radio interferometry with PURIFY. (arXiv:1903.04502).