Microwave Scattering from Submerged Object Induced Wake over Rough Sea Surface

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Abstract

This paper presents a numerical simulation of electromagnetic scattering from surface wakes excited by a submerged object over two-dimensional sea surface. Based on the thin ship theory, an open software is presented for the geometric modeling of wake including Bernoulli hump and Kelvin wake. Then the scattering coefficient calculations are carried out by using the second order small slope approximation (SSA-II) in monostatic and bistatic configurations. Numerical results show that the radar scattering from sea surface is decreased with the presence of wake in the vicinity of the specular direction. And the effect will be reduced by decreasing the speed and increasing the submergence depth. Moreover, the texture of wake pattern can be observed from the distribution of backscattering coefficients under low sea states.

Keywords—electromagnetic scattering, submerged object, wake, small slope approximation

1. Introduction

Study of electromagnetic (EM) scattering characteristics from oceanic surfaces has received considerable attention for its wide applications in target detection and recognition. For a moving body under water, conventional detection technologies include active and passive sonar technologies, radar technology, magnetic anomaly detection, and thermal infrared detection, etc [1] [2]. As for the Synthetic Aperture Radar (SAR) technique, surface wake produced by a submerged object is of great value for the radar scattering from sea surface waves. In microwave band, EM wave can hardly penetrate through the surface to detect a submerged body for the high conductivity of sea water. Similar to a surface ship, an object moving under water will leave hydrodynamic disturbance on the surface. In general, three major disturbances are either directly or indirectly produced including the Bernoulli hump, the Kelvin waves and the internal waves [3], all of which are usually static with respect to the body’s reference frame. Of these waves, the Bernoulli hump performs as a characteristic hump of water in the near field and the Kelvin waves are responsible for the characteristic V-shaped wake in the far field. Both of them rapidly decay with decreasing the speed and increasing the submergence depth. As for the internal wave, its appearance attributes to the existence of a stratification of water density in horizontal layers. Unlike a surface ship, submerged body itself makes almost no contribution to the radar scattering for the negligible penetration depth. These motivate us to conduct the research on the EM scattering characteristics from the wake over sea surface.

The rest of this paper is organized as follows. In Section II, the modeling of wake excited by submerged body is combined with sea surface waves for the formulation of total free surface waves. In Section III, the tapered incident wave revised SSA-II model is presented for the evaluation of scattering coefficient. Comparisons of numerical results of monostatic and bistatic scattering coefficients under various parameters are analyzed and discussed in Section IV. The last section concludes this paper.

2. Geometric Model of Sea Surface Wave with Wake Excited by Submerged Object

For a fully developed infinite-depth sea, the rough sea surface can be modeled on the basis of spectral method through a linear superposition of harmonic waves. Then, the sea surface elevation $h_{sea}(r, t)$ can be expressed as [4] [5]

$$h_{sea}(r, t) = \sum_{k} H_{sea}(k, t) \exp(\mathbf{k} \cdot \mathbf{r})$$

where $\mathbf{k} = (k_x, k_y)$ is a 2-D wavenumber vector, and $H_{sea}(k, t)$ is the Fourier amplitudes of a sea surface elevation.

For the estimation of surface wave wake induced by a submerged body, a Rankine ovoid is firstly introduced to represent a simple submerged model similar to an ellipsoid. When it is submerged in a uniform stream, the resulting fluid motion may be viewed as that due to a pair of point source and point sink moving along its main axis.
with strength $M$, as shown in Fig.1. The surface wave elevation generated by a moving ovoid with speed $U_i$ at a submergence $D$ may be shown approximately to be [6]

$$h_{n_e}(x,y) = \sum_{n}\frac{M}{2\pi U_i} \text{Re} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ik\sec\phi\phi^2 \phi - i\mu sec\phi} \phi d\phi \right]$$

(2)

where $\mu$ is a fictitious frictional constant.

For a real complex submerged body model, the wave elevation is calculated using an open software instead, which is based on the theory of thin ship with an inclusion of viscous effects. To study the hydrodynamic characteristics of a real submerged body, its geometry structure must be transformed into a table of offsets, which lists the distance from the center plane to the outline of the model at every station and waterline. A linear superposition of the wind-driven sea surface wave and the wake gives the instantaneous wave elevation of free surface

$$h = h_{wave} + h_{wake}$$

(3)

The surface wave elevation produced by a submerged moving ovoid model with a length to diameter ratio of 7 had been measured in a towing tank of DTMB for various conditions shown in TABLE I [7]. The offset indicates a position at some distance off the centerline.

