

An improved Equivalent Circuit Model for Nonlinearly Loaded Receiving Structures subject to High Power Electromagnetic Pulses

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Abstract

Equivalent circuit models of receiving structures are commonly used to include lumped elements with nonlinear current-voltage characteristics into an EMC analysis. In this contribution an efficient method will be introduced to derive an equivalent circuit model of a loop-antenna that includes a diode as a nonlinear load. The relevance of this circuit model will be illustrated by studying the response of this antenna to repetitive HPEM pulses.

1 Introduction

Methods that predict the response of receiving structures are often based on transfer functions in frequency domain and presuppose linearity. Linear transfer functions can be generalized to nonlinear systems using Volterra series [1]. However, this requires that the nonlinearities are sufficiently small [2]. Other techniques, such as the harmonic-balance technique, are based on splitting linear from nonlinear parts. If the nonlinear parts can be treated as lumped elements it is meaningful to model the linear parts by equivalent circuits, often characterized by port impedances. This approach was used in [3], [4], and [5], for example. The equivalent circuit models can be derived as Foster equivalent networks of the first type [6]. These allow the inclusion of radiation effects, if sufficiently small [7]. As an example, such an equivalent circuit model was utilized in [5] to study the response of a nonlinearly loaded loop-antenna if excited by an HPEM-pulse. It is the aim of this contribution to improve this model to obtain a better correlation if compared to full wave simulations. To this end, Section II shortly introduces the considered loop nonlinearly loaded loop antenna. Then, an efficient method for deriving an equivalent circuit model of the antenna is proposed. For the study of the response with respect to HPEM-pulses, results from equivalent circuit simulations are compared to results from full-wave simulations in Section III, yielding a very good agreement.

2 Antenna model

A nonlinearly loaded loop-antenna is considered, as sketched in Fig. 1. For this antenna, an appropriate equivalent circuit model to model its input impedance Z_{in} has to be derived first.

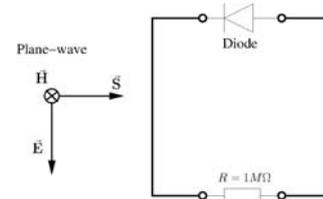


Figure 1. A nonlinearly loaded loop-antenna, being subject to a plane wave excitation. For later purposes, the antenna load is chosen as $1\text{ M}\Omega$.

2.1 Derivation of an equivalent circuit for the input impedance

The first step is to calculate the input impedance Z_{in} of the linear antenna part, as seen by the nonlinear load, compare Fig. 2. In order to avoid numerical inconveniences, the input admittance Y_{in} will be considered in the following.

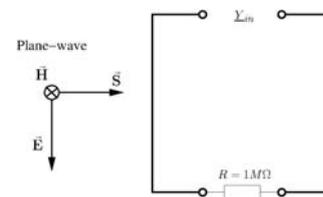


Figure 2. Input admittance of the linear part

The numerical calculation of Y_{in} is performed with a standard frequency domain solver [8] that returns the magnitude and phase results. The magnitude is not of primary interest here, the focus is on the phase in Fig. 3. Three resonant frequencies are considered, as shown in Fig. 3. In order to match the first resonance frequency $f_{res0} = 364.4\text{MHz}$, the equivalent of Fig. 4 is considered. For the lower frequency

range, the only relevant element in the circuit is the capacitance C_0 . This becomes evident when the imaginary part of the previously calculated admittance is considered, see Fig. 5, since the susceptance, for low frequencies, appears as positive and linear.

