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Adaptive Beamforming Synthesis for Thinned Fractal Antenna Arrays

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Abstract

Fractal antenna array is a compact and multi-band antenna-element design technique. One of the major challenges of this array design is the possibility of radiation pattern synthesis. In this work, the Least Mean Square (LMS) algorithm is investigated as an adaptive beamforming method in the design of thinned fractal antenna arrays for the first time. This array design is developed by estimating the active fractal array elements corresponding to the desired radiation pattern main lobe and nulls, while maintaining the same Side Lobe Level (SLL) at multiple frequency bands. The capability of the proposed method is demonstrated by considering linear cantor and Sierpinski carpet fractal antenna arrays. The results show that the proposed antenna array design is much superior in terms of multi- band frequency operation, array element reduction, and beamforming accuracy. This reveals the effectiveness of the proposed algorithm as a promising design technique in the smart antenna technology.

1. Introduction

One of the prime purposes in wireless communication systems is to design wideband or multi-band small-size antennas. This type of antenna behavior can be achieved by self-scalable fractal antenna array [1]. Fractal is a self-similar geometry with a non-integer dimension, self-similar means that a magnified section of the shape has the same structure as the whole geometry [2]. Only few design methodologies are available for the generation of self -scalable linear, planar, and conformal fractal antenna arrays [3]. Concentric circular ring subarray generator is the most commonly used methodology for the generation of linear and planar fractal array antennas. This method can be used to design multiple types of fractal array antennas such as cantor linear, planar, Sierpinski triangular, and hexagonal fractal antenna arrays [4]. One of the most important applications of fractal antenna design is the thinned fractal arrays, where the elements are arranged in fractal geometry to obtain wideband or multi-band performance with small number of elements [5].

Another one of the main targets in modern communication systems is the array pattern synthesis, i.e., the design of antenna arrays with dynamically shaped radiation pattern according to certain requirements, which are called smart antennas [6, 7]. These antenna arrays allow the peak of the radiation main lobe to be steered towards a desired signal, while the radiation pattern nulls can be formed in the direction of the undesired signals or the interference. All elements of the smart antenna arrays should be weighted in order to adapt to the desired pattern requirements. Several adaptive beamforming techniques were developed to calculate these excitation weights for the normal antenna array case [8]. To the author's knowledge, the adaptive beamforming has not been used before in fractal thinned array synthesis.

The aim of this paper is to design multi-band thinned fractal antenna arrays with adaptive beamforming. The organization of this paper is as follows. Section 2 introduces the fractal cantor linear array and the Sierpinski carpet arrays. Section 3 illustrates the Least Mean Square (LMS) technique for adaptive beamforming. Section 4 discusses the results of the proposed thinned fractal array design with and without adaptive beamforming. The conclusions are provided in Section 5.

2. Cantor and Sierpinski Antenna Arrays

The basic triadic Cantor array is one of the most common design methods to construct fractal arrays and it can be formed by starting with a three-element generating subarray, and then repeating it over P scales of growth [4]. The generator, in this case, is a subarray where elements are uniformly arranged with turning off the center element, i.e., 101. The array is formed recursively by replacing 1 by 101 and 0 by 000 at each growth stage of the construction, i.e., the array pattern will be 101 000101 at the second growth stage (P = 2), and it will be 101000101000000000101000101 at the third stage (P = 3) and so on.

The array factor of the three-element generating subarray with the representation 101 is given by

$$GA(\psi) = 2cos(\psi) \tag{1}$$

where ψ is defined as $\psi = kdcos(\theta) + \beta$, d is the spacing of the generator array, θ is the angle between the direction of propagation and the axis of the array, β is the progressive phase-shift of the generator array, and $k = \frac{2\pi}{\lambda}$ is the wavenumber.

Assuming an expansion factor of three $\delta = 3$, the array factor can be expressed as:

$$AF_p(\psi) = \prod_{p=1}^{P} COS(3^{p-1}\psi) \qquad (2)$$

where P is the number of growth stages.

