On the resonant interaction of relativistic electrons with oblique whistler-mode waves.

Abstract

1. Introduction

The Earth’s radiation belts were discovered experimentally in 1958 by Explorer mission lead by Vernov. Since the discovery of the radiation belts the particle energization and precipitation processes in the Earth’s magnetosphere are one of the most important problems of the physics of the Earth’s magnetosphere. The Hamiltonian approach to the description of the resonant interaction is to use the following Hamiltonian:

\[ H = mc^2 \sqrt{1 + \frac{1}{m^2c^2} \left( P + \frac{\mathbf{e}}{c}(A + A_0) \right)} \]

where:\n- \( A \) is the vector potential of the Earth’s magnetic field, \( A_0 \) is the vector potential of the initial external magnetic fieldovan, \( P \) is the particle momentum in the initial external magnetic field, \( E \) is the particle energy in the initial external magnetic field.

2. Layout

In Section 3, we present the main equations of the theory of such an interaction under the threat to the dangerous effects on the electronic equipment on the satellites posed by highly energetic electrons. Since the discovery of the radiation belts the energetic electrons of the outer radiation belt have become one of the most important mechanisms play an important role in the dynamics of the Earth’s radiation belts located at 3.5 Earth radii (RE) is highly important. Electron dynamics and resonance condition

3. Particle dynamics and resonance condition

The motion of trapped electron in the external electromagnetic field is given by the Hamiltonian:

\[ H = mc^2 \gamma_0 \left( \mathbf{p} + \frac{\mathbf{e}}{c} \mathbf{A} \right) - \frac{e}{m} \mathbf{A} \cdot \mathbf{v} \sin \xi \]

where:
- \( \gamma_0 = \frac{1}{\sqrt{1 - \frac{2 \mu \omega_c}{m c^2} + \frac{P_{\|}^2}{m^2 c^2}}} \)
- \( \xi \) is the angle with respect to the Earth’s magnetic field.
- \( \sqrt{1 + \frac{2 \mu \omega_c}{m c^2} + \frac{P_{\|}^2}{m^2 c^2}} \)
\[ \xi_n = \int k_{\parallel} ds + n\nu - \omega t \]

\[ p_{\parallel}^{(n)} = m \frac{\omega_0(p_{\parallel}, s, \mu) - n\omega_0(s)}{\xi_n(s)} \]

\[ h_n = nH - \mu \omega \]

\[ F[\gamma_0] = \gamma_0^2 - 2n \frac{\omega_c}{\omega} \gamma_0 + \left( 2 \frac{\omega_c}{\omega} \frac{h_n}{mc^2} - 1 \right) - \left( \frac{n\omega_c}{ck_{\parallel}} \right)^2 \frac{1}{1 - \omega^2/k_{\parallel}^2c^2} = 0 \]

\[ \beta = \frac{k_{\parallel}^2 m}{\gamma_0 c^2} \epsilon A_\omega V_n \left( 1 - \frac{\omega^2}{k_{\parallel}^2c^2} \right) \]

4. Numerical simulations
Figure 2. The wave parameters on the particle trajectory \((L = 3.5)\) as a function of geomagnetic latitude. \(N\) is the refractive index, \(\theta\) is the wave normal angle.

Having calculated the wave parameters along the particle trajectory, we can find the resonance momenta for the particles with different energies in different resonances. The results for the first cyclotron resonance are presented in Figure 3.

Figure 3. The particle resonance parameters as functions of geomagnetic latitude \((L = 3.5)\). Upper panels: Lorentz factor, which is dimensionless energy. Lower panels: inhomogeneity parameter and effective amplitude. Green color corresponds to the negative solution for the resonance momentum (particle moves towards the wave), magenta color corresponds to the positive solution (5).

When \(\beta < \alpha\), the particle escapes from the resonance. Figure 3 presents the following situation. Initially, there is a particle moving from positive latitudes to the equator (and towards the wave). When the effective amplitude becomes larger than inhomogeneity parameter, the particle can be trapped into the resonance. Then, the particle energy variation follows the resonance Lorentz factor. One can see that the energy increases towards the equator.

Let us discuss the left panels of Figure 3. They correspond to the case when there is no resonance turning point on the trapped particle trajectory. The inhomogeneity parameter is almost symmetric with respect to the equator. But the effective amplitude is not: after the equator it starts oscillating. Oscillations of the effective amplitude lead to the particle escape from the resonance, preventing the energy exchange between the particle and the wave.

But after crossing the equator such exchange would lead to particle deceleration, as can be seen from upper left panel of Figure 3. So such an asymmetry of the effective amplitude with respect to the equator results in efficient energization of the trapped particles. The opposite holds for the case, when there is a resonance turning point on the particle trajectory (see right panels of Figure 3). Now, the effective amplitude is asymmetric with respect to the resonance turning point, so the trapped particle escapes the resonance after the turning point, and is not being energized further. It can be shown that such an asymmetry arises for the relativistic effective amplitude due to the obliquity of wave propagation. The effective amplitude \((6)\) depends on the Bessel functions of dimensionless gyroradius, and this dependency gives rise to the oscillations of the effective amplitude.

5. Conclusion

We investigated the structure of the resonant surface for relativistic resonant interaction of electrons with oblique whistler-mode wave. There is an asymmetry of the effective amplitude with respect to equator and resonance turning point. This asymmetry leads to efficient trapped particle energization when there is no resonance turning point, but reduces the efficiency of relativistic turning acceleration process.

6. Acknowledgements

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7. References