**TABLE I.** Simulation parameters for the moving ovoid

<table>
<thead>
<tr>
<th>Fig. 2</th>
<th>Length (FT)</th>
<th>Submergence (FT)</th>
<th>Speed (FT)</th>
<th>Offset (FT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>4.5</td>
<td>1.5</td>
<td>7.3</td>
<td>0.0</td>
</tr>
<tr>
<td>(b)</td>
<td>4.5</td>
<td>1.5</td>
<td>10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

![Fig.2](image)

**Fig.2** Comparison between theoretical and experimental wave profiles under conditions in Table I

![Fig.3](image)

**Fig.3** Bernoulli hump and Kelvin pattern waves generated by submerged body at different moving speed at submergence $D' = 0.5m$

In general, the comparisons in Fig.2 between simulated and measured wave elevations show excellent performance of Yim's theory and the software in both near field and far field. The results from a scaled submerged object model are shown in Fig.3 with different moving speed. The model is with an overall length of 55.0 m, a beam of 4.8 m, and an overall draft of 8.0 m. It can be clearly seen that the Bernoulli hump is right above the bow followed by the Kelvin pattern of waves. As increase of the moving speed, the Bernoulli hump becomes apparent due to its larger wave elevation. For the Kelvin waves, the increase of the wavelength is much more obvious than the wave elevation.

3. Scattering Calculation with SSA-II Model

For the scattering from 2-D sea surface with the wake, the tapered incident wave is introduced to diminish the undesired edge effect. The geometry configuration is shown in Fig.4. The tapered incident wave can be expressed as [8] [9]

$$\psi_{inc}(\hat{R}) = T(\hat{R}) \exp\left(ik \cdot \hat{R}\right)$$

(4)

![Fig.4](image)

**Fig.4** Geometry configuration for scattering from rough sea surface

With the correction of taper function, the scattering amplitude of the SSA-II can be expressed as

$$s(\hat{k},\hat{k}_0,\hat{t}) = -\frac{2}{(2\pi)^2} \int d\hat{r} T(\hat{R}) \exp\left[-i(\hat{k} - \hat{k}_0) \cdot \hat{r} - i(\hat{k}_0 + \hat{q}_1) h(\hat{r},\hat{t})\right] \times$$

$$B(\hat{k},\hat{k}_0) - \frac{i}{4} \int M(\hat{k},\hat{k}_0,\hat{\xi}) H(\hat{\xi},\hat{t}) \exp\left(-i\hat{\xi} \cdot \hat{r}\right) d\xi$$

(5)

where,

$$H(\hat{\xi},\hat{t}) = \frac{1}{(2\pi)^2} \int h(\hat{r},\hat{t}) \exp\left(-i\hat{\xi} \cdot \hat{r}\right) d\hat{r}$$

(6)

is the Fourier transform of free surface wave elevation. In terms of the scattering amplitude given by SSA-II, the scattering coefficient is defined as

$$\sigma^0 = 4\pi q_0 q_1 \left\{ \langle s(\hat{k},\hat{k}_0,\hat{t}) \rangle \right\}$$

(7)

where the angle bracket $\langle \rangle$ denotes the ensemble average over sea surface realizations.

4. Numerical results and analysis

In the following, the radar works at a frequency of 1.2GHz. And the free surface size is $L_x = L_y = 256\lambda$ with a sampling interval of $\lambda/8$ in each direction. The tapering parameter is chosen to be $L_x / 6$ to eliminate the edge.
effect caused by the limited free surface size. The scaled object model is used to simulate the surface wave wake.

As an important parameter of a moving submerged object, the effects of submergence on radar backscattering coefficient are shown in Fig.5. The wind speed is \( U_{10} = 3.0 \text{ m/s} \) and the object speed is \( U_s = 5.0 \text{ m/s} \). With the wake, the scattering coefficients for both VV and HH polarizations become smaller than those from sea surface alone in the vicinity of the normal incidence. As the incident angle increases, they tend to be less sensitive to the submergence. In addition, the influence of wake on scattering coefficient decreases with increasing submergence in the quasi-specular region, so is the range of incidence angles where the scattering coefficients get affected. These are attributed to the interaction of short and long waves on sea surface. In the vicinity of the normal incidence, radar scattering are controlled by the specular scattering from long waves.