A Sierpinski carpet is a two-dimensional version of the Cantor set [9], and it can be utilized for thinning planar fractal array. An example of the Sierpinski generating subarray is

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Figure 1 illustrate the first four stages of growth for Sierpinski carpet fractal antenna array. Assuming an expansion factor of δ = 3 and spacing of $d_x = d_y =$ /2, the normalized fractal array at stage P can be expressed as:

$$AF_p(u_x, u_y) = \prod_{p=1}^{P} COS(3^{p-1}\pi u_x) + COS(3^{p-1}\pi u_y) + 2COS(3^{p-1}\pi u_x)COS(3^{p-1}\pi u_y) \qquad (3)$$

Where

$$u_x = \sin(\theta)\cos(\phi) - \sin(\theta_o)\cos(\phi_o)$$
$$u_y \sin(\theta)\, s\,in(\phi) - \sin(\theta_o)\sin(\phi_o).$$

θ_0 and ϕ_o are the steering angles.

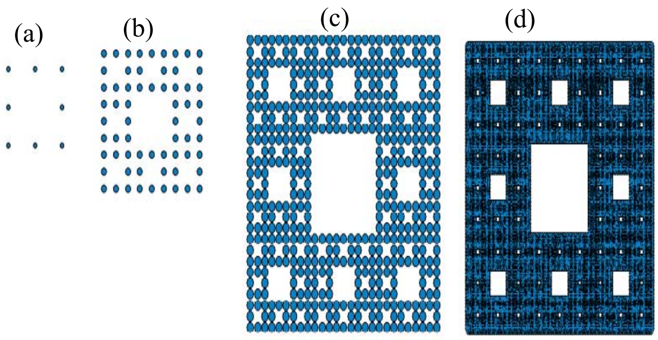


Figure 1. The first four growth stages for self-scalable Sierpinski carpet arrays: (a) P = 1, (b) P = 2, (c) P = 3, and (d) P = 4.

3. LMS Adaptive Beamforming

In the Least-Mean-Square (LMS) algorithm, the steepest-descent method recursively computes and updates the array element weights [6]. The element weights are updated using the instantaneous gradient vector ∇J(n) instead of the matrix inverse operation as follows:

$$W(n+1) = W(n) + \frac{1}{2}\mu[-\nabla J(n)] \qquad (4)$$

where μ is the convergence factor which controls the speed of convergence and its value lies between 0 and 1 [6].

The instantaneous gradient vector ∇J(n) is given by

$$\hat{\nabla} J(n) = -2\hat{P}(n) + 2\hat{R}(n)W(n) \qquad (5)$$

where $\hat{R}(n) = X(n)X^H(n)$ is the correlation matrix of the array inputs X(n) and $\hat{P}(n) = X(n)d^*(n)$ is the cross-correlation vector between the array inputs X(n) and the desired signal d(n).

Substituting from (5) into (4), we get

$$\begin{aligned} \widehat{W}(n+1) &= \widehat{W}(n) + \mu[\widehat{P}(n) - \widehat{R}(n)\widehat{W}(n)] \\ &= \widehat{W}(n) + \mu X(n)[d^*(n) - X^H(n)\widehat{W}(n)] \qquad (6) \\ &= \widehat{W}(n) + \mu X(n)e^*(n) \end{aligned}$$

where *e(n)* is the error estimate. The Mean Square Error (MSE) is decreasing with the step size decrease. In order to ensure convergence for the array weight vector, the step size should lie in the range 0< μ <1/ $_{max}$ [10].

