Cross-Sectional Thermoacoustic Imaging Using Multi-Layer Cylindrical Media

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Abstract

For cross-sectional two-dimensional thermoacoustic imaging of breast and brain, we explored solution of the wave equation using layered tissue model consisting of concentric annular layers on a cylindrical cross-section. To obtain the forward and inverse solutions of the thermoacoustic wave equation, we derived the Green’s function involving Bessel and Hankel functions by employing the geometrical and acoustic parameters (densities and velocities) of layered media together with temporal initial condition, radiation conditions and continuity conditions on the layers’ boundaries. The image reconstruction based on this approach involves the layer parameters as the apriori information which can be estimated from the acquired thermoacoustic data. To test and compare our layered solution with conventional solution based on homogeneous medium assumption, we performed simulations using numerical test phantoms consisting of sources distributed in the layered structure.

1 Introduction

In thermoacoustic imaging, short-duration RF or microwave or laser pulses are applied to the tissue, where the absorbed pulsed energy is converted to heat which causes thermoelastic expansion and thus acoustic emission. Thermoacoustic imaging is based on reconstruction of source distribution, which is function of electromagnetic or optical absorption, thermal and acoustic parameters, from acoustic data acquired by transducers located over the surface enclosing the tissue to be imaged. Thermoacoustic imaging is a non-ionizing, new modality for medical applications such as early detection of cancer and follow up during the therapy.

Most of the research studies reported in the literature were based on inverse solution of thermoacoustic wave equation using homogeneous medium assumption [1, 2]. In a recent study, the boundary conditions for thermoacoustic imaging have been investigated by Yang and Wang [5]. In a more recent study, Schoonover and Anastasio [4] have presented an inverse solution based on piecewise homogeneous structure consisting of layers between the source distribution and the transducers. Here our approach is based on the fact that two-dimensional cross-sections of many organs and tissue structures such as breast and brain can efficiently be modelled by piecewise homogeneous layers for accurate inverse solution of the thermoacoustic wave equation. In this study, for cross-sectional two-dimensional thermoacoustic imaging of breast and brain, we explored solution of the wave equation using layered tissue model consisting of concentric annular layers on a cylindrical cross-section. To test and compare our layered solution with conventional solution based on homogeneous medium assumption, we performed simulations using numerical test phantoms consisting of sources distributed in the layered structure.

2 Thermoacoustic Wave Equation

We consider a region containing N-concentric cylindric layers with different acoustic properties in the space \( \mathbb{R}^3 \). The z-cross section of configuration is depicted in the Figure 1. We call the region between consecutive boundaries \( S_{m-1} \) and \( S_m \) as \( R_m \). Suppose there is a cylindrical transducer, called as \( S_N \) in Layer \( N \) closing the other regions as in the Figure 1. We call the volume covered by transducer \( S_N \) as \( V \). We want to determine the source distribution of the region covered by transducer.

The acoustic waves are measured by the transducer for a sufficiently long time interval so that the waves emitted from every source location reach to the transducer. When the regions are different, there will be reflections and transmissions at the layers’ boundaries \( S_m \) for \( 1 \leq m \leq N - 1 \). The thermoacoustic wave propagation is governed by the nonhomogeneous wave equation

![Figure 1. Cross-section of the cylindrical structure consisting of multiple concentric layers with different parameters.](image-url)
\[ \nabla^2 p(r, t) - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -p_0(r) \delta'(t) \]  

(1)

with 2(N-1) boundary conditions

\[ p_m(r, t) = p_{m+1}(r, t) \bigg|_{r \in S_m} \]

and

\[ \frac{1}{\rho_m} \frac{\partial p_m(r, t)}{\partial n} = \frac{1}{\rho_{m+1}} \frac{\partial p_{m+1}(r, t)}{\partial n} \bigg|_{r \in S_m} \]

on each boundary \( S_m \). Here, \( p, p_m \) and \( p_{m+1} \) are the acoustic waves, \( p_0(r) \) is the source term, \( c \) is the acoustic speed and \( \rho_m \) and \( \rho_{m+1} \) are the densities for Layer \( m \) and Layer \( m + 1 \), respectively. Also, as a nature of the problem, nonhomogeneous thermoacoustic wave equation must satisfy the following initial condition

\[ p(r, 0) = p_0(r). \]  