![Figure 5](image)

**Fig.5** Monostatic scattering coefficients from sea surface with and without wake at different submergence, wind speed \( U_{10} = 3.0 \text{ m/s} \) and the object speed \( U_s = 5.0 \text{ m/s} \)

Changing the wind speed from 3.0 m/s to 5.0 m/s for a certain objects speed \( U_s = 5.0 \text{ m/s} \) at submergence \( D' = 0.5 \text{ m} \), the bistatic scattering coefficients from sea surface with and without wake are shown in Fig.6. The incidence angle is 60°. Positive angles indicate forward scattering, while negative angles represent scattering toward the quadrant where the radar transmitter is located. It is interesting to notice that there are obvious peaks in the specular direction from sea surface alone at wind speed \( U_{10} = 3.0 \text{ m/s} \). This is due to the fact that the coherent scattering is very strong and the scattering mechanism is mainly from specular reflection. However, the effect would be diminished with the superposition of the wake over sea surface. This implies that the wake has a strong influence on the specular scattering, which is small away from the specular direction. As the wind speed increases to 5.0 m/s, the scattering intensity in specular direction becomes weaker and a backscattering coefficient enhancement appears due to the increase of sea surface roughness.

To study the impact of object speed on radar scattering, Fig.7 shows the variation of bistatic scattering coefficients with object speed, which is presented to check the bistatic performances along with scattering azimuth angle \( \phi_i \) from 0° to 180° under the bistatic configuration \( \theta_i = \theta_s = 30° \), \( \phi_i = 0° \). The wind speed is \( U_{10} = 3.0 \text{ m/s} \) and the submergence is \( D' = 0.5 \text{ m} \). The scattering coefficient lines reach the nadirs when the receiving polarization vector locates in the orthogonal plane of incident polarization vector. As observed, for HH polarization, the position of the nadir always appears at \( \phi_s = 90° \); for VV polarization, it locates at \( \phi_s = 75° \) depending on the incident angle. Moreover, the effect of the wake on the scattering coefficient is obviously larger at higher body speed, especially for the scattering azimuth angles smaller than the nadir angle. However, in small scattering azimuth angles (0°~10°), there is almost no difference between two speeds, both of which show a decrease compared to that from sea surface alone.

![Figure 6](image)

**Fig.6** Bistatic scattering coefficients versus scattering angle \( \phi_i \) for 60° incidence angle from sea surface with and without wake at different wind speed, object speed \( U_s = 5.0 \text{ m/s} \) at submergence \( D' = 0.5 \text{ m} \)

![Figure 7](image)

**Fig.7** Bistatic scattering coefficients versus scattering azimuth angle \( \phi_i \) from sea surface with and without wake at different object speed, wind speed \( U_{10} = 3.0 \text{ m/s} \) and submergence \( D' = 0.5 \text{ m} \)

![Figure 8](image)

**Fig.8** Distribution of facet backscattering coefficients at incidence angle \( \theta_i = 40° \),wind speed \( U_{10} = 3.0 \text{ m/s} \) and the object speed \( U_s = 5.0 \text{ m/s} \) at submergence \( D' = 0.5 \text{ m} \)

Corresponding to the wake scenario in Fig.3 (b), Fig.8 shows the distribution of backscattering coefficients for every single facet at incidence angle \( \theta_i = 40° \). The wind
speed is \( U_{\text{in}} = 3.0 \text{ m/s} \) and the object speed is \( U_i = 5.0 \text{ m/s} \) at submergence \( D' = 0.5 \text{ m} \). The texture of the wake pattern can be readily observed in both polarizations. However, the difference of scattering coefficients between crest and trough is more notable in HH polarization than that in VV polarization. This explains that HH polarization could to some extent reflect the texture feature more clearly.

5. Conclusion

The second order SSA method has been employed to study the monostatic and bistatic scattering from two-dimensional sea surface with wake excited by submerged body at L-band. From the wake, it is seen that the submerged object produces surface waves including Bernoulli hump and Kelvin wake, which are closely related to the object movements. As for the scattering coefficients, we can find that a decrease appears with the presence of the wake in the vicinity of the specular direction, where the difference is small far away from the specular direction. Moreover, the texture of wake pattern can be readily observed from the distribution of backscattering coefficients for every single facet. The analysis presented in this paper helps to understand the scattering characteristics of wake excited by submerged object over sea surface, and then provide a theoretical basis for the detection of submerged object from SAR imaging.

6. Acknowledgements

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7. References