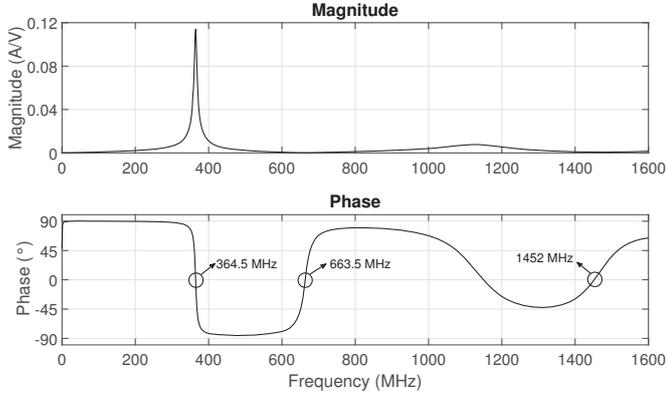


Figure 3. Magnitude and phase of the input admittance \underline{Y}_{in}

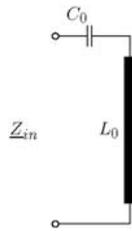


Figure 4. Equivalent circuit for low frequencies

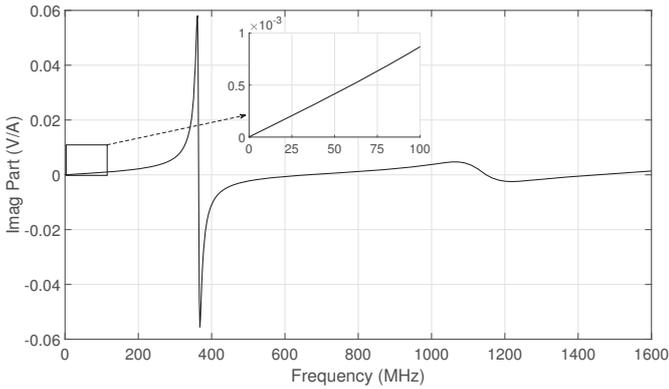


Figure 5. Imaginary part of the input admittance at low frequencies

The value of C_0 can be determined from

$$C_0 = \frac{\text{Imag}(\underline{Y}_{in})}{2\pi f} \quad (1)$$

Since the first resonance frequency $f_{res1} = 364.5\text{MHz}$ should be matched, L_0 is determined from

$$f_{res1} = \frac{1}{2\pi\sqrt{C_0 L_0}} \quad (2)$$

In the next step, a higher frequency range is considered and two additional resonant circuits are added to the equivalent circuit model proposed in Fig. 4. This leads to the equivalent shown in Fig. 6. The inductance L_1 is coupled via the

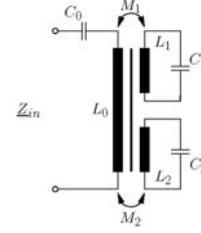


Figure 6. Equivalent circuit model with two resonant circuits added

coupling inductance M_1 to L_0 while L_2 is coupled to L_0 via M_2 . The corresponding input admittance \underline{Y}_{in} is given by

$$\underline{Y}_{in}(\omega) = \frac{1}{\frac{1}{j\omega C_0} + j\omega L_0 - j\sum_{n=1}^2 \omega M_n \frac{C_n M_n \omega^2}{C_n L_n \omega^2 - 1}} \quad (3)$$

with $\omega = 2\pi f$. The coupling inductances M_1 and M_2 can be expressed by coupling factors k_1 and k_2 :

$$M_n = k_n \cdot \sqrt{L_0 L_n} \quad n = 1, 2 \quad (4)$$

For the further design of the equivalent circuit model, the two capacitances C_1 and C_2 are now arbitrarily set to $C_n = 0.1\text{pF}$. The values of these capacitances are not critical, nevertheless, it is recommended to choose a small value, as will be discussed later. With this choice the inductances L_1 and L_2 are again determined via $f_{res} = \frac{1}{2\pi\sqrt{CL}}$ in order to match the resonance frequencies $f_{res1} = 636.5\text{MHz}$ and $f_{res2} = 1452\text{MHz}$, compare Fig. 3. The remaining unknowns are the coupling factors k_1 and k_2 . If no coupling losses are assumed, the two factors are related by

