Design of multi-locus transcranial magnetic stimulation coil with single driver

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Abstract

Transcranial magnetic stimulation (TMS) has been widely used for research and clinical applications of neurological disorders. In order to satisfy the requirement of clinical treatment and scientific research for understanding synergy between different brain regions, people have been trying to achieve multi-point stimulation of TMS. However, it is still not possible to use multi-locus stimulation of TMS at an acceptable cost. In this study, A design method of a multi-locus TMS stimulation coil with a single current driving device is proposed. The key concept of this method is to solve the stream function directly from the target electric field by Tikhonov regularization, then the corresponding coil shape can be obtained by drawing the contour lines of the stream function. At the same time, we have designed several multi-locus stimulation coils. By simulating the electric field generated by these coils, we have confirmed that this method can produce a multi-locus stimulation coil.

1 Introduction

Transcranial magnetic stimulation (TMS) is a non-invasive biological stimulation technology that has been widely used for research and clinical applications of neurological disorders[1]. The principle of TMS stimulation is based on Faraday's law, that is, a strong pulse magnetic field(1 Tesla in 300 us) generates a strong electric field to depolarize nerve cells. The original TMS coil is mainly designed to study the function of the brain, so the focality and stimulation depth of the coil is the most concerned. With the popularity of the clinical treatment, the TMS coil is also currently designed to generate a wide range of magnetic fields or the minimum energy consumption. Recently, to investigate the synergy between brain regions, some studies[2, 3] have used two TMS coils to stimulate the left and right motor cortex. Due to the needs of the research purpose, some studies have verified the feasibility of multi-focus TMS coils using simulations[4, 5], and in 2018 the first multi-focus TMS coils have been implemented by combining multiple basic mode coils [6]. However, Because different high-current (3kA) TMS drivers need to be controlled separately, the complexity of the entire system is high. In comparison, with the mature of 3D printing technology, designing a TMS coil that can generate fixed multi-locus is relatively more stable and available.

In order to meet the needs of designing different multi-locus TMS coil in the future. We propose a multi-locus TMS coil design method based on solving the stream function directly from the target induced Eddy current field.

2 Methods

Theory of Coil Design

Stream function (Ψ) is a scalar field orthogonal to the potential field (Φ), which is also the imaginary part of the complex potential function shown in eq 1. If this complex potential function is analytic in $\mathbb{R}^2$, then the Cauchy-Riemann Equation is satisfied, which is shown in eq. 2.

$$\Omega = \Phi + i\Psi \tag{1}$$

$$\begin{cases} \dfrac{\partial \Phi}{\partial x} = \dfrac{\partial \Psi}{\partial y} \\ \dfrac{\partial \Phi}{\partial y} = -\dfrac{\partial \Psi}{\partial x} \end{cases} \tag{2}$$

With these relationships, the current density vector $\vec{j}$ of TMS coil and electrical field $\vec{E}_{coil}$ in $\mathbb{R}^2$ can be derived as eq.3 shows.

$$\vec{E}_{coil}(x,y) = \frac{1}{\sigma}\vec{J}(x,y) = \left(-\frac{\partial \Phi}{\partial x}, -\frac{\partial \Phi}{\partial y} \right) \tag{3}$$

by substitute eq 2 into eq 3, The current density can be represented as:

$$\vec{J}(x,y) = \sigma \left(-\frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial x} \right) \tag{4}$$

The contour lines of stream function are the streamline which can be treated as the current density flow or the winding path of the coil[7]. In order to design the TMS coil based on target eddy current field E_{eddy}, First, we derive the relationship between the induced electric field in the brain and the stream function of coil analytically. The vector potential can be represented in eq.5.

$$\vec{A}(x,y,z,t) = \frac{\mu_0}{4\pi} \iint \frac{\vec{j}(x',y',t)}{|\vec{r}' - \vec{r}|} dx'dy' \tag{5}$$

Where the $\vec{A}$ is the vector potential at the target area and $\vec{j}$ is the current density at the TMS coil plane. The electrical

field inducted by TMS coil can be represented in eq 6.

$$\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \qquad (6)$$

In actual TMS stimulation, the first electrostatic potential energy will not be ignored due to the different conductivity of different brain tissues. But for coil design, in order to investigate the property of the coil itself, we assume that the coil is discharged in the air, so the first electrostatic term in eq 6 can be regarded as 0. In other word, $\varphi = 0$. Also in the TMS system, the current density $\vec{j}(x', y', t)$ can be represented as $\vec{j}(x', y') * sin(\omega_0 t)$. Then the induced electrical field is shown in eq. 7:

$$\vec{E}(x,y,z) = -\frac{\mu_0}{4\pi} \omega_0 \cos(\omega_0 t) \iint \frac{\vec{J}(x', y')}{|\vec{r}' - \vec{r}|} dx' dy' \qquad (7)$$