4. Results and discussion

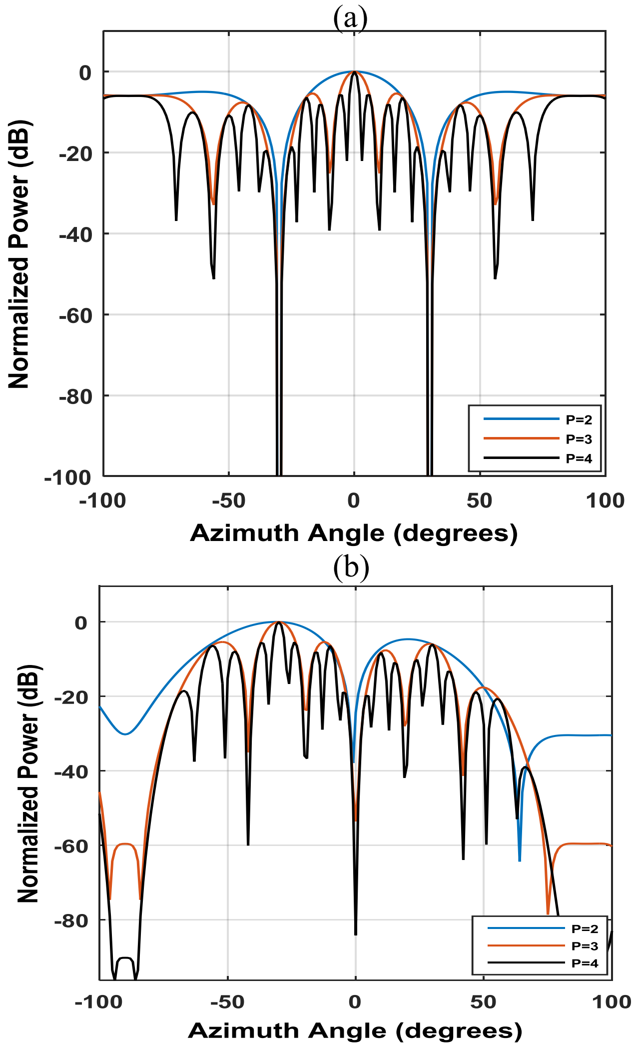


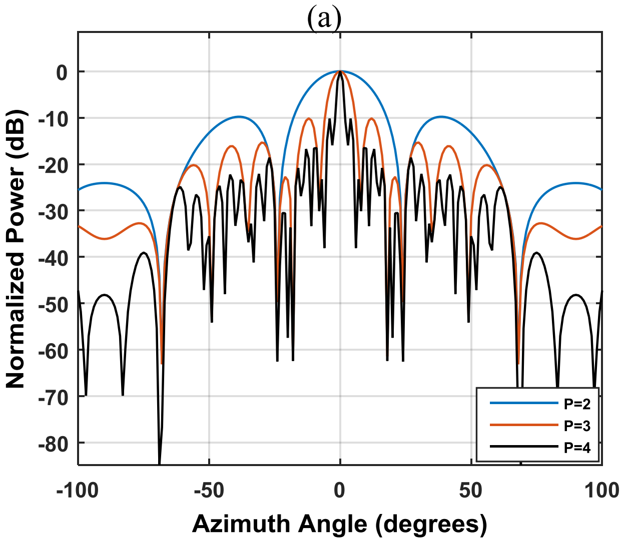
Figure 2.The normalized patterns of linear cantor fractal antenna arrays (a) without beamforming and (b) with beamforming at frequency 100 MHz.

In this work, linear cantor and Sierpinski carpet fractal antenna arrays are investigated using the following parameters: element spacing d = /4 and expansion factor

$\delta = 3$, where   is the wavelength. Computer simulations are performed to investigate the performance of the proposed LMS pattern synthesis method for cantor fractal antenna array. Figure 2(a) shows the normalized radiation patterns of the linear cantor fractal antenna arrays at three iterations p = 2, 3, and 4. It can be noted from Figure 2(a) that the SLL is almost constant around −5.5 dB (in conventional array , it is around -13.4 dB) and its value slowly decreases with the increase of iteration number.

Figure 2(b) shows the normalized radiation patterns of the linear cantor with LMS beamforming at three iterations p = 2, 3, and 4, assuming that the desired signal is received at an angle of -30° and that interference signals are received at an angle of 0° with Signal to Noise Ratio(SNR) of 10 dB. It can be shown from Figure 2(b) that the desired main lobe and nulls radiation pattern characteristics are kept the same for all the iterations, revealing the successful application of adaptive beamforming in fractal cantor array.

