Outage Probability of Two-Way Relaying Systems Over Mixed Fluctuating Two-Ray and Nakagami-$m$ Fading Channels

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Abstract

Two-way relaying systems can significantly save time resource and increase the throughput for wireless communication systems. In this paper, we carry out the performance analysis of two-way relaying systems over mixed fluctuating two-ray (FTR) and Nakagami-$m$ fading channels. The FTR fading promises a good fit for small-scale experimental data in millimeter wave communications. More specifically, we derive novel and exact analytical expressions for outage probability (OP) and simple asymptotic OP at high signal-to-noise-ratio regimes to show important physical insights into the impact of parameters on the system performance. It is interesting to find that larger values of fading severity parameters of both links help to decrease the OP. Finally, the correctness of our derived analytical results is validated by Monte Carlo simulations.

1 Introduction

Recently, two-way relaying systems have received a lot of attention for bringing huge effective system throughput in wireless communications. With the aid of a relay node, two-way relaying can significantly save the time resource. In contrast to half-duplex relaying scheme, two-way relaying enables the source and destination communicate within only two time slots. The performance of two-way interference-limited amplify-and-forward (AF) relaying systems over independent, non-identically distributed Nakagami-$m$ fading channels was investigated in [1]. Moreover, performance analysis and relay selection of two-way hybrid terrestrial-satellite relaying systems were proposed in [2], which analyzes the performance of systems over Nakagami-$m$ and k-$\mu$ shadowed fading channels.

The FTR channel fading model recently proposed in [3] has been proved to be very useful, since it is in good agreement with the experimental data of millimeter wave (mmWave) communications. Therefore, considerable research has been carried out on the FTR fading channel. For example, exact probability distribution function (PDF) and cumulative distribution function (CDF) for the FTR distribution with arbitrary parameters were proposed in [4]. To the best of our knowledge, however, the mmWave bands have not been considered in the two-way relaying system.

s. To this end, for the successful deployment of mmWave bands in two-way relaying systems, a complete understanding of its performance when operating under FTR fading channels becomes essential.

Motivated by the above discussion, in this paper we present novel analytical expressions for outage probability (OP) of two-way relaying systems over mixed FTR and Nakagami-$m$ fading channels. Moreover, simple asymptotic expressions for the OP are derived to obtain important engineering insights in the high-SNR regime. Finally, our results can generalize most of previous results in the literature.

2 Two-Way Relaying Systems

2.1 System Model

Similar to [2], let us consider two transmitter nodes $A$ and $B$ communicating with each other through an AF relay $R$. Due to poor communication quality in the direct link, we adopt a relay $R$ to assist the transmission of links as $A \rightarrow R \rightarrow B$ and $B \rightarrow R \rightarrow A$. All nodes are equipped with one antenna. The transmission process between $A$ and $B$ is divided into two time phases. Specifically, in the first phase, both $A$ and $B$ transmit symbols to the relay $R$ simultaneously. In the second phase, the relay $R$ processes the received signal to forward the combined symbols to both $A$ and $B$. We assume that the channel state information is completely known at each node, and perfect synchronization between $A$, $B$ and $R$ is achieved. Then, the instantaneously received SNRs at $A$ and $B$ can be given by [5]

$$\gamma_A = \frac{P_2 Q \gamma_2}{(P_1 + Q) \gamma_1 + P_2 \gamma_2}, \quad (1)$$

$$\gamma_B = \frac{P_1 Q \gamma_2}{(P_2 + Q) \gamma_1 + P_1 \gamma_2}, \quad (2)$$

where $\gamma_1 = |h_1|^2/\mathcal{N}_0$, $\gamma_2 = |h_2|^2/\mathcal{N}_0$, and $\mathcal{N}_0$ denotes the variance of the additive white Gaussian noise (AWGN) at all nodes. On the other hand, we use $h_1$ and $h_2$ to denote the channel coefficients of $A \rightarrow R$ and $B \rightarrow R$ links. $P_1$, $P_2$, and $Q$ denote the transmit power at $A$, $B$, and $R$, respectively. Without loss of generality, we introduce power coefficients as $p_{\ell} \triangleq P_{\ell}/Q$, $p_{2} \triangleq P_{2}/Q$, $b_{\ell} \triangleq 1/Q$. Then, we can rewrite
\(\gamma_1 = \frac{\gamma_1 \gamma_2}{p_d \gamma_1 + b_2 \gamma_2},\) \hspace{1cm} (3)