(2)

### 2.1 Forward Solution

The equation (1) in frequency domain corresponds to the nonhomogenous Helmholtz equation

\[ \nabla^2 P(r, w) + k^2 P(r, w) = -iw p_0(r), \]  

(3)

where \( P(r, w) \) is the temporal Fourier Transform of \( p(r, t) \). In this study, we make use of Green’s function to solve nonhomogeneous wave equation (3). Green’s function is the unit impulse response of a medium. When a point source is located at \( r = r' \), Green’s function is the solution of the equation

\[ \nabla^2 G(r'; r, w) + k^2 G(r'; r, w) = \delta(r - r') \]  

(4)

where \( \delta(\cdot) \) is the Dirac delta function. It is convenient to study in cylindrical coordinates for the N-layered cylindrical configuration. So we transform the equation (4) in to cylindrical coordinates and derive outgoing Green’s function by using boundary conditions and Radiation conditions. After that we obtain the forward solution of thermoacoustic wave equation in a layered media. If the source distributions in a layered media is given by

\[ p_0(r) = p_0'(r), \quad r \in R_i \]

then the forward solution is

\[ \hat{P}(r, w) = -iw \int_{r'} p_0(r') G^{out}(r'; r, w) dr' \]  

(5)

where \( V^* \) is the whole space, \( G^{out} \) is the outgoing Green’s function corresponding to layered structure.

### 2.2 Inverse Solution

In an inverse source problem, \( p_0(r) \) is to be reconstructed given that acoustic field is measured by the transducer and is known on the surface \( S_0 \). Inverse source problem has been studied for homogeneous medium by Wang et al. [1] for specific measurement geometries: two parallel planes, an infinitely long circular cylinder and a sphere, and this solution extended to the arbitrary measurement geometry by Idemen and Alkumru [2]. In these studies, the source distribution inside the medium is determined by the following integral equation:

\[ p_0(r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(r_s, w) \frac{\partial G^m_n(r; r_s, w)}{\partial n_s} dSdw \]  

(6)

where \( S \) is a measurement surface and \( G^m_n \) is a free space Green function.

For N-layered cylinder configuration, we prove that

\[ p_{i0}(r) = \frac{\rho(r)}{2\pi} \int_{-\infty}^{\infty} P(r_s, w) \frac{\partial G^{in}_i(r; r_s, w)}{\partial n_s} dSdw, \quad r \in R_i \]

(7)

where \( P(r_s, w) \) is the acoustic pressure measured on the surface \( S_N \), \( G^{in}_i \) is the corresponding incoming Green’s function \( (G^m = (G^{in})^*) \) and \( \rho(r) \) is a density function such that

\[ \rho(r) = \rho_i, \quad r \in R_i. \]  

(8)

for \( 1 \leq i \leq N \).

### 3 Numerical Simulations

To test and compare our layered solution with conventional solution based on homogeneous medium assumption, we performed simulations using numerical test phantoms consisting of thermoacoustic sources distributed in the layered structure. In the simulations, we generated synthetic data by using forward solution of thermoacoustic wave equation for the layered media. Then we reconstructed the thermoacoustic source distribution from this data using our layered inverse solution and the existing homogeneous inverse solution. Here we present an illustrative test result in the figure, where the numerical phantom and the reconstructed inverse source distribution are displayed. The test results ensure that the homogeneous inverse solution produces incorrect source locations and poor point-spread-functions. Our layered solution produces point-spread-functions with relatively narrower main-lobe and lower side-lobes. Also, the numerical test results show that the layered solution enable reconstruction of both the low and high contrast cyst-like structures with more accurate features compared to the homogeneous solution.
Figure 2. In numerical simulation skin, fat tissue acoustic parameters and water acoustic parameters are used. Layer 1, 2 and 3 considered as fat, skin and water media, respectively.

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4 Acknowledgements

This work was supported by TUBITAK of Turkey through ARDEB-1003 Program under Grant 213E038.

References