by representing $\sigma \frac{\mu_0}{4\pi} \omega_0 \cos(\omega_0 t)$ as constant C, and substitute eq 4 into eq. 7. The relationship between induced electrical field and stream function Ψ is:

$$\begin{cases} E_x(x,y,z) = C \iint \frac{\partial \Psi(x', y')}{\partial y'} \frac{1}{|\vec{r}' - \vec{r}|} dx' dy' \\ E_y(x,y,z) = -C \iint \frac{\partial \Psi(x', y')}{\partial x'} \frac{1}{|\vec{r}' - \vec{r}|} dx' dy' \end{cases} \qquad (8)$$

where the $E(x,y,z)$ is the electrical field at the target position and the $\Psi(x', y')$ is the stream function at the particular curved surface, The C is a constant. After obtaining this relationship, we can establish a discrete numerical method and inversely solve the stream function through the induced electric field.

Numerical implementation

Based on the derivation based on electromagnetic theory shown in eq.8, we established the relationship between the stream function and the induced electric field. This section mainly introduces how to implement by the finite-difference grid, and how to calculate the coil winding pattern by the target electric field. Although there are many forms of discretization of differences, in this work, considering the implementation complexity and accuracy requirements, we choose a forward difference which is:

$$\frac{\partial \Psi(x', y')}{\partial y'} = \frac{\Psi(i, j+1) - \Psi(i, j)}{\Delta y'} \qquad (9)$$

where $\Delta y'$ is the grid length in the y-direction of finite difference grid, and the $\Psi(i,j)$ represent the Ψ matrix value at row index i and column index j. The setting of Ψ matrix is shown in fig 1 (a).

Also in order to design a practical coil, as shown in fig 1 (b). We implemented a finite-difference grid on a plane. The grid is construed in a plane of 20cm*10cm with 100*50
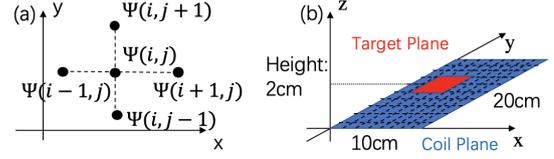
Figure 1. Diagram of Finite Difference grid. (a) is the relationship between index of matrix and cartesian coordinates. (b) is the geometric setup for the coil designing.

points, and the target electric field is set to 2cm away from the plane in an out-of-plane direction with 25*25 points. The reason for setting 2cm is that the magnetic field generated by the TMS coil needs to penetrate the 2cm thick skull and CSF on average. With this gird setting and eq.9, the discretization form of eq. 8 can be represented as eq.10.

$$E_x(x,y,2) = \frac{C}{\Delta y} \sum_{j=0}^{N_y} \sum_{i=0}^{N_x} \frac{\Psi(i,j+1) - \Psi(i,j)}{\sqrt{(x - i\Delta x)^2 + (y - j\Delta y)^2 + (2-0)^2}} \qquad (10)$$

With this discretization a linear equation $E = A\Psi$ can be obtained, In order to generate the actual coil, the solution of this ill-posed equation is calculated and obtained directly from the target electric field by Tikhonov regularization. In order to design a multi-locus TMS coil, the electric field strength of the desired stimulus location can be set to nonzero and the other locations to 0. The algorithm for solute stream function is shown in eq.11. The hyperparameter λ can be selected by L-curve[8].

$$\underset{\Psi}{\mathrm{argmin}} \|A\Psi - E\|_2^2 + \lambda \|\Psi\|_2^2 \qquad (11)$$

Finally, the corresponding coil shape can be obtained by drawing the contour lines of the stream function Ψ[7]. The number of turns of the coil is the number of contour lines. We set the number of turns of the coil to 10, this value is consistent with the general TMS coil. This numerical implementation program is written in Matlab (MathWorks co., Ltd).

3   Results

In this section, we have shown the design result of such multi-locus coils. For example, in order to design a TMS coil that can stimulate three target areas, first, by setting the target electric field, as shown in fig 2, we can set three nonzero regions to represent the locus of TMS coil. With this setup, we can obtain the three-locus TMS coil as shown in fig. 3. This coil achieved three similar electrical field regions in fig 3(b). At the same time, by analyzing the relative strength of the electric field, we found that although the ratio of the electric field strength is set to 1: 1: 1, the ratio of the electric field strength from left to right of the TMS coil actually designed is 1.00: 2.02: 1.20.
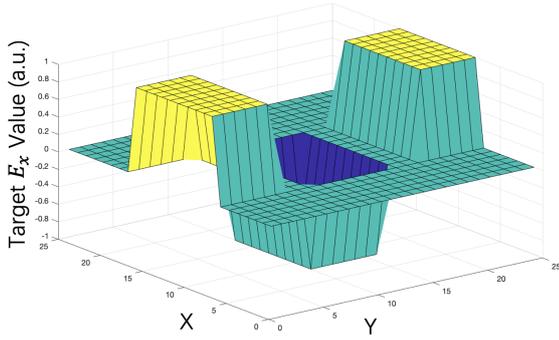
Figure 2. Design of Target electrical field, The value represents the intensity of E_x, positive value represent E-field along the x direction.

