



## A New Method of SAR Radio Frequency Interference Mitigation Based On Maximum A Posterior Estimation

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### Abstract

Radio Frequency Interference (RFI) is a common kind of interference in synthetic aperture radar (SAR). The existence of RFI will significantly affect the quality of SAR image and the ability of SAR image interpretation. How to effectively reduce the bad effects of RFI is one of the important research questions in the field of active microwave remote sensing. According to the statistical distribution of RFI and the SAR echo, we propose a new method of RFI estimation and mitigation based on maximum a posterior (MAP) and Bayesian inference. This method is established in the basis of the probability distribution model of RFI and SAR echo. It converts the problem of RFI estimation to the question of MAP estimation and Bayesian inference of given observation and probability distribution and uses the gradient descent method which can cause rapidly iteration to solve the parameters of probability distribution model and accurately reconstructs the RFI. Then we use RFI cancellation under data domain to get SAR data without interference and obtain high-quality SAR imaging results through SAR processing. Finally, the the results of the measured data demonstrate the validity of the proposed method.

### 1. Introduction

SAR has the advantages of all-day, all-weather, long range and high resolution two dimensional imaging, so it becomes the important means of active microwave remote sensing nowadays. However, with the widespread use of SAR, it is hugely affected by the threat and challenge of interference under the complex electromagnetic environment. In particular, the great use of radiation sources such as telecommunication base stations, airport surveillance radars, ground based radars, and other co-frequencies remote sensing telemetry devices makes RFI the most common interference pattern for SAR [1-2]. Ref. [1] gives the example of the global distribution of radio frequency to prove that RFI is ubiquitous. Ref. [2-5] show the result of the SAR image comparison before and after RFI mitigation. It demonstrates that the existence of RFI will reduce the SNR of the SAR image. If RFI is strong, it will produce interference bands in SAR images and submerge the signal of SAR targets and reduce the ability of SAR image interpretation. Therefore, RFI is a difficult problem in the field of microwave remote sensing imaging and applications. For this reason, the IEEE GRS

Frequency Allocations in Remote Sensing Committee considers RFI identification and mitigation techniques as an open-ended problem to research in order to improve the efficiency and quality of remote sensors [1].

At present, SAR RFI mitigation techniques can be divided into two categories which are parametric and non-parametric methods according to the difference of interference modeling and interference mitigation processing [3-5]. SAR RFI mitigation techniques can also be divided into data domain and image domain RFI mitigation according to difference of Interference processing stage [6]. Parametric methods often take RFI modeling as a combination of multiple single frequency models and then use parameter estimation to conduct precise RFI reconstruction and mitigation. Non-parametric methods are to analyze the different characteristics between the interference and the data. Non-parametric methods tend to amplify the difference of interference and the SAR echo through a certain mapping transformation and then suppress the RFI effectively by adaptive filtering. In general, parametric methods require the precise modeling of the RFI. When the modeling is relatively accurate, parametric methods can achieve great interference mitigation effects.

However, if the modeling is not able to fit the situation, a larger RFI estimation error will occur. Although the non-parametric RFI estimation and mitigation method is not limited by the RFI model, it's not easy to obtain the appropriate mapping transformation method which can effectively distinguish the interference and the SAR echo signal. Besides, during the adaptive filtering process, it can cause some certain loss of signal when we are trying to suppress the RFI. Data domain interference mitigation is one kind of RFI interference mitigation method which we usually use. We choose to do the RFI mitigation before SAR imaging processing because it can reduce the impact of RFI during imaging process. Meanwhile, we can also combine interference mitigation and imaging algorithms together to get well-focused SAR images effectively. Methods processed in image domain is to use the image processing method to detect and suppress the RFI from the SAR images which are polluted by the interference. This method can effectively separate the information of RFI and SAR target under certain conditions, but the method is also easy to cause certain loss of image quality and cannot reduce the impact of RFI on the imaging process.

