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Modeling the Fine Structure of Natural Discrete Plasma Wave Emissions in the Earth's Radiation Belts

Miroslav Hanzelka*(1)(2), Ondřej Santolík(1)(2), Yoshiharu Omura(3), and Ivana Kolmašová(1)(2)
(1) Department of Space Physics, Institute of Atmospheric Physics, Czech Academy of Sciences, Prague, Czech Republic
(2) Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic
(3) Research Institute for Sustainable Humanosphere, Kyoto University, Uji, Japan

Abstract

Whistler mode chorus emission is an important type of natural discrete plasma wave emissions that influence the energetic particles in the Earth's outer radiation belt through nonlinear wave-particle interactions. In this work we modify a previously developed theoretical model of the growth of amplitude and frequency in the discrete chorus elements to incorporate the experimentally confirmed formation of short wave packets inside the elements. Numerical solution of the model equations produces a chorus element with this subpacket structure, also featuring an upstream shift of the source region away from the magnetic equator and an irregular growth of wave frequency. The model well captures basic features of instantaneous frequency and amplitude measurements provided by the Van Allen Probes spacecraft. The modeled wave field can be used in future particle acceleration studies.

1 Introduction

Chorus emissions are coherent electromagnetic waves propagating in the right-hand polarized whistler mode which are frequently observed in the inner magnetosphere, typically in the range from 4 to 8 Earth's radii [1]. A characteristic feature of these emissions are the discrete elements (wave packets) in their frequency-time spectra, exhibiting rising or falling frequencies. They can induce both acceleration and losses of energetic electrons in the radiation belts through nonlinear interactions [2]. These processes are sensitive to the frequency-time structure of the chorus wave packets [3]. The fine structure of rising tone chorus elements has been discovered from high resolution measurements of the Cluster spacecraft [4] which show that each element of the discrete emission consists of several subpackets with growing wave frequencies. The subpacket structure of chorus has been confirmed by recent analyses of multi-component measurements of chorus by Van Allen Probes [5, 6]. This fine structure has also been observed in self-consistent particle simulations [7].

In the present study we use the nonlinear growth theory of Omura *et al.* [8] to develop a simple model of the fine structure of rising tone chorus emissions. The evolution of the wave amplitude and wave frequency inside a single subpacket in the source region is described by the so-called chorus equations [8]. Wave propagation and convective growth is modeled with advection equations. The fundamental assumption employed in the present model is that the resonant current, produced through wave-particle interaction, carries the information about the wave vector and frequency of the emission and can act as a helical antenna and radiate a new coherent wave during their upstream propagation. Similar idea (i.e., the resonant current acting as an antenna) already appeared in the seminal paper of R. A. Helliwell [9], but they did not connect it with the nonlinear growth theory, which was not yet developed at that time. In the present study further assumptions are made to separate the newly radiated wave from the previous subpacket, and the optimum amplitude derived by Omura *et al.* [10] is used to introduce saturation effects into the model. Chorus elements obtained from the numerical solution show that between adjacent subpacket, there are small, local drops in the otherwise growing frequency, which is a feature that seems to be also indicated by the measurements of the Van Allen Probes [5, 6]. The upstream shift of the source region, previously obtained in some full-particle simulations [7], is also present in the model.

2 Description of the model

We are studying the evolution of wave frequency $\omega(h,t)$ and wave amplitude $B_\mathrm{w}(h,t)$ of a coherent electromagnetic whistler mode wave propagating parallel to a background dipole magnetic field through a one-component plasma with a constant number density of electrons. Distance h is measured along a magnetic field line, starting at the equator, t is the time. Following [11], we describe the evolution with two coupled advection equations

$$\frac{\partial\omega}{\partial t}+V_\mathrm{g}\frac{\partial\omega}{\partial h}=0\,,\quad \frac{\partial B_\mathrm{w}}{\partial t}+V_\mathrm{g}\frac{\partial B_\mathrm{w}}{\partial h}=-\frac{\mu_0 V_\mathrm{g}}{2}J_E\,, \tag{1}$$

where V_g is the group velocity of a whistler mode wave, μ_0 is the permeability of vacuum and J_E is the resonant current density component parallel to the wave electric field. A detailed derivation of the second equation has been given by, e.g., [12] or [13]. Further we will assume a parabolic approximation of the magnetic field strength along field lines

$$B_0 = B_{\mathrm{eq}}\left(1 + ah^2\right), \quad \Omega_{\mathrm{e}} = \Omega_{\mathrm{e0}}\left(1 + ah^2\right), \qquad (2)$$

where B_{eq} is the equatorial magnetic field and a comes from the small-latitude expansion of the magnetic field and is given by $a = 4.5/(LR_{\mathrm{E}})^2$, with R_{E} being the Earth's radius and Ω_{e0} the equatorial electron gyrofrequency.