$$k_1 + k_2 = 1 \quad (5)$$

In order to study the influence of these coupling factors, Eq. 3 is used to calculate \underline{Y}_{in} for the coupling factors $k_1 = 0.5$ and $k_2 = 0.5$. The corresponding phases are displayed in Fig. 7. At the resonance frequencies $f_{res1} = 636.5\text{MHz}$ and $f_{res2} = 1452\text{MHz}$ a perfect match is achieved. Next, in Fig. 8 the k -factors are adapted in a way to best match the remaining resonance frequency. Unfortunately, this leads to a deterioration of the first resonance frequency at $f_{res0} = 364.5\text{MHz}$. However, the k -factors and L_0 should be adjusted such that all resonance frequencies are matched, of course. This is achieved in Fig. 9 with the values shown in the figure. Now all the components of the equivalent circuit model introduced in Fig. 6 are determined. Explicit values are $L_0 = 115.45\text{nH}$, $C_0 = 1.3\text{pF}$, $L_1 = 576.3\text{nH}$, $L_2 = 120.2\text{nH}$, $C_1 = C_2 = 0.1\text{pF}$, $k_1 = 0.795$, and $k_2 = 0.205$.

A further point to be considered are radiation losses. In the equivalent circuit model, these losses are modeled by two

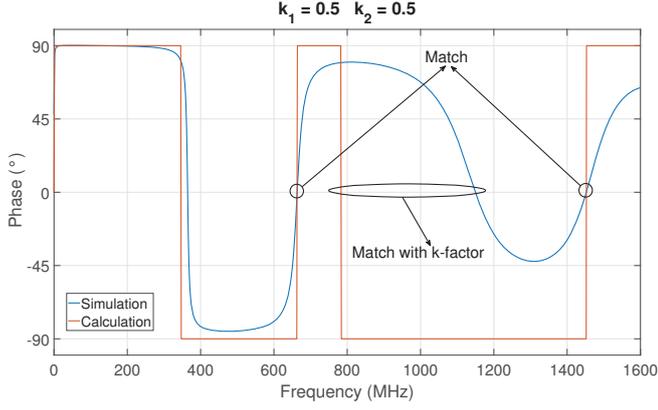


Figure 7. Phase of the input admittance Y_{in} compared to the calculated phase with $k_1 = 0.5$ and $k_2 = 0.5$.

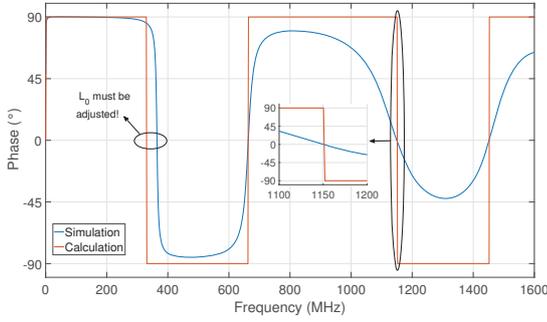


Figure 8. Phase of the input admittance Y_{in} with adapted coupling factors k_1 and k_2

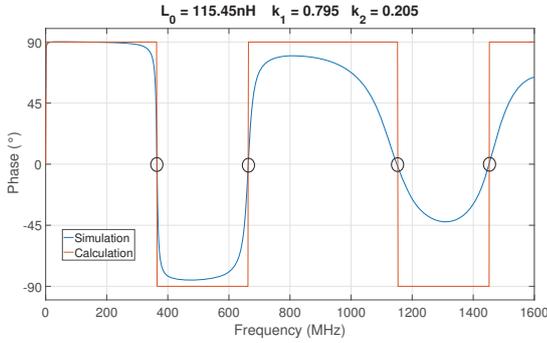


Figure 9. Phase of the input admittance Y_{in} with adapted inductance L_0 and coupling factors k_1, k_2

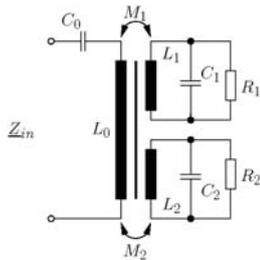


Figure 10. Equivalent circuit model with the inclusion of radiation losses

resistors R_1 and R_2 added to the resonance circuits, as proposed in Fig. 10. In order to determine the values of these resistances, the bandwidths of the resonances are taken into account. These are given by, compare Fig. 11,

$$b_n = f|_{\text{Phase}=+45^\circ} - f|_{\text{Phase}=-45^\circ}, \quad n = 1, 2 \quad (6)$$

such that the values of the resistors R_1 and R_2 can now be determined from

$$b_n = \frac{1}{2\pi R_n C_n}, \quad n = 1, 2, \quad (7)$$

leading to $R_1 = 60.0\text{k}\Omega$ and $R_2 = 12.4\text{k}\Omega$. As mentioned