\(\gamma_2 = \frac{\gamma_1 \gamma_2}{b_2 \gamma_1 + p_d \gamma_2}.\) \hspace{1cm} (4)

2.2 Channel Model

The Nakagami-\(m\) fading is assumed for the \(A \leftrightarrow R\) link. The instantaneous SNR of the \(A \leftrightarrow R\) link, \(\gamma_1\), is a gamma distributed RV. Then the probability distribution function (PDF) of \(\gamma_1\) is given by [2, Eq. (7)]

\[f_{\gamma_1}(\gamma) = \frac{m_1^{m_1} \gamma^{m_1-1} e^{-\frac{m_1}{\gamma_1}}}{\Gamma(m_1)} \frac{1}{\gamma_1},\] \hspace{1cm} (5)

where \(\Gamma(\cdot)\) is the gamma function [6, Eq. (8.310.1)] and \(m_1\) is the Nakagami-\(m\) fading parameter. \(\gamma_1 = \sigma_2^2 \gamma_0\) is the average SNR of the \(A \leftrightarrow R\) links, and \(\sigma_2^2 = E[|h_1|^2]\) is the expectation operator. Moreover, we introduce \(\gamma_0 \triangleq P_t / N_0\) as the average transmit SNR of the \(A \rightarrow R\) link, and introduce \(\gamma_2 \triangleq Q/N_0\) for the \(R \rightarrow A\) link, respectively. Hence, the CDF of \(\gamma_1\) can be written as

\[F_{\gamma_1}(\gamma) = 1 - \frac{\Gamma(m_1, (m_1/\gamma_1) \gamma)}{\Gamma(m_1)},\] \hspace{1cm} (6)

where \(\Gamma(\cdot, \cdot)\) is the upper incomplete gamma function [7, Eq. (8.350.2)]. Furthermore, the \(B \leftrightarrow R\) link is modeled as the FTR distribution, which provides better fit than other channel models with small-scale fading measurements in mmWave communications [3, 4]. The PDF of the instantaneous SNR of \(S \leftrightarrow R\) links, \(\gamma_2\), is given as [4, Eqs. (6)]

\[f_{\gamma_2}(\gamma) = \frac{m_2^{m_2} \gamma^{m_2-1} e^{-\frac{m_2}{\gamma_2}}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K_j d_j}{j!} F\left(\gamma, j + 1, 1.2\sigma_2^2\right),\] \hspace{1cm} (7)

where \(F(\gamma, j + 1, 1.2\sigma_2^2)\) and \(d_j\) is given in [4, Eqs. (8-9)]. \(K\) denotes the ratio of the average power of the dominant wave to the remaining diffuse multipath and \(m_2\) is the fading severity parameter. Moreover, \(F(\cdot)\) is the Legendre function of the first kind [7, Eq. (8.702)], while \(\Delta\) is a variable from 0 to 1, representing the correlation between the two dominant waves. Furthermore, \(\gamma_2 = \sigma_2^2 \gamma_0\) represents the average SNR of the \(B \leftrightarrow R\) links. Similarly, \(\sigma_2^2 = E[|h_2|^2]\) is the variance of the channel coefficient \(h_2\), where \(\gamma_0 \triangleq P_t / N_0\) and \(\gamma_1 \triangleq Q/N_0\) denotes the average transmit SNR of the \(B \rightarrow R\) link and of the \(R \rightarrow B\) link, respectively. Then, the CDF of \(\gamma_2\) is given as

\[F_{\gamma_2}(\gamma) = \frac{m_2^{m_2} \gamma^{m_2-1} e^{-\frac{m_2}{\gamma_2}}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K_j d_j}{j!} F\left(\gamma, j + 1, 1.2\sigma_2^2\right),\] \hspace{1cm} (8)

where

\[F\left(\gamma, j + 1, 1.2\sigma_2^2\right) \triangleq \frac{1}{\Gamma(j+1)} P\left(j + 1, \frac{\gamma}{2\sigma_2^2}\right),\] \hspace{1cm} (9)

where \(P(\cdot, \cdot, \cdot)\) is the lower incomplete gamma function [7, Eq. (8.350.1)].