Following [6], we use Equations 1 to describe the evolution of a single subpacket, not the whole chorus element. Furthermore, we also allow the subpackets to start at some position h_i upstream of the equator, instead of fixing the source region to $h = 0$. This changes the chorus equations [8] which specify the evolution of frequency and amplitude at h_i and thus serve as the boundary conditions. With the assumption in Equation 2, the boundary condition for frequency will change to

$$\left.\frac{\partial \omega}{\partial t}\right|_{\mathrm{modif}} = \left.\frac{\partial \omega}{\partial t}\right|_{\mathrm{equatorial}} - 2c\Omega_{\mathrm{e0}}\frac{as_2}{s_1}h_i, \qquad (3)$$

where s_i are additional parameters describing the properties of the wave and the environment, see [8] for details. Similarly, the optimum amplitude, which controls the saturation of the amplitude growth [10], will change to

$$B_{\mathrm{opt}}\big|_{\mathrm{modif}} = B_{\mathrm{opt}}\big|_{\mathrm{equatorial}} + \frac{2cB_{\mathrm{eq}}}{S_{\mathrm{max}}}\frac{as_2}{s_0\omega}h_i \qquad (4)$$

The extra term is negative, which causes the optimum amplitude to decrease faster and it can stop the growth if B_{opt} falls below a certain threshold B_{thr}. S_{max} is a constant which defines the magnetic field inhomogeneity perceived by the rising frequency element.

Having the complete set of evolution equations and boundary conditions, the evolution of the chorus element is modeled in the following way. Initially, the electromagnetic emissions in the equatorial region are dominated by incoherent noise in the whistler mode. Through interaction with hot electrons, the amplitude of the noise grows according to the linear growth theory with a rate γ_{L}, which maximizes at the equator. After some time the linear growth produces a coherent emission. Once this seed wave reaches a certain amplitude threshold, it will start to grow in frequency and amplitude according to the chorus equations. This is accompanied by the release of resonant current into the upstream region – this current comes from the phase bunching of electrons and has a helical spatial structure that was formed by the whistler wave. The emission also propagates away from the equator, experiencing further convective growth. The growth in the source is limited by the optimum amplitude. After the wave amplitude reaches B_{opt}, the nonlinear growth mechanism breaks down. At the same time, the strongest resonant current is released into the upstream. This current acts as a helical antenna and releases a new whistler wave with the frequency $\omega_1 = \omega_0 + \Delta\omega_1$, where ω_0 is the wave frequency of the initial subpacket and $\Delta\omega_1$ is the frequency difference measured at the point where the optimum amplitude was reached. To model a smooth decrease in amplitude of the initial subpacket, we simply switch the sign in the chorus equation for amplitude, reversing thus the growth rate. It is further assumed that the new wave, produced by the radiation from the helical current, cannot replace the previous subpacket until its amplitude drops below B_{thr}. The ensures the existence of well separated subpackets.

3 Numerical Results and Comparison to Observations

We solve the partial differential equations 1 with an upwind integration scheme, with the chorus equations acting as the boundary conditions at h_i. As the initial conditions we choose $B_{\mathrm{w}}(0,0) \equiv B_{\mathrm{w0}} = 2B_{\mathrm{thr}}(0,0)$ and $\omega(0,0) \equiv \omega_0 = 0.25\,\Omega_{\mathrm{e0}}$. For each new subpacket the initial amplitude is always set to the double of the local threshold frequency. Other model parameters (mainly the properties of the hot electron velocity distribution) were chosen to represent the typical values measured in the radiation belts. The process is stopped when $B_{\mathrm{thr}}(h_i) > B_{\mathrm{opt}}(h_i)$ or when the initial frequency of the next subpacket exceeds a limiting frequency $\omega_{\mathrm{fin}} = 0.5\,\Omega_{\mathrm{e0}}$.

