Phase distribution of the response in chaotic reverberation chambers

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1 Extended Abstract

We predict the phase distribution of the electromagnetic response (EM) in chaotic reverberation chambers (RC). In particular, like for the intensity distribution in a chaotic RC, this prediction relies on the knowledge of a single parameter, namely the modal overlap. We use the theoretical approach for the statistical distribution of the electric field in a chaotic RC deduced from a Random Matrix model introduced in [1, 2]. For a given frequency of excitation, the EM response is built upon a sum over resonant modes with mean spacing Δf between adjacent resonant frequencies and average width Γ. One defines the mean modal overlap $d = \Gamma/\Delta f \simeq 8\pi V f^3 c^{-3} Q^{-1}$ where V is the volume of the cavity and Q the mean quality factor. For a given excitation frequency and a given configuration of the RC, it can be shown that the distribution of the phase of each component of the response depends on a single parameter ρ, called the *phase rigidity* [1]. More precisely, due to the *ergodicity* of the modes contributing to the response, for a given realization, the probability distribution of the phase φ of each component depends on the sole modulus of ρ and is given by equation (21) of [3]:

$$P(\varphi;q) = (q/\pi)(1+q^2)/\left[4q^2 + (1-q^2)^2 \sin^2\varphi\right] \quad (1)$$

where the parameter q is related to ρ through $q^2 = (1-|\rho|)/(1+|\rho|)$. Since q is itself a distributed quantity, the distribution of the phase for an ensemble of responses resulting from stirring reads

$$P(\varphi) = \int_0^1 P_q(q)P(\varphi;q)dq \quad (2)$$

where P_q is the distribution of q. From the Antsatz given in [2] for the distribution of ρ, one deduces the Ansatz for the distribution of q: $P_q(q) = 2(B/q^3)\exp[-B(1-q^2)/q^2]$, where an empirical estimation of B is given by $B(d) = ad^2/(1+bd)$, with $a = 0.58 \pm 0.04$ and $b = 2.3 \pm 0.3$ [2]. An example of the remarkable agreement between the prediction (2) and EM responses calculated for the chaotic RC studied in [2] is shown in Fig. 1 for two different modal overlaps d.

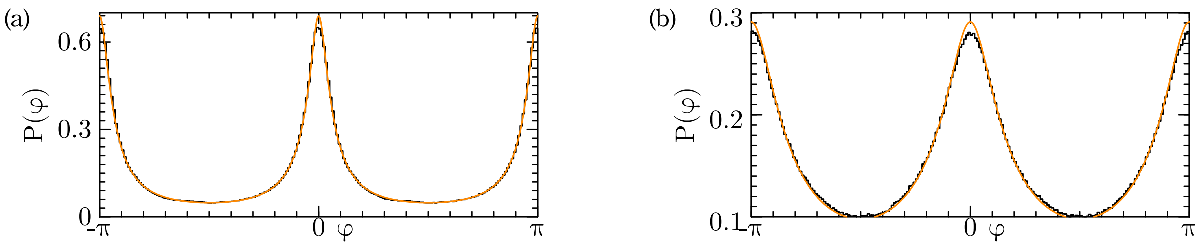

Figure 1. Histogram of the phase of the calculated response around (a) 200 MHz with $d = 0.16$, (b) 400 MHz with $d = 0.58$, in the chaotic RC studied in [2]. The continuous line is the theoretical prediction (2).

References

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