Therefore, current parameterized interference mitigation technique based on data domain is an effective method

and a hotspot for RFI mitigation research. At now, the least squares method is the most commonly used parametric approach [3], which takes RFI modeling as a combination of multiple single frequency components. This kind of method reconstructs RFI by the least squares method. However, this method is essentially a maximum likelihood solution under Gaussian noise. There is no constraint on the RFI form and it's easy to cause estimation error in the case of low JSR(Jamming-signal-ratio). Due to the rapid development of CS (Compressed Sensing) theory, [7] use the CS theory to conduct high probability RFI reconstruction and suppression. This method only considers the sparse prior information of RFI, but it needs to use prior information and manual selection to get key parameters like sparsity and others. The question is that it's difficult to achieve key parameters under different JSRs. For this reason, this paper not only uses the sparse prior information of RFI, but also introduces the parametric model of probability density distribution of RFI sparse constraints and establishes the SAR echo observation model with RFI. Then, we use the MAP and Bayesian inference to estimate the model parameters and precisely reconstruct the RFI. Besides, we use the interference cancellation in data domain to obtain the original data after interference suppression. Finally, we use traditional SAR imaging algorithms to obtain high-quality SAR images. This proposed method makes full use of prior parametric model of RFI to reconstruct RFI with high accuracy by the method of MAP and Bayesian inference. Moreover, this method can be well combined with SAR imaging processing methods and achieve high-quality SAR image results after interference mitigation.

## 2. SAR echo probability distribution model with RFI and the method of RFI estimation and mitigation

RFI is often represented by a combination of multiple single frequency interference model and it is often generated by other radiation sources which has the same frequency with SAR. That means RFI is generally independent with SAR echo. Therefore, SAR echo model of one pulse with RFI can be expressed as a combination of RFI, SAR echo and additive observation noise [3-5], that is,

$$X(\hat{t}) = \sum_{k=1}^K J_{0k} \exp(j(2\pi f_k \hat{t} + \theta_k)) + S(\hat{t}) + N(\hat{t}) \quad (1)$$

Where  $J_{0k}$ ,  $f_k$ ,  $\theta_k$  denote module, frequency and initial phase of  $k$ th interference component, respectively.  $S(\hat{t})$  is the valid SAR echo signal,  $N(\hat{t})$  represents observation noise,  $\hat{t}$  represents sampling time in range, i.e, fast time.

In order to obtain a high range resolution, the signal which SAR transmits is generally Linear Frequency Modulation (LFM) signal with large and wide bandwidth product. The echo of target is the convolution between a large number of ground scattering points in the observed

scene and LFM signal. RFI is often generated by other radiation sources which have the same frequencies with SAR and its signal bandwidth is relatively narrow compared to SAR signal bandwidth. So RFI is usually sparse in frequency domain. Meanwhile, only the RFI with strong energy can have obvious interference effect in SAR images after two dimensional coherent process of SAR imaging process. Thus, the SAR echo signal and observation noise presents Gauss-like distribution relative to strong RFI [3-4]. If observation noise is the complex Gaussian distribution, the SAR target echo can also be regarded as the complex Gaussian distribution. Thus, after discrete sampling (1) can be rewritten as

$$X = AJ + N_s \quad (2)$$

where  $X = [x_1, x_2, \dots, x_n]^T$ ,  $[\cdot]^T$  represents transpose operator.  $n$  represents sampling points in range.  $A \in C^{n \times k}$  represents RFI frequency dictionary matrix.  $J = [J_1, J_2, \dots, J_k]^T$  represents complex coefficients of RFI, where  $J_i = J_{oi} \exp(j\theta_i)$ ,  $i = 1, 2, \dots, k$ .

$N_s = [N_{s1}, N_{s2}, \dots, N_{sn}]^T$  represents the combination of SAR echo and noise and  $N_s$  is the complex Gaussian distribution, that is  $N_s \sim \mathcal{CN}(N_s | 0, \Sigma_{N_s})$  where covariance matrix is  $\Sigma_{N_s} = \text{diag}(d_1, d_2, \dots, d_n)$ . So the complex multivariate Gaussian distribution of SAR echo with RFI is  $X \sim \mathcal{CN}(X | AJ, \Sigma_{N_s})$ . Its specific probability density function is written as [8]

$$p(X | AJ, \Sigma_{N_s}) = \frac{1}{\pi^n |\Sigma_{N_s}|} \exp(-(X - AJ)^H \Sigma_{N_s}^{-1} (X - AJ)) \quad (3)$$

where  $[\cdot]^H$  represents conjugate transpose operator. Meanwhile, suppose that the multidimensional complex Gaussian distribution of RFI is expressed as  $J \sim \mathcal{CN}(J | 0, \Sigma_J)$ , where the covariance matrix is  $\Sigma_J = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_k)$ . In order to cater to sparse conditions through restraining RFI, we introduce hyper parameters  $\alpha$  and  $\beta$  ( $\alpha \ll \beta$ ). Let  $\sigma_i$ ,  $i = 1, 2, \dots, k$  in order to satisfy the gamma distribution and let gamma distribution and Gaussian distribution become conjugate prior distribution, which is written as

$$\Gamma(\Sigma_J | \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} |\Sigma_J|^{\alpha-1} \exp(-\beta \text{Tr}(\Sigma_J)) \quad (4)$$

Where  $\Gamma(\cdot)$  denotes gamma distribution,  $\text{Tr}(\cdot)$  denotes the trace of matrix.

