Dyadic Green’s Function Studies for the Three-shell Head Model

Grigoris P. Zouros(1), Georgios D. Kolezas(1), George Fikioris(1), and Nikolaos L. Tsitsas*(2)
(1) School of Electrical and Computer Engineering, National Technical University of Athens, 15780 Athens, Greece
(2) School of Informatics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Abstract

In this work, we employ analytical expressions for the calculation of the dyadic Green’s function (DGF) of the isotropic three-shell head model, due to an infinitesimal electric dipole located in the internal core that corresponds to the brain. We compute the electric field obtained from this DGF and compare and validate our analytical results with HFSS commercial software when the dipole is inside a single sphere. Then, we numerically examine the behavior of the electric field due to the placement of the dipole inside a three-shell spherical head model. Finally, we discuss how the isotropic model can be extended to the more complicated anisotropic one by considering a volume integral equation approach.

1 Introduction

The accurate computation of the interaction of electromagnetic waves with biological tissues has direct biomedical applications in modern deep brain stimulation (DBS) treatment, with the latter being used to treat Parkinson’s disease [1], depression [2], or in applications like pacemakers [3]. DBS is achieved through surgical implants of electrodes, which act as electrical sources and render local parts of the brain inactive for treatment needs. The measurement of the electric or magnetic activity of this internal source interaction is carried out through electroencephalography (EEG) or magnetoencephalography (MEG). In biological imaging, EEG or MEG is performed via multilayered spherical models, where, in each layer, the conductivity changes significantly. The most realistic head model routinely adopted is the three-layered one, consisting of the brain (the core in the three-layered configuration), the skull and the scalp (the outer layer) [4], while other, more complicated models exist, in which conductivity is considered anisotropic [5].

The use of finite element and finite difference methods revealed that the errors obtained from the spherical shape are of the same order of magnitude with those obtained due to the inaccurate knowledge of the electrical properties of the head [6]. Numerical methods, however, face high computational load due to the large discretization they require, otherwise they cannot accurately approximate the multilayered and anisotropic properties of the head, especially when the stimulation is within the region of inhomogeneity (i.e., inside the brain) [7]. This is because, for accurate computations, a minimum number of samples per wavelength is required in each individual layer. For multilayered structures, this results in a high number of finite elements, and as a consequence, in high memory consumption and time-consuming execution of the algorithm. Therefore, what is commonly adopted in practice, is to reduce the discretization to obtain a low computational time, but with an impact on reduced accuracy.

In this work, we focus our study on the three-shell model [4], and extract analytical formulas for expressing the DGF due to an infinitesimal electric dipole located in the internal core, which corresponds to the brain [8]. In particular, the DGF of the multilayered configuration is decomposed in two terms: a singular term that corresponds to the DGF with constitutive parameters of the respective isotropic layer, and a regular dyadic term to be computed. We present and discuss numerical results on the electric field generated by the dipole when the latter is placed inside the core-brain of the three-shell model. To establish validity, the electric field obtained from the analytical solution is compared with HFSS, first for the simplified single sphere, and second for the three-layered sphere.

2 DGF Formulation for the Head Model

We consider a spherical human head model consisting of three concentric spheres, each having radius \( r = a_p \) (\( p = 1, 2, 3 \)), with \( a_3 < a_2 < a_1 \). The spheres divide the head into three homogeneous layers, namely, \( V_3 \) (\( 0 \leq r < a_3 \)), \( V_2 \) (\( a_3 < r < a_2 \)), and \( V_1 \) (\( a_2 < r < a_1 \)). Each layer \( V_p \) has permittivity \( \varepsilon_p \) and permeability \( \mu_p \). The surrounding medium is free space, which is assumed as an unbounded outer layer \( V_0 \) (\( r > a_1 \)), with permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \). The configuration is excited by an infinitesimal electric dipole with dipole moment \( \mathbf{p} \), located in layer \( V_3 \). A time dependence of the form \( \exp(-i\omega t) \) is assumed and suppressed throughout.

The electric field induced in layer \( V_p \) by the electric dipole located at \( \mathbf{r} = \mathbf{r}_d \) in layer \( V_3 \) is given by [8]

\[
E^p(\mathbf{r}; \mathbf{r}_d) = i\omega\mu_p\tilde{G}^p(\mathbf{r}; \mathbf{r}_d) \cdot \mathbf{p}, \quad \mathbf{r} \in V_p, \quad (1)
\]

where \( \tilde{G}^p \) is the component of the configuration’s DGF in layer \( V_p \). For layers \( V_p \) (\( p = 0, 1, 2 \)), \( \tilde{G}^p \) can be expressed
As a series of spherical vector wave functions as
\[
\mathbf{G}^r(r,r') = \frac{i k_0}{4\pi} \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{\sigma=\pm1} \frac{2n+1}{n(n+1)} \epsilon_{m} (n-m)! \times \left\{ \mathbf{M}^{(1)}_{\sigma mn}(k p, r) \left[ \alpha_{p}^{\sigma} \mathbf{M}^{(3)}_{\sigma mn}(k_{3}, r') + \beta_{p}^{\sigma} \mathbf{M}^{(3)}_{\sigma mn}(k_{3}, r') \right] + \mathbf{N}^{(1)}_{\sigma mn}(k p, r) \left[ \delta_{p}^{\sigma} \mathbf{N}^{(3)}_{\sigma mn}(k_{3}, r') + \delta_{p}^{\sigma} \mathbf{N}^{(3)}_{\sigma mn}(k_{3}, r') \right] \right\}.
\]