To describe the properties of the modeled chorus element, we computed the number of subpackets $N_{\mathrm{S}} = 10$, upstream shift of the source location across the whole chorus element $h_{\mathrm{elm}} = 1300\,\mathrm{km}$, the time duration $t_{\mathrm{elm}} = 200\,\mathrm{ms}$, frequency sweep rate $\Delta\omega/\Delta t = 8.9\,\mathrm{kHz/s}$ and the maximum amplitude $B_{\mathrm{w,max}} = 0.014B_{\mathrm{eq}}$. Sweep rate, time duration and maximum amplitudes were calculated for $h = 1400\,\mathrm{km}$ (magnetic latitude of $2^\circ - 3^\circ$). If we measured the maximum amplitudes at larger h, they would grow steadily up to unreasonable values ($B_{\mathrm{w,max}}/B_{\mathrm{eq}} > 0.1$), which is caused by the assumption of parallel propagation of whistler modes.

High quality electromagnetic wave measurements provided by the two Van Allen Probes were used to identify large amplitude chorus events in the radiation belts. One such event, detected by the Van Allen Probe B spacecraft on 08 October 2012, is used here for comparison with our model. The measurement was done by the EMFISIS Waves instrument in the morning sector at McIlwain's $L = 4.91$ and magnetic latitude $\lambda_{\mathrm{m}} = 1.74^\circ$ northward from the equator. Figures 2a and 2b show detailed waveforms of a large amplitude chorus element. The calibrated waveform is pass-band filtered between $0.4\,\mathrm{kHz}$ and $3\,\mathrm{kHz}$ and analytic signals are constructed using the Hilbert transform. Their instantaneous amplitudes are shown in Fig. 2c. The instantaneous frequencies plotted in Fig. 2d are obtained as time derivatives of the phases of the complex analytic signals [5].

The analyzed chorus element is composed of several subpackets. The instantaneous frequency is globally rising with time but sometimes it steps back at the boundaries