According to Bayesian law, the posterior probability distribution of RFI can be represented by the probability distribution of SAR echo and prior probability distribution of interference, which is written as

$$p(J, \Sigma_J | X) \propto p(X | J) p(J | \Sigma_J) p(\Sigma_J | \alpha, \beta) \quad (5)$$

Suppose that  $\theta = \{J, \Sigma_{N_s}, \Sigma_J\}$ , the maximum a posteriori probability method is used to obtain the complex coefficients of RFI and the parameters of SAR echo

distribution model. According to (3), (4) and (5), (6) can be written as

$$\theta_{MAP} = \arg \max_{\theta} -(X - AJ)^H \sum_{N_s}^{-1} (X - AJ) - J^H \sum_J^{-1} J - \ln |\sum_{N_s}| + (\alpha - 2) \ln |\sum_J| - \beta \text{Tr}(\sum_J) + C \quad (6)$$

where  $C = -2n \ln \pi + k\alpha \ln \beta - k \ln \Gamma(\alpha)$  is a constant term which has nothing to do with  $\theta$ . Suppose that  $\sigma_i = \sigma$ ,  $i = 1, 2, \dots, k$ ;  $\lambda_j = \lambda$ ,  $j = 1, 2, \dots, n$ , and the constant term  $C$  is ignored. (6) can be rewritten as

$$\hat{J} = \arg \min_J \|X - AJ\|_p + \frac{\lambda}{\sigma} \|J\|_p + f_1(\lambda) + f_2(\sigma) \quad (7)$$

where  $f_1(\lambda) = n\lambda \ln \lambda$ ,  $f_2(\sigma) = \lambda k(2 - \alpha) \ln \sigma + \lambda k \beta \sigma$ ,  $\|\cdot\|_p$  represents the norm  $p$ , ( $p=2$ ). It is known from (7) that the first term is the objective function term and the latter three terms are penalty terms (constraint terms). If we only consider the first term, the outcome is the least-squares solution of the interference. If we consider the first and second term, we just solve this problem based on the method of ridge regression. If  $p=1$ , the method for RFI solution is based on the compression perception theory. It can be seen from the above analysis that (7) makes full use of the statistical properties and prior information of RFI and SAR echo data and conduct adaptation adjustment to the sparseness and maximum a posteriori solution of RFI according to the statistical distribution characteristics of RFI and SAR echo through penalty terms. Therefore, it can be seen from (7) that this method can obtain better RFI reconstruction estimation and its performance is superior to the traditional RFI mitigation method.

When hyper parameters  $\alpha = 2.00001$ ,  $\beta \gg \alpha$ , the covariance matrix, the covariance matrix and the observation noise covariance matrix can be obtained separately by calculating the gradient by the parameter set of  $\theta$  according to (6). The maximum likelihood estimation of the parameter set of  $\theta$  can also be obtained by iterative processing, the specific solution results are as follows:

$$\hat{J}^{(m+1)} = \left( A^H \sum_{N_s}^{-1(m)} A + \sum_J^{-1(m)} \right)^{-1} A^H \sum_{N_s}^{-1(m)} X \quad (8)$$

where  $m$  represents the estimation of parameter set for the current step and  $m+1$  represents the estimation of parameter set for the next step. In practical applications, the iteration can be terminated by the convergence of the parameter estimation process, which is evaluated by the difference between two adjacent steps. Suppose that the  $M$ th step is the final step, then, the maximum a posteriori complex coefficient estimate of RFI is  $J_{MAP} = \hat{J}^{(M)}$ , and we can get high-precision RFI reconstruction results using the RFI dictionary  $A$ . From the previous analysis, the frequencies of RFI can be coarsely estimated by jamming detection, then we can construct the refined dictionary by choosing the elements around the jamming frequencies on a fine grid.