In (2), \(\mathbf{M}^{(j)}_{\sigma mn}(k p, r)\) and \(\mathbf{N}^{(j)}_{\sigma mn}(k p, r)\) are the even/odd spherical vector wave functions of the first \((j = 1)\) and third \((j = 3)\) kind [9], \(k p = \omega \sqrt{\varepsilon_p \mu_p}\), \(k_3 = \omega \sqrt{\varepsilon_3 \mu_3}\), and \(\epsilon_{m} = 1\) for \(m = 0\), while \(\epsilon_{m} = 2\) for \(m \neq 0\).

In the case of layer \(V_3\) which contains the dipole source, the DGF component is given by
\[
\mathbf{G}^3(r,r') = \mathbf{G}^r(r,r') + \mathbf{G}^{sec}(r,r').
\]

In (3), \(\mathbf{G}^{pr}\) is the primary contribution that corresponds to the DGF of the unbounded medium filled with the material of layer \(V_3\), and has the expression [8]
\[
\mathbf{G}^{pr}(r,r') = -\frac{\mu_0}{k_0^2} \delta(r-r') + \frac{i k_0}{4\pi} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \sum_{\sigma=\pm1} \frac{2n+1}{n(n+1)} \epsilon_{m} (n-m)! \times \left\{ \mathbf{M}^{(3)}_{\sigma mn}(k_{3}, r) \mathbf{M}^{(1)}_{\sigma mn}(k_{3}, r') + \mathbf{N}^{(3)}_{\sigma mn}(k_{3}, r) \mathbf{N}^{(1)}_{\sigma mn}(k_{3}, r') \right\}.
\]
three-shell sphere, when the dipole is $z$-oriented and located at $r/a_1 = r_5/a_1 = 0.2$. Values of parameters: $k_0 a_1 = 0.6 \pi$, $a_2/a_1 = 0.95$, $a_3/a_1 = 0.8$, $\varepsilon_1 = 4$, $\varepsilon_2 = 5$, $\varepsilon_3 = 2.54$, $\mu_1 = \mu_2 = \mu_3 = 1$. Blue curve: analytical solution; red curve: HFSS.

In Fig. 2 we plot the electric field components $E_r$ and $E_\theta$ for the same configuration as in Fig. 1 and for the same observation path, but when the dipole is located at $r/a_1 = r_j/a_1 = 0.2$. Obviously the agreement with HFSS is evident. Since $|E_\theta|$ is tangential to the spherical surface, it is continuous as it crosses the $r/a_1 = 1$ point, while $|E_r|$ clearly depicts a step discontinuity. The $|E_\theta|$ component is from one up to two orders of magnitude smaller than $|E_r|$, $|E_\theta|$, thus negligible, and for that reason is not depicted.

Having established the validity for the single sphere, we now proceed to the three-shell model. The values of the parameters used are gathered in the caption of Fig. 3, with the latter depicting results for $|E|$ for the same observation points as in Fig. 1. Now, due to the change of layers at $r/a_1 = a_3/a_1 = 0.8$ and at $r/a_1 = a_5/a_1 = 0.95$, there are two discontinuities at these points, in addition to the one at the outer interface at $r/a_1 = 1$. It should be noted that the discontinuity at $r/a_1 = 0.95$ is not clearly depicted in Fig. 3 and one should zoom in to distinguish it. Again, the comparison with HFSS clearly shows the validity of the analytical solution.

Finally, to complement the study for the three-layered sphere, in Fig. 4 we present plots for the magnitude of $E_r$ and $E_\theta$ components. Now, all three discontinuities of the $E_r$ are clearly observed, while $E_\theta$ is continuous as it passes through all layers. These properties are well captured by both the analytical solution and HFSS, thus verifying the validity of the proposed analytical method.

4 Conclusion and Future Work

We have computed and examined the behavior of the electric field in a three-shell spherical head model, excited by an infinitesimal electric dipole located in the core of the configuration. This task has been carried out by first computing the DGF in each layer. The proposed analytical solution has been validated versus HFSS, showing a very good agreement.

Extensions of the present work consist the use of more than three layers to model the head, and the improvement of the simplified model presented here to the more complicated anisotropic one [5]. Obviously, this latter task is quite challenging. A workaround may be the consideration of a volume integral equation method for the evaluation of the unknown DGF, while the anisotropic multilayers will be handled by integration in a similar manner as in the electromagnetic scattering by anisotropic spheres [10].

5 Acknowledgements

This research is co-financed by Greece and the European Union (European Social Fund-ESF) through the Operational Programme “Human Resources Development, Education and Lifelong Learning 2014–2020” in the context of the project “Deep brain stimulation modeling through accurate electrodynamic techniques” (MIS 5047942).
References