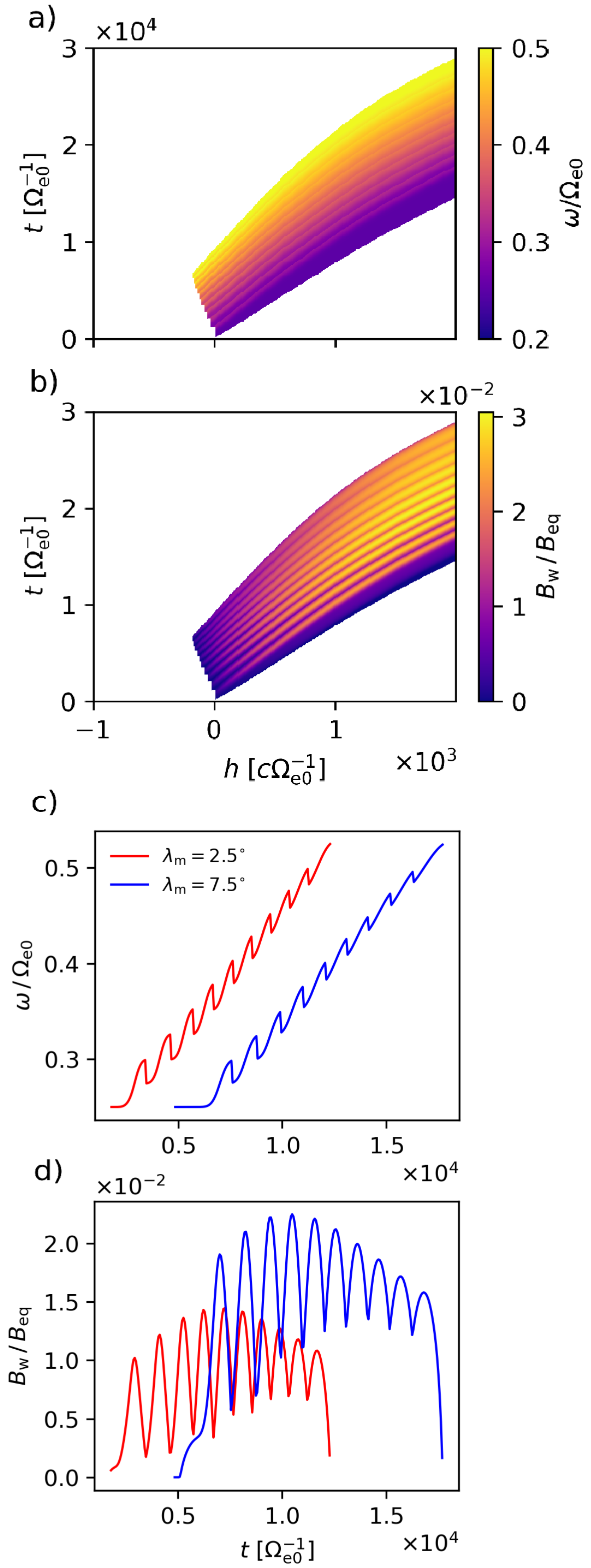

Figure 1. Modeled evolution of a) wave frequency and b) wave amplitude in time and space. The range of axes converted to the SI and derived units is $t = (0, 650)\,\mathrm{ms}$, $h = (-6500, 13\,000)\,\mathrm{km}$. c,d) The profiles of frequency and amplitude in time as we would see them at magnetic latitudes 2.5° (red lines) and 7.5° (blue lines).

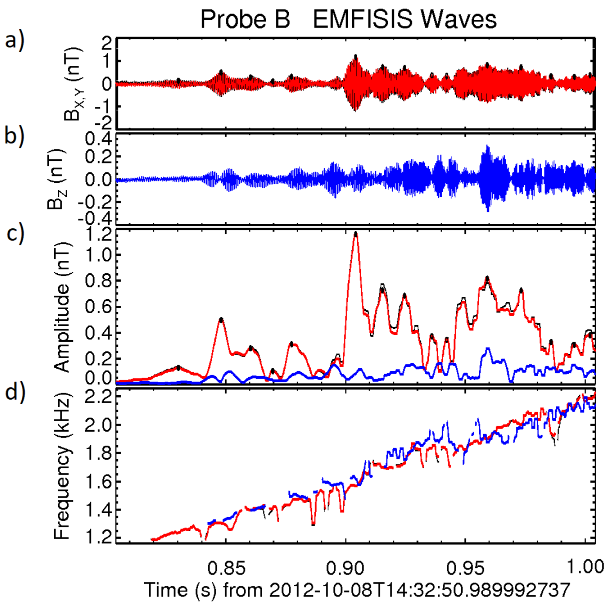

Figure 2. Detailed analysis of a large amplitude chorus element measured by EMFISIS. a) Waveform of perpendicular magnetic field fluctuations, b) waveform of parallel magnetic field fluctuations, c) instantaneous amplitudes for the perpendicular and parallel components and for the modulus, shown respectively by red, blue, and black lines, d) instantaneous frequency with the same color coding plotted for the instantaneous amplitudes larger than 50 pT. Black dots show the local maxima of amplitude of the dominant perpendicular component larger than 50 pT relative to adjacent minima.

of the subpackets. This is consistent with the simulation results. The properties of the analyzed element described by the quantities $N_S \approx 15$, $\Delta\omega/\Delta t \approx 5.0\,\mathrm{kHz/s}$ and $t_{\mathrm{elm}} \approx 200\,\mathrm{ms}$ are within a multiple of two from the output parameters obtained in the simulation, in which we used values of $L = 4.91$, $\omega_{\mathrm{pe}}/\Omega_{\mathrm{e0}} = 6.45$ corresponding to the EMFISIS data.

4 Discussion and Conclusion

The comparison of simulation results with observations of the Van Allen Probes confirms that the drops in frequency between subpackets can be observed in large amplitude chorus elements. The upstream shift of the source cannot be determined from measurements of a single spacecraft, but indirect indications of a similar effect have been reported by Taubenschuss *et al.* [14] for bidirectional chorus wave packets. Two satellites with a small spatial separation (hundreds of kilometers) should be in principle able to directly intercept one chorus element inside the source at different stages of its development. Short distances between spacecraft with highly sensitive wave instruments were achieved during several close separation campaigns of the four-spacecraft Cluster mission [15], and additional work is needed to identify signatures of this effect for special configurations when different spacecraft are located close to a single magnetic field line, at transverse separations lower than a typical transverse size of generation regions of separate chorus wave packets, i.e. on the order of 100 km [16].

To summarize, we have shown that a model based on the nonlinear growth theory and the antenna effect can be used to simulate growth and propagation of single chorus elements with subpacket structure. The model features steep drops in frequency at the point where one subpacket transitions to the next one, and an upstream drift of the source region with increasing wave frequency. The first feature was confirmed by observations of the Van Allen Probes spacecraft, the second one appears in self-consistent particle simulations. Time duration and frequency sweep rate of the element and the number of subpackets obtained through simulations are comparable to those observed in a typical event of intense chorus recorded by the Van Allen Probe B spacecraft. The model can be used in test particle simulations to determine the effect of subpackets on particle acceleration – this is left for future studies.