(a)



(b)

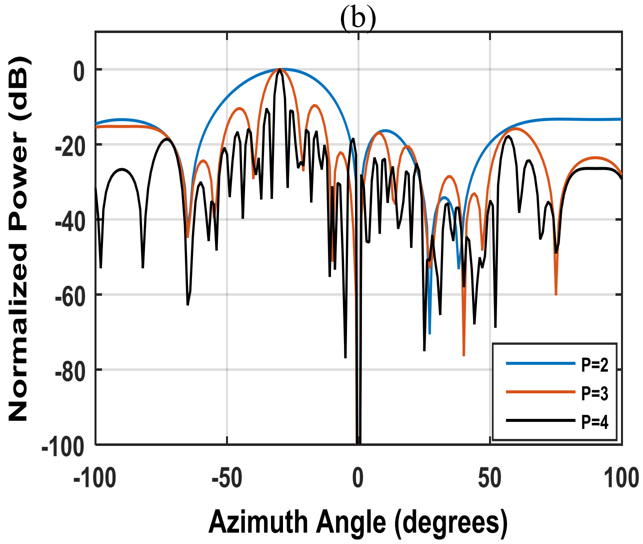
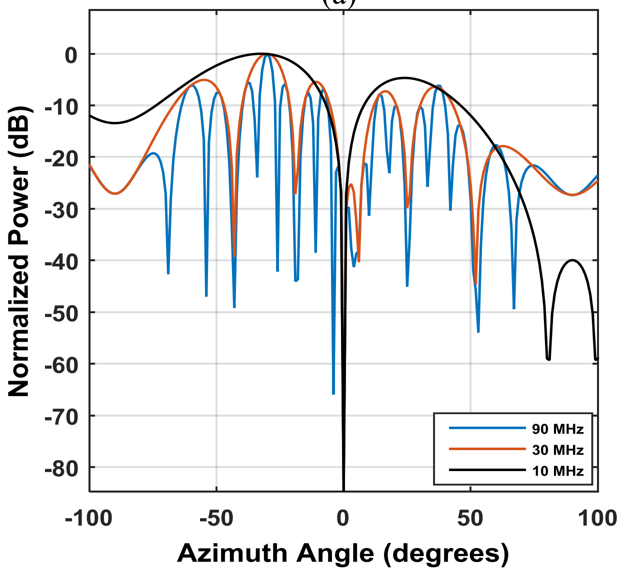


Figure 3. The normalized patterns of Sierpinski carpet fractal antenna arrays (a) without beamforming and (b) with beamforming at frequency 300 MHz.

Similarly, computer simulations are performed to validate the adaptive beamforming algorithm for thinned Sierpinski fractal antenna arrays and the results are shown in Figure 3 using the same parameters of element spacing expansion factor, SNR, and the required radiation pattern nulls and main lobe. Figures 3(a) and 3(b) show the normalized radiation patterns of the Sierpinski fractal arrays without and with LMS beamforming, respectively, at three iterations p=2, 3, and 4. As shown in Figure 3(b), the desired array pattern synthesis is the same for all iterations of Sierpinski carpet.

(a)



(b)