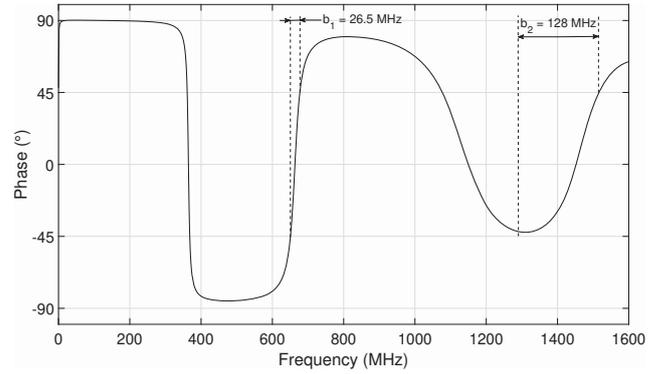


Figure 11. Bandwidths of the resonances

before, the values of C_n can be chosen randomly. Nevertheless, a high value of C_1 leads to a small value of R_1 which complicates the task to derive an equivalent circuit model that is valid for a wide frequency range.

2.2 Completion of the equivalent circuit model

In order to complete the whole equivalent circuit model, the nonlinear load and the terminating resistance R , compare Fig. 1, are added as shown in Fig. 12. The nonlinear

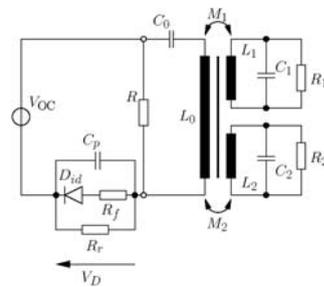


Figure 12. Complete equivalent circuit model of a non-linearly loaded loop-antenna

load is chosen as a diode witch consists of an ideal diode D_{id} , a forward resistance $R_f = 190\Omega$, a reverse resistance $R_r = 40\text{M}\Omega$, and a parasitic capacitance $C_p = 0.284\text{pF}$.

3 Testing the equivalent circuit model

In order to validate the proposed equivalent circuit model, a 3D free field simulation in time domain is performed [8]. The excitation is carried by a plane wave as indicated in Fig. 1. The excitation-signal consists of several consecutive pulses, separated by time intervals of 300ns, see Fig. 13. Each single pulse can be defined according to

$$E(t) = E_0 K(a, b) \Theta(t_0) \left(e^{-a(t-t_0)} - e^{-b(t-t_0)} \right) \quad (8)$$

with the Heavyside-function $\Theta(t_0)$ and the correction factor

$$K(a, b) = \left(e^{-a \frac{\ln \frac{a}{b}}{a-b}} - e^{-b \frac{\ln \frac{a}{b}}{a-b}} \right)^{-1} \quad (9)$$

Explicit parameters are: $a = \frac{1}{t_f}$ with $t_f = 4ns$ and $b = \frac{1}{t_r}$ with $t_r = 3ns$. Finally, t_0 is set to $t_0 = 2.5ns$.

