Diffractive and Focusing Properties of Bessel-Gauss Beams in Electromagnetics

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Abstract

Bessel and Bessel-Gauss beams are extensively used in optics because of their remarkable focusing character. At microwaves, where the paraxial approximation no longer holds, the generation of Bessel beams appeared only recently, whereas Bessel-Gauss beams have still not found an experimental validation. Here, we discuss the generation of Bessel-Gauss beams in electromagnetics, accounting for the vectorial nature of electromagnetic fields and without resorting to the paraxial approximation. Numerical results are shown to discuss the focusing and diffractive properties of Bessel-Gauss beams. A possible realization that takes advantage of cylindrical leaky waves is finally commented.

1 Introduction

Bessel beams (BBs) are limited-diffraction solutions of Helmholtz equation. They were experimentally generated in optics thanks to the work of Durnin and his collaborators, more than thirty years ago [1]. Successively, a new type of solution, the so-called Bessel-Gauss beam (BGB), was proposed in [2] to derive an analytical description of a limited-diffraction beam, under the paraxial approximation. There, it was shown that the Gaussian apodization function on the Bessel aperture distribution not only allowed for deriving an analytical expression for a BGB, but also turned to be useful to reduce the undesirable on-axis oscillations commonly experienced by BBs due to diffraction from edges. A comprehensive discussion about the beam features of BGBs was finally provided in [3], where analytical and numerical results were obtained under the frame of a *scalar theory*.

The previous results hold in the optical range, but are not strictly valid at lower frequencies, e.g., at microwaves, where the *fully-vectorial nature* of electromagnetic fields has to accounted for. Interestingly, while BBs are now widespread at microwaves (see, e.g., [4, 5] and refs. therein) BGBs have not been experimentally generated yet.

In this contribution, we discuss the diffractive and focusing properties of BGBs in electromagnetics, without taking advantage of either the scalar theory or the paraxial approximation. In Section 2, we present the theoretical framework. In Section 3, we comment the focusing and diffractive properties of BGBs through the evaluation of the axial flux of the Poynting vector. Finally, a possible realization of BGBs through cylindrical leaky waves is discussed in Section 4.

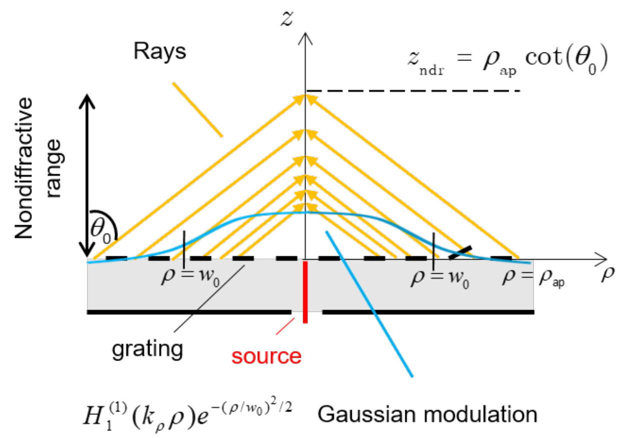

Figure 1. Ray interpretation of Bessel-Gauss beam generation from an inward cylindrical wave aperture field. In contrast with *classical* BBs, BGBs exhibit an amplitude Gaussian modulation. The density of the rays is proportional to the field amplitude, thus the nondiffractive range of a BGB is expected to be smaller than that of a BB as the beam waist parameter decreases.

2 Theoretical Framework

We start by considering a canonical structure capable of generating BBs at microwaves, i.e., a circular radiating aperture centrally fed by a coaxial feed. Since this kind of source can be modeled by a vertical electric dipole, we consider only transverse magnetic (TM) electromagnetic fields (see Fig.1). In order to focus a BB in the near-field region, the aperture field must exhibit an inward character [6]. Therefore, we assume the cylindrical tangential components of a TM-polarized aperture field of an azimuthally-invariant structure to take the following expressions:

$$E_\rho(\rho) \propto j\frac{k_z}{k_\rho}H_1^{(1)}(k_\rho\rho), \quad H_\phi(\rho) \propto \frac{1}{k_\rho}H_1^{(1)}(k_\rho\rho) \quad (1)$$

where k_z and k_ρ are the vertical and radial wavenumbers related to the free-space wavenumber k_0 through $k_0^2 = k_z^2 + k_\rho^2$, and $H_1^{(1)}(\cdot)$ is the Hankel function of the first kind.