After reconstructing the RFI, the RFI can be suppressed in this SAR echo according to time domain cancellation. Using (2) and (8), we can get

$$\hat{X} = X - AJ_{MAP} \quad (9)$$

where  $\hat{X}$  represents SAR echo signals after RFI mitigation. At the same time, in order to prevent the saturation problem from which the amplitude of target echoes is too large, the SAR system is generally provided with automatic gain control (AGC) function. Therefore, when the RFI energy is too strong, the SAR echo signal containing RFI can be depressed. So the SAR echo signal needs to be recovered after RFI mitigation, as is shown in (10)

$$X' = \varepsilon \hat{X} \quad (10)$$

Where  $X'$  represents the SAR echo signal after amplitude recovery,  $\varepsilon$  represents the ratio of the mean amplitude of SAR echo without RFI to the average amplitude of SAR echo after interference mitigation. Since RFI is often nonparametric in different pulses of SAR echoes, it is possible to perform RFI mitigation pulse-by-pulse of SAR echoes according to (9) and (10).

After SAR interference mitigation, SAR echo data with high quality can be obtained. According to SAR echo signal the well-focused SAR imaging results can be obtained by range compression, range azimuth decoupling, motion compensation and azimuth matching filtering, etc. Therefore, the proposed method of RFI mitigation based on MAP in this paper belongs to the method of RFI mitigation in data domain. It can effectively reduce the influence of RFI to the Doppler parameter estimation in the following motion compensation and azimuth matching filtering or other imaging processes. What's more, the proposed method can be well combined with SAR imaging algorithms.

## 4. Experimental Results

In this part, a measured SAR data contaminated by RFI is utilized for algorithm validation. This data was collected by an X-band, i.e., whose theoretical resolution is  $1\text{m} \times 1\text{m}$ . The illuminated scene is a suburban area consisting of apartment buildings and bare lands. Fig1 plots the range spectrum for a particular pulse. It is shown that the radar echoes are contaminated with multiple radio frequency interferences.

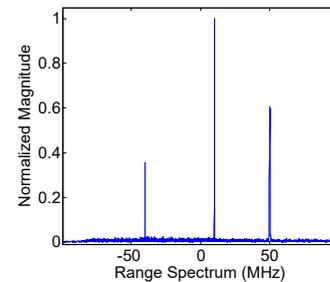
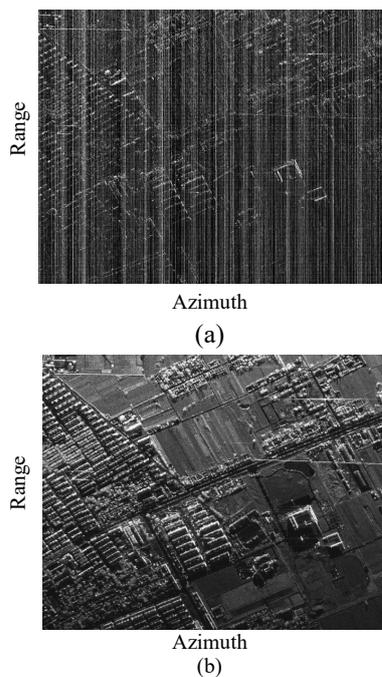


Fig 1. Illustration of the spectrum for a particular echo including the RFI.

Fig2 (a) shows the imaging result using the interference corrupted data without interference mitigation. Due to the presence of strong interference, many bright lines appear on the image, which would definitely have adverse impacts for the image interpretation and post processing. Fig2 (b) illustrates the imaging result after RFI mitigation by the the proposed MAP method in (7) - (10). It is shown that the proposed method is capable of suppressing the strong RFI while maintaining the SAR image with good quality, and well focused.



**Fig 2.** Imaging result of the measured data (a) without NBI mitigation, (b) using the proposed MAP method.

## 5. Conclusion

RFI is a common kind of interference in SAR, the existence of RFI seriously affects the applications like interested target imaging interpretation and quantitative remote sensing for SAR images. In this paper, we make full use of sparse characteristic of RFI and complex Gaussian distribution of SAR echo to establish SAR probability distribution model with RFI. Besides, we introduce the Bayesian law and use the method of maximum posteriori probability to realize the precise reconstruction of RFI. The results of the data process show the validity of the method proposed in this paper. Our next work is to find the probability distribution model of broadband interference and the effective method of interference mitigation.

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## 7. References

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