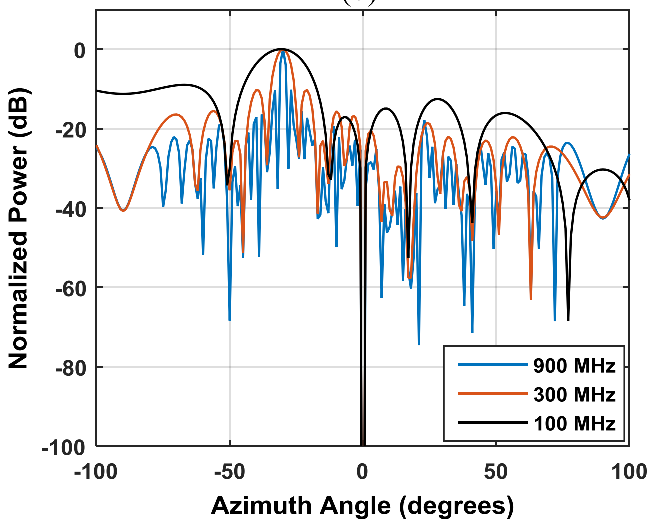


Figure 4. The normalized patterns with LMS beamforming of (a) linear cantor and (b) Sierpinski carpet fractal antenna arrays for different operating frequencies.

One of the main advantages of the proposed technique is the multi-band behavior, in which the SLL and the radiation pattern charactreistics remain the same at different frequency bands scaled by the expansion factor δ. This is illustrated in Figure 4. Figures 4(a) and 4(b) show the normalized radiation patterns with LMS beamforming of linear cantor and Sierpinski fractal antenna arrays, respectively, at iteration p = 4 and scaling factor δ =3 for different frequency bands. The fixed design frequency is assumed to be 900 MHz for the Sierpinski array and 90 MHz for the linear cantor array. In fractal antenna array, the operating frequency can be reduced by a factor of δ_n from the fixed design frequency, where n = 1, 2,⋯, P−1. In the current simulation, δ =3 and n = 1, 2, 3, which leads to possible operating frequencies of 900, 300, and 100 MHz for Sierpinski array, and frequencies of 90, 30, and 10 MHz for linear cantor array as shown in Figure 4 and Table 1.

(a) Linear cantor array

Band	90MHz	30MHz	10MHz
SLL(dB)	-5.86	-5.42	-5.17
HPBW	3.78	11.04	34.40

(b) Sierpinski carpet array

Band	900MHz	300MHz	100MHz
SLL(dB)	-10.32	-10.24	-9.83
HPBW	0.21	2.51	8.26

Table 1. The SLL and HPBW at different operating frequency bands for (a) linear cantor and (b) Sierpinski carpet fractal antenna arrays.

Figure 4 illustrates that the adaptive fractal array patterns maintain the same radiation pattern characteristics and the same SLL at the three frequency bands for both the cantor and Sierpinski arrays. Tables 1(a) and 1(b) show the LMS beamforming results of SLL and Half Power Beam Width (HPBW) for both cantor and Sierpinski fractal arrays, respectively, at growth stage P = 4. As shown in Table 1, the SLL is kept almost constant at the three frequency bands of each case, and the HPBW is increasing with the frequency decrease. Comparing Tables 1(a) and 1(b), it can be noted that the HPBW of Sierpinski carpet arrays is narrower than the linear cantor. As illustrated in Figure 4 and Table 1, with the proposed algorithm, the designer can efficiently control both radiation pattern main lobe and nulls, while maintaining the same Side Lobe Level (SLL) at at multiple frequency bands.

5. Conclusions

This paper introduces a new design methodology that combines the unique multi-band features of thinned fractal antenna arrays with the adaptive beamforming requirements. The LMS algorithm is employed to estimate the required excitation weights of the fractal array elements assuming that a desired signal and several undesired signals are received with certain power level of additive Gaussian noise by the array at respective directions of arrival. The pattern array synthesis is made by estimating the active elements needed to achieve the desired radiation pattern. In order to demonstrate the effectiveness of the proposed method, linear cantor and Sierpinski carpet fractal antenna arrays are investigated with and without the LMS adaptive beamforming method. The simulation results show that the proposed algorithm succeeds not only to steer the main lobe towards the desired signal and form nulls in the direction of arrival of all interference signals but also to maintain the same radiation pattern characteristics and the same SLL at at several frequency bands. As a future work, applying advanced beamforming techniques to the fractal antenna array layouts can provide various new design methodologies for multi-band, thinned, low cost, frequency agile, and shaped-beam smart antenna systems.

6. References

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