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Differential Total Electron Content Tomography of the Ionosphere

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1 Extended Abstract

The ionosphere is the prevailing nuisance of the deep radio interferometry imaging of the universe. The spatially and temporally varying electron density of the ionosphere causes complex amplitude distortions to passing wavefronts, becoming more severe at lower frequencies. These translate into both shape and intensity distortions of radio sources, and in some cases completely render data useless. Borrowing ideas from seismic waveform inversion techniques [1], we present here a method for inverting the total electron content (TEC), for the spatial distribution of the electron density. To start, we represent the electron density in lognormal, $\mu(\vec{x}) = \log(n_e(\vec{x})/K)$, and then define the differential total electron content (dTEC) for a given ray, $\mathscr{R}^i$ from an antenna,

$$g^i(\mu(\vec{x}),\rho) \equiv dTEC(\mathscr{R}^i) = \int_{\mathscr{R}^i} ds(\vec{x}) K \exp \mu(\vec{x}) - \rho. \tag{1}$$

Equation 1 is the forward model, equal to the TEC minus a baseline, ρ. We then derive the Fréchet derivative which is the tangent application, and integral kernel that maps changes in the model manifold to changes in the data manifold,

$$G^i(\vec{x})_{[\mu,\rho]} = \partial_{[\mu,\rho]} g^i(\mu(\vec{x}),\rho) = (K \exp \mu(\vec{x}), 1) \tag{2}$$

Defining a weighted misfit function $2S(\mu(\vec{x}),\rho) = |dTEC(\mathscr{R}^i) - g(\mu(\vec{x}),\rho)|_D^2 + |\mu_{\text{prior}} - \mu|_\mu^2 + |\rho_{\text{prior}} - \rho|_\rho^2$, we can linearize with Eq. 2 and derive an iterative inversion solution,

$$\mu_{n+1}(\vec{x}) = \mu_n(\vec{x}) - \int_{\mathscr{R}^i} ds(\vec{x'}) C_\mu(\vec{x},\vec{x'}) G^i_{\mu_n}(\vec{x'}) T_{ij} (g^j(\mu_n(\vec{x}),\rho_n) - dTEC(\mathscr{R}^j)), \tag{3}$$

where $C_\mu(\vec{x},\vec{x'})$ is a desired *a priori* estimate of the covariance of the electron density, which we often take an exponential, and,

$$(T_{ij})^{-1} = S^{ij} = C_D^{ij} + \sigma_\rho^2 + K^2 \int_{\mathscr{R}^i} ds(\vec{x}) \int_{\mathscr{R}^j} ds(\vec{x'}) \exp \mu(\vec{x}) C_\mu(\vec{x},\vec{x'}) \exp \mu(\vec{x'}), \tag{4}$$

where C_D is the covariance of the measured dTEC and σ_ρ^2 is the estimated variance of the baseline.

Equation 4 can be integrated numerically with a tricubic interpolator, and then inverted quite quickly. Equation 3 likewise can be integrated numerically, and it corresponds to weighted backprojections of the data residuals. It should be noted that at no point does the covariance matrix or the kernel need to be evaluated at everywhere and stored in a matrix in computer memory, as this would be prohibitive.

Determining the rays requires ray tracing. As changes in the electron content model also causes changes in the refractive index of the ionosphere, this can lead to large non-linearities. We find that refractive effects are minimal to first order, which makes this problem quasi-linear and solvable using iterative applications of Eq. 3.

Clear benefits of this method are that 1) the ionosphere's structure is measured and can be studied, 2) phase screens can be created easily for all directions (even those lacking a bright calibrator), and 3) the method can largely be parallelised. A draw back at the moment is that this technique requires good measurements of dTEC in order to perform the inversion, which at the moment requires a facet-based calibration, and good separation of delay error ($\propto \nu$) and propagation error ($\propto \nu^{-1}$).

References

[1] A. Tarantola, "Inversion Problem Theory and Methods for Model Parameter Estimation," *Siam*, 2nd ed., April 2005.