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References

[1] B. T. Tsurutani and E. J. Smith, “Postmidnight chorus: A substorm phenomenon,” *Journal of Geophysical Research*, vol. 79, no. 1, pp. 118–127, Jan 1974.

[2] D. L. Turner, V. Angelopoulos, W. Li, M. D. Hartinger, M. Usanova, I. R. Mann, J. Bortnik, and Y. Shprits, “On the storm-time evolution of relativistic electron phase space density in Earth’s outer radiation belt,” *Journal of Geophysical Research (Space Physics)*, vol. 118, no. 5, pp. 2196–2212, May 2013.

[3] X. Tao, J. Bortnik, J. M. Albert, R. M. Thorne, and W. Li, “The importance of amplitude modulation in nonlinear interactions between electrons and large amplitude whistler waves,” *Journal of Atmospheric and Terrestrial Physics*, vol. 99, pp. 67–72, Jul. 2013.

[4] O. Santolík, D. A. Gurnett, J. S. Pickett, M. Parrot, and N. Cornilleau-Wehrlin, “A microscopic and nanoscopic view of storm-time chorus on 31 March 2001,” *Geophysical Research Letters*, vol. 31, no. 2, p. L02801, Jan 2004.

[5] O. Santolík, C. A. Kletzing, W. S. Kurth, G. B. Hospodarsky, and S. R. Bounds, “Fine structure of large-amplitude chorus wave packets,” *Geophysical Research Letters*, vol. 41, pp. 293–299, Jan. 2014.

[6] J. C. Foster, P. J. Erickson, Y. Omura, D. N. Baker, C. A. Kletzing, and S. G. Claudepierre, “Van Allen Probes observations of prompt MeV radiation belt electron acceleration in nonlinear interactions with VLF chorus,” *Journal of Geophysical Research (Space Physics)*, vol. 122, no. 1, pp. 324–339, Jan 2017.

[7] M. Hikishima, S. Yagitani, Y. Omura, and I. Nagano, “Full particle simulation of whistler-mode rising chorus emissions in the magnetosphere,” *Journal of Geophysical Research (Space Physics)*, vol. 114, p. A01203, Jan. 2009.

[8] Y. Omura, M. Hikishima, Y. Katoh, D. Summers, and S. Yagitani, “Nonlinear mechanisms of lower-band and upper-band VLF chorus emissions in the magnetosphere,” *Journal of Geophysical Research (Space Physics)*, vol. 114, no. A7, p. A07217, Jul 2009.

[9] R. A. Helliwell, “A theory of discrete VLF emissions from the magnetosphere,” *Journal of Geophysical Research*, vol. 72, no. 19, pp. 4773–4790, Oct 1967.

[10] Y. Omura and D. Nunn, “Triggering process of whistler mode chorus emissions in the magnetosphere,” *Journal of Geophysical Research (Space Physics)*, vol. 116, no. A5, p. A05205, May 2011.

[11] D. Summers, Y. Omura, Y. Miyashita, and D.-H. Lee, “Nonlinear spatiotemporal evolution of whistler mode chorus waves in Earth’s inner magnetosphere,” *Journal of Geophysical Research (Space Physics)*, vol. 117, no. A9, p. A09206, Sep 2012.

[12] Y. Omura, Y. Katoh, and D. Summers, “Theory and simulation of the generation of whistler-mode chorus,” *Journal of Geophysical Research (Space Physics)*, vol. 113, p. A04223, Apr. 2008.

[13] D. Nunn, “A self-consistent theory of triggered VLF emissions,” *Planetary and Space Science*, vol. 22, no. 3, pp. 349–378, Mar 1974.

[14] U. Taubenschuss, A. G. Demekhov, and O. Santolik, “Interpretation of whistler mode chorus observations with the backward wave oscillator model,” in *Planetary Radio Emissions VIII*, Jan 2017, pp. 233–242.

[15] F. Němec, J. S. Pickett, and O. Santolík, “Multispacecraft Cluster observations of quasiperiodic emissions close to the geomagnetic equator,” *Journal of Geophysical Research (Space Physics)*, vol. 119, pp. 9101–9112, Nov. 2014.

[16] O. Santolík and D. A. Gurnett, “Transverse dimensions of chorus in the source region,” *Geophysical Research Letters*, vol. 30, no. 2, p. 1031, Jan 2003.