### 3 Performance Analysis

#### 3.1 Outage Probability

The outage probability \(P_o(\gamma_0)\) is defined as the probability that the instantaneous SNR is below a given threshold, and it can be written as \(P_o(\gamma_0) = \Pr(\gamma < \gamma_0)\). Therefore, we can derive the OP at A as

\[P_o(\gamma_0) = F_{\gamma_1}(\gamma_0),\] \hspace{1cm} (10)

\[P_o(\gamma_b) = F_{\gamma_2}(\gamma_b),\] \hspace{1cm} (11)

where \(F_{\gamma_1}(\gamma_b)\) and \(F_{\gamma_2}(\gamma_b)\) represent the CDFs at A and B in the case of instantaneous SNR, respectively. Then the exact OP is given by the following Lemma.

**Lemma 1.** The exact OP at A and B of two-way relaying systems are given as (12) and (13) at the bottom of the next page, where \(m_1\) is a positive integer, \(\Phi_1 \triangleq m_1 b_q/\gamma_1\), \(\Phi_2 \triangleq P_d/2\sigma^2\), \(\Psi_1 \triangleq m_1 p_s/\gamma_1\), \(\Psi_2 \triangleq b_q/2\sigma^2\), \(c_j \triangleq m_2^{m_2} K_j d_j/\Gamma(m_2) \Gamma(j+1)\), and \(K_j(\cdot)\) denotes the \(j\)-th order modified Bessel function of the second kind [8, Eq. (9.6.2)].

**Proof.** Please see Appendix.

Tables I and II present the required terms of infinite series in (12) and (13) for a required convergence. Although there are infinite terms in the formulas, it requires a small number of terms to reduce the error to smaller than \(10^{-3}\) for all considered cases. As shown in the Tables, the truncation error is related to different channel fading parameters.

Considering Rician shadowed fading model (by setting \(\Delta = 0\)), (12) and (13) can reduce to the ones for mixed Nakagami-\(m\) and Rician shadowed fading channels in [2, Eqs. (15-16)]. Moreover, (12) and (13) can reduce to the OP of two-way AF relaying systems over Nakagami-\(m\) (by setting \(K \rightarrow \infty\) and \(\Delta = 0\)) fading channels in [9, Eq. (15)].
3.2 Asymptotic Outage Probability

In order to reveal more engineering insights, we elaborate on the asymptotic high-SNR regime.

**Lemma 2.** With high-SNR regime, the asymptotic OP of two-way relaying systems at A and B can be expressed as

\[
P_{\text{O, th}}^{\infty} \approx 1 - \sum_{i=0}^{\infty} \sum_{j=0}^{i} \sum_{k=0}^{j} \left( \frac{m_1}{\gamma_l} \right)^{m_1-1} c^j \frac{\Gamma(r+1)b_q m_1 \Gamma(m_1)}{i! \Gamma(m_1)} \exp\left(-\Phi_1 \gamma\right)^j\exp\left(-\Phi_2 \gamma\right)^k \exp\left(-\Psi_1 \gamma \right)^{i-j-k}\exp\left(-\Psi_2 \gamma \right)^{i-j-k+1} K_{r-s+1} \left(2 \gamma \sqrt{\Phi_1 \Phi_2} \right),
\]

\[
K_{r-s+1} \left(2 \gamma \sqrt{\Phi_1 \Phi_2} \right) \approx (r-s+1) \left( \frac{m_1 p b_q \gamma^2}{2 \gamma_l \sigma^2} \right)^{r-s+1}.
\]

Proof. For high SNRs, according to [8, Eq. (9.6.6)] and [8, Eq. (9.6.9)], we can express \( K(\gamma) \) as

\[
K_{r-s+1} \left(2 \gamma \sqrt{\Phi_1 \Phi_2} \right) \approx (r-s+1) \left( \frac{m_1 p b_q \gamma^2}{2 \gamma_l \sigma^2} \right)^{r-s+1}.
\]

According to [7, Eq. (1.211.1)], the exponential function \( e^t \) can be represented by the main decisive terms (here we use 3 smallest exponents). After some mathematical simplifications, (14) can be obtained. Following similar steps, we can obtain (15) to finish the proof.

