Investigation of Accelerated Monostatic Radar Cross Section Computations by Using Block and Projection Iterative Methods

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Abstract

Accurate numerical solutions of scattering phenomena can be achieved by method of moments (MoM) solutions of integral equations. For the evaluation of the monostatic radar cross section (RCS) of an object, the same system of equations has to be solved many times for different excitation fields incident from many directions. Especially for electrically large bodies involving many millions of unknowns, this becomes computationally demanding. In this work, we study and compare the performance of block and projection iterative methods for MoM solutions of surface integral equations related to perfect electrically conducting objects. Both methods are capable of reducing the overall number of matrix-vector products in the considered iterative solution process.

1 Introduction

Monostatic RCS computations play an important role in many modern electromagnetic design processes. An accurate numerical representation of the physical scattering phenomena of PEC objects is provided by MoM solutions of surface integral equations [1]. Electrically large objects can lead to systems of equations with many millions of unknowns, thus, rendering such full-wave methods computationally very demanding. Over the last decades, a variety of methods have been proposed to tackle this problem. These methods can be divided into two main categories, namely direct and iterative methods [2].

Direct methods aim at computing an explicit inverse of the matrix of the linear system of equations, which arises in the discretization process of an integral equation. Classical approaches compute the inverse via the Gaussian elimination or the LU-decomposition, whose computational and storage complexities scale with \( \Theta(N^3) \) and \( \Theta(N^2) \), respectively, where \( N \) refers to the number of unknowns. Especially for large structures, computations with those methods become too cumbersome. For this reason, accelerated techniques have been designed, which construct a low-rank approximation for the inverse of the system matrix, e.g., see [3].

Iterative methods avoid the explicit inversion of the system matrix by consecutively performing matrix-vector products, in order to generate a search space, in which an approximate solution is found. This process is repeated until a certain error criterion is met.

The main issue in computing the monostatic RCS of an object is the fact that the linear system of equations has to be solved all over again, whenever the incident direction of the electromagnetic wave is changed, i.e., for every single monostatic observation direction.

It is possible to directly invoke all incident wave for all incident directions as a collection of right-hand sides (RHSs) in the solution process. This is achieved by so-called block methods, which are augmented versions of the classic iterative methods [4]. The expanded search space, in which the overall solution is found, helps to improve the convergence of the overall solution process, thus, leading to fewer overall matrix-vector products and, in turn, also to less computation time. The expanded search space leads to an increase in storage requirements, which can be a problem for the computation of large objects with many RHSs. This can be remedied by using projection methods [5], which treat only one RHS at a time, but generate a new initial solution by utilizing previously obtained solutions for different incident directions. The new initial solution may lead to a lower initial residual, such that fewer iteration steps are needed in order to meet the error criterion.

In this study, we work with an augmented version of the Flexible Generalized Minimal Residual Method (FGMRES) [6], the Block FGMRES (BFGMRES), and compare it to Galerkin projection methods (GPMs) as well as to the standard FGMRES.

2 Problem Statement

A scattering object is characterized by its radar-cross-section (RCS), which is given as

\[
\sigma(\delta_i, \varphi_i, \delta_r, \varphi_r) = \lim_{r \to \infty} 4\pi r^2 \frac{|E^{\text{scattered}}(\delta_i, \varphi_i)|^2}{|E^{\text{incident}}(\delta_i, \varphi_i)|^2}
\]

(1)

with scattered electric field \( E^{\text{scattered}} \) for the scattering angles \( \delta_i, \varphi_i \), for an incident plane wave with electric field \( E^{\text{incident}} \) with incident angles \( \delta_i, \varphi_i \). A suppressed time factor
\begin{equation}
\mathbf{x} = \mathbf{b}, \quad (6)
\end{equation}

where $i$ is the index of a RHS, based on previously solved systems with $k = 1, \ldots, i - 1$ [5]. The whole solution process can, therefore, be described as a sequence

\begin{equation}
P_i : \quad \mathbf{A}\mathbf{x}^{(i)} = \mathbf{b}^{(i)} \quad i \in [1, \ldots] \quad (7)
\end{equation}

of linear systems of equations. Since the success of the projection methods depends on the previously generated solutions, it is required that the solutions share a certain degree of similarity among each other. In the monostatic RCS case, this is accomplished by ensuring that the angular sampling criterion is met, which is required to accurately reconstruct the 3D monostatic RCS [10]. Also, subsequent solutions should work with excitations according to neighboring incident angles on the monostatic RCS sampling grid. Similar requirements should, of course, also be fulfilled for the block of RHSs considered in one BFGMRES solution, as discussed in Section 3.1.
We consider the realistic metallic model of a MiG-29 air-4 Numerical Results

mance, before the Galerkin method is employed.

ify the already obtained solutions towards a better perfor-

mation scheme [8, 11]. The solution process was stopped at a

relative residual error of $10^{-4}$ as stopping criterion. The BFGMRES and the GPM are

compared to the standard FGMRES. The numbers after BFGM-

RES and GPM indicate how many RHS and previous solutions

have been taken into account, respectively. In the case of BFGM-

RES, deflation reduced the 35 RHSs to 24 before the first restart

after 100 iterations, which was considered to correspondingly
down the number of iterations.

search space during the actual system of equations solu-

tions.

5 Conclusion

The block flexible generalized minimal residual (BFGM-

RES) solver and the Galerkin-based projection methods

considered in this work managed both to reduce the over-

all number of iterations in the monostatic radar cross sec-

tion solution process of scattering integral equations with

multiple incident directions, in comparison to the standard

FGMRES solver. Both methods do require a certain simi-

larity between the different considered excitations, which

may be achieved by considering neighboring incident an-
gles and fulfill the corresponding sampling requirements.

Overall, it is even a bit amazing that the found projection

method could well compete with the BFGMRES solver,

which needs considerably more computer memory and also

slightly more computation time per iteration due to the or-

thogonalization of a considerably larger search space.

**References**


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