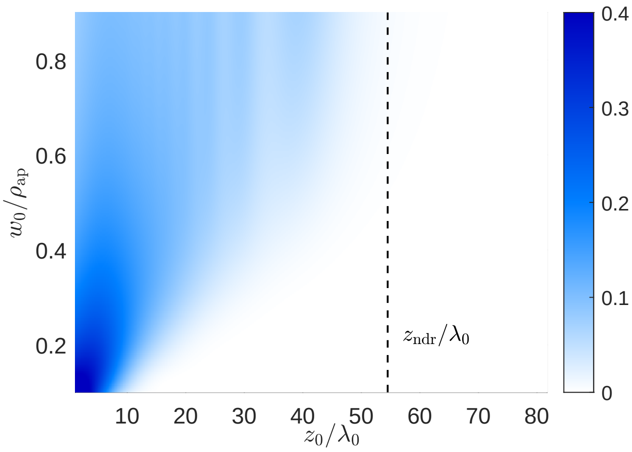

Figure 2. The power flux (normalized to the input power) along the propagation axis is shown as a function of z/λ_0 and w_0/ρ_{ap}. The black dashed line represents the limit set by z_{ndr}/λ_0.

As well known [6], such an aperture field is capable of focusing a zeroth-order BB on the vertical electric field component within a diamond-shaped region whose vertex is determined by the nondiffractive range $z_{ndr} = \rho_{ap} \cot\theta_0$, where ρ_{ap} is the aperture radius, and $\theta_0 = \arctan(k_\rho/k_z)$ is known as the *axicon angle*. In order to generate a BGB over the longitudinal component of the electric field, an additional Gaussian taper has to be accounted for in the expression of the aperture fields. Therefore, (1) is multiplied by the factor $\exp(-\rho^2/2w_0^2)$, with w_0 representing the beam waist parameter.

3 Numerical Results

We evaluated the near-field distribution of all electromagnetic field components by integrating the aperture field through the Huygens-Fresnel radiation formulas. Results are obtained considering $\rho_{ap} = 25\lambda_0$ and $\theta_0 \simeq 25°$. Then, the Poynting vector along the vertical component $S_z = 1/2\Re\{E_\rho H_\phi\}$ has been integrated over a disk orthogonal to the propagation z-axis, centered at $z = z_0$, and with $\bar{\rho}$ such that $k_\rho \bar{\rho}$ is equal to the first null of the zeroth-order Bessel function of the first kind, in order to obtain the power flux carried by the main beam of the BGB. The power flux has been evaluated at different distances z_0 from the radiating aperture, even beyond z_{ndr} and for different w_0.

As expected from theory, for $z_0 > z_{ndr}$ a negligible power is transported. Interestingly, for $w_0 \to 0$, BGBs show focusing features similar to Gaussian beams (GBs): the power flux transported along the z-axis is efficiently confined, but only for few wavelengths. Conversely, for $w_0 \to \rho_{ap}$, the power flux transported along the z-axis is more uniformly distributed along z_0, never reaching the efficiency of a GBs, but remaining stable for several wavelengths, up to z_{ndr}. We notice that Fig. 2 suggests to use w_0 in the range $0.4 < w_0/\rho_{ap} < 0.6$; in this range, the power transport efficiency is moderately good and evenly distributed along z_0.

4 Realization through Cylindrical Leaky Waves

It has been recently been shown [4, 5] that cylindrical leaky waves (CLWs) can profitably be used to generate Bessel-like beams. Differently from *classical* Bessel beams, those generated through CLWs are characterized by an exponentially decaying modulation of the type $\exp(-\alpha\rho)$, where α is the attenuation constant of the radial wavenumber k_ρ. Although this behavior considerably differs from that required by BGBs, it is possible to convert such an exponential modulation to a Gaussian modulation, by suitably *tapering* the aperture distribution, i.e., locally modifying the value of the leakage rate α along the radial direction. This tapering procedure has been extensively used in the past for 1-D leaky-wave antennas (LWAs) [7] and has been more recently proposed for radially-periodic LWAs [8]. A possible implementation of the synthesis procedure aimed at producing a BGB will be shown at the conference.

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