\[
P_{\text{O, th}}^{\infty} \approx 1 - \sum_{i=0}^{\infty} \sum_{j=0}^{i} \sum_{k=0}^{j} \left( \frac{m_1}{\gamma_l} \right)^{m_1-1} c^j \frac{\Gamma(r+1)b_q m_1 \Gamma(m_1)}{i! \Gamma(m_1)} \exp\left(-\Phi_1 \gamma\right)^j\exp\left(-\Phi_2 \gamma\right)^k \exp\left(-\Psi_1 \gamma \right)^{i-j-k}\exp\left(-\Psi_2 \gamma \right)^{i-j-k+1} K_{r-s+1} \left(2 \gamma \sqrt{\Phi_1 \Phi_2} \right),
\]

\[
P_{\text{O, th}}^{\infty} \approx 1 - \sum_{i=0}^{\infty} \sum_{j=0}^{i} \sum_{k=0}^{j} \left( \frac{m_1}{\gamma_l} \right)^{m_1-1} c^j \frac{\Gamma(r+1)b_q m_1 \Gamma(m_1)}{i! \Gamma(m_1)} \exp\left(-\Phi_1 \gamma\right)^j\exp\left(-\Phi_2 \gamma\right)^k \exp\left(-\Psi_1 \gamma \right)^{i-j-k}\exp\left(-\Psi_2 \gamma \right)^{i-j-k+1} K_{r-s+1} \left(2 \gamma \sqrt{\Phi_1 \Phi_2} \right).
obtain important physical insights and simplify the calculation, closed-form expressions for high-SNR regime have been presented. Our results reveals the relationship between channel parameters and system performance. For instance, OP performance can be improved by increasing the values of $m_1$ and/or $m_2$. Furthermore, the OP performance of the two-way relaying systems can also be improved by increasing the values of the transmit power at $A, B$ and/or the average SNR at either link. Ultimately, the presented results are quite useful for satisfying the performance requirements of a practical two-way relaying mmWave system.

6 Appendix

With the help of (5), we can express the $F_{\gamma_{th}}(\gamma)$ as

$$F_{\gamma_{th}}(\gamma) = \int_{0}^{\infty} \Pr(\gamma_{th} \leq \gamma | \gamma_{th}) f_{\gamma_{th}}(\gamma_{th}) d\gamma_{th}$$

$$= \int_{0}^{b_1} \Pr(\gamma_{th} \leq \gamma_{th} | \gamma_{th} = b_1) f_{\gamma_{th}}(b_1) d\gamma_{th}$$

$$+ \int_{b_1}^{\infty} \Pr(\gamma_{th} \leq \gamma_{th} | \gamma_{th} = b_2) f_{\gamma_{th}}(b_2) d\gamma_{th}$$

$$= F_{\gamma_{th}}(b_1 \gamma) + \int_{b_2}^{\infty} F_{\gamma_{th}} \left( \frac{p_d \gamma_{th}}{\gamma_{th} - b_2 \gamma} \right) f_{\gamma_{th}}(\gamma_{th}) d\gamma_{th}.$$  \hspace{1cm} (17)

Substituting (6) into (17), and with the help of [7, Eq. (8.356.3)], $F_{\gamma_{th}}(b_1 \gamma)$ can be solved. According to (5), (8) and (9), we can simplify the integral $A_1$ as

$$A_1 = \frac{m_1^{m_1-1} m_2^{m_2-1}}{\gamma_{th} \Gamma(m_1) \Gamma(m_2)} \gamma_{th} \exp \left( -\frac{m_1 b_1 \gamma}{\gamma_{th}} \right) \sum_{j=0}^{m_2-1} \sum_{r=0}^{m_1-1}$$

$$\times \frac{K \Gamma(j+1) \Gamma(j+1)}{\Gamma(j+1)} (b_2 \gamma)^{m_1-1-r} \left( \frac{m_1}{r} \right) \int_{0}^{\infty} x^{j+1} \gamma_{th}^{j+1} \left( \frac{p_d \gamma_{th} (x+b_2 \gamma)}{2 \sigma^2 + x} \right) \exp \left( \frac{m_2 x}{\gamma_{th}} \right) dx.$$  \hspace{1cm} (18)

With the help of [7, Eq. (3.471.9)], the CDF of $\gamma_{th}$ can be obtained with some mathematical simplifications.

References