In addition to the three-locus TMS coils, we have designed four-locus and five-locus coils at the same time. Like the three-locus coils, we set the target electric field and then solve the corresponding coil winding. The four-locus coil is shown in fig 4, When designing this coil, we hope that there is no stimulus at the center point and a similar stimulus intensity around it. It may be used to stimulate the left and right brains at the same time without affecting the central region. The ratio of the intensity of each electric field from the clockwise direction is 1:1.03:0.97:0.95. At the same time, the relative electric field strength at the center is only 0.14, which is very low compared to the surrounding electric field strength. From this proportional relationship, The multi-locus coil that only stimulates the surrounding 4 points and does not stimulate the center has been achieved.

In order to show that our method can adjust the locus easily of the coil by setting the target electric field, we have implemented a five-locus coil that includes a central stimulus on the basis of a four-locus coil. We set the electric field value at the center point to obtain this coil. The coil and field distribution is shown in fig.5. The ratio of the intensity of each electric field strat from topleft and with the clockwise direction is 1:0.96:0.93:0.99, and at the center is 0.96. These values prove that our method can easily adjust the relative intensity of multifocal stimuli.

First of all, Our results show that a multi-locus TMS coil can be solved directly by setting the target electric field. At the same time, in our numerical implementation, the dimension of target E-field is much smaller than the dimension of the unknown variable, even it is a very ill-posed problem, but with proper regularization parameter selection, our method can solve the stream function and obtain a result. Therefore, for more control points or other surfaces, our method can also solve the stream function to obtain the optimal coil.

In the results, we only show 3, 4 and 5 locus TMS coils respectively. But for any number of locus TMS coils, our method can quickly solve the corresponding coil winding
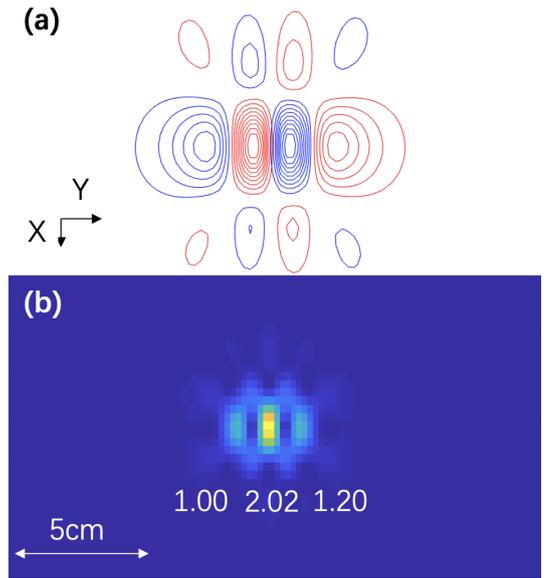
Figure 3. Diagram of TMS coils that can simultaneously stimulate three positions. (a)The TMS coil generated by our numarical method, red and blue color indicate that the current direction in the coil is clockwise and counterclockwise, respectively.(b)Distribution of electric field strength generated by this coil
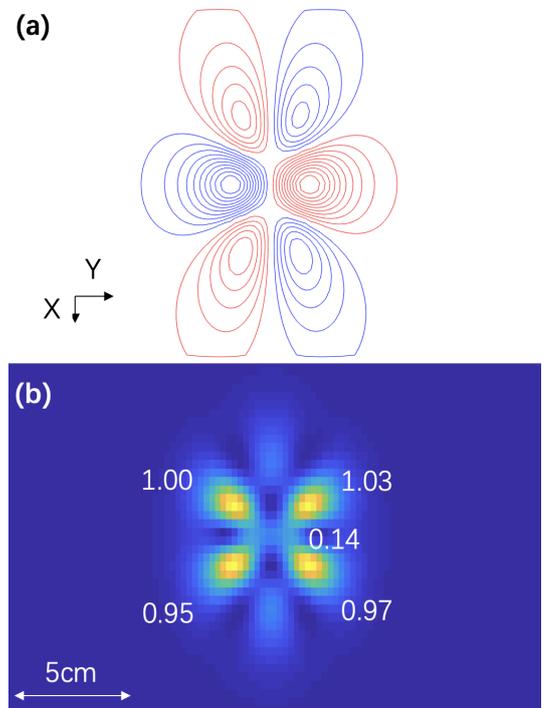
Figure 4. Diagram of clover-like TMS coils that can simultaneously stimulate four separate regions **without** center region. (a) coil wiring pattern. (b)Distribution of electric field strength generated by this coil.