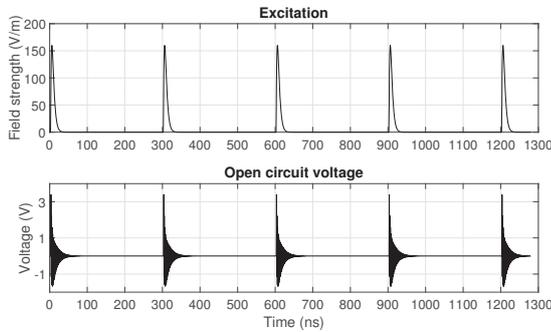


Figure 13. Excitation signal and open circuit voltage for consecutive pulses at intervals of 300 ns

As observable, the voltage drop across the diode is recorded. An analogous circuit simulation, using LTSpice [9], is additionally performed for comparison. For this simulation, the open circuit voltage V_{OC} is required, compare Fig. 12. It is calculated in a full wave simulation with the linear part of the structure only, according to Fig. 14. Once the open circuit voltage is found, compare Fig. 13, it can be used within the equivalent circuit model in LTSpice. The results concerning the voltage drop V_D

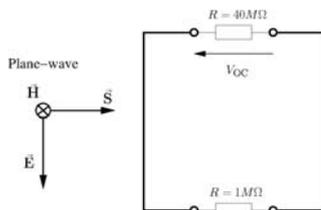


Figure 14. Simulation setup for the calculation of the open circuit voltage V_{OC}

across the diode of both the full wave (CST) and electric circuit (LTSpice) simulations are compared, leading to the result displayed in Fig. 15. Both voltages show a very satisfying correlation.

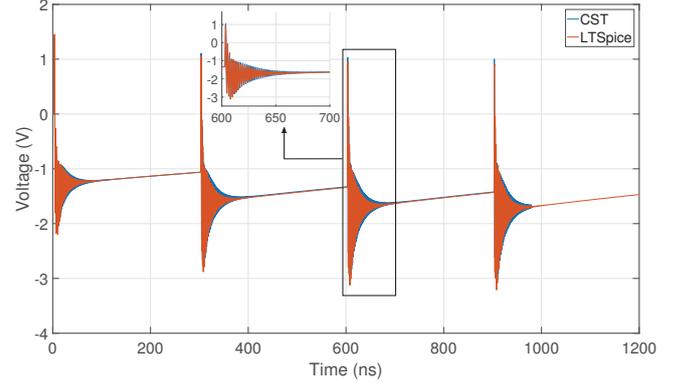


Figure 15. Voltage drop across the diode (V_D) calculated by full wave (CST) and electric circuit (LTSpice) simulations

4 Acknowledgement

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References

- [1] T. Sarkar and D. Weiner, "Scattering Analysis of Nonlinearly Loaded Antennas," *IEEE Trans. Antennas Propagat.*, **24**, 2, March 1976, pp. 125–131, doi: 10.1109/TAP.1976.1141327.
- [2] S. A. Mass, "Nonlinear Microwave Circuits," *Artech House*, 1988, ISBN: 0-89006-251-X.
- [3] T. Liu and F. Tesche, "Analysis of Antennas and Scatterers with Nonlinear Loads," *IEEE Trans. Antennas Propagat.*, **24**, 2, March 1976, pp. 131–139, doi: 10.1109/TAP.1976.1141328.
- [4] K.-C. Lee, "Two Efficient Algorithms for the Analyses of a Nonlinearly Loaded Antenna and Antenna Array in the Frequency Domain," *IEEE Trans. Electromagn. Compat.*, **42**, 4, November 2000, pp. 339–346, doi: 10.1109/15.902303.
- [5] M. Kotzev, X. Bi, M. Kreitlow, and F. Gronwald, "Equivalent Circuit Simulation of HPEM-Induced Transient Responses at Nonlinear Loads," *Adv. Radio Sci.*, **15**, September 2017, pp. 175–180, doi: 10.5194/ars-15-175-2017.
- [6] R. F. Harrington, "Time-Harmonic Electromagnetic Fields," *McGraw-Hill Book Company*, 1961, ISBN: 07-026745-6.
- [7] S. Ramo, J.R. Whinnery, and T. Van Duzer, "Fields and Waves in Communication Electronics," 3rd ed., *Wiley-India*, 2010, ISBN: 978-81-265-1525-7.
- [8] CST Microwave Studio Suite 2018, online: <http://www.cst.com>
- [9] LTSpice: Linear Technology, LTSpice IV, 2016, Available: <http://www.linear.com/designtools/software/>