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Radiation Efficiency of an Individual Antenna in a System of Multiple Non-Identical Antennas

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Abstract

The challenge of calculating the radiation efficiency of each antenna in a system/device, consisting of multiple non-identical antennas, is addressed in this paper. A model, which accounts for mutual coupling between antennas is proposed and subsequently the radiation efficiency of each individual antenna was calculated. The proposed model was validated using simulations and experimental outcomes.

1 Introduction

Many devices and systems such as smart phones and autonomous vehicles contain different antennas, which are used to perform different tasks. Radiation efficiency of each antenna directly impacts the performance of each task and the power usage. Mutual coupling between the antennas alters expected radiation efficiency of each antenna measured in free space. Thus, it is important to calculate the radiation efficiency of each antenna as they are intended to be positioned in the system. Also, this information can be used to optimize the layout of antennas in a system/device to yield the highest radiation efficiency of each antenna.

The conventional approach to this challenge would be to place the system in an anechoic chamber and measure the total radiated power exciting each antenna one at a time while other antennas are connected to matched loads. The radiation efficiency of i th antenna of the system (η_i) is calculated using,

$$\eta_i = \frac{\text{Radiated Power}}{\text{Net Accepted Power to } i \text{ th Antenna}}$$

while other anetnnas are terminated with matched loads. This is costly as it involves the use of an anechoic chamber and is time consuming.

This paper presents a model to calculate the radiation efficiency of each individual antenna in a system of multiple non-identical antennas. The proposed model accounts for mutual coupling using a two-port device connecting the antenna two-port devices. In order to demonstrate the operation of the model, this paper uses a simple scenario of two non-identical antennas. A signal flow graph of the model is presented and equations were derived to calculate the radiation efficiency of each antenna. The model was verified using measurements and simulations using a prototype of two non-identical monopoles.

2 Proposed Model to Represent Mutual Coupling

Figure 1 presents the signal flow graph of the proposed model. Considering Figure 1, a_1, b_1 nodes and a_2, b_2 nodes are associated with the inputs to the Antenna 1 and Antenna 2 respectively. The outputs of Antenna 1 and Antenna 2 are associated with a_3, b_3 nodes and a_4, b_4 nodes. Thus, the 2-port device which denotes mutual coupling between Antenna 1 and Antenna 2 is connected between a_3, b_3 nodes and a_4, b_4 nodes.

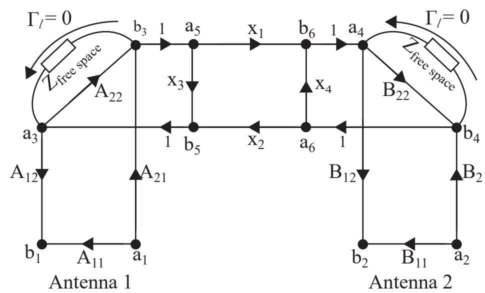

Figure 1. Signal flow graph of the proposed model

Antenna 1 and Antenna 2 S-parameters (A-matrix and B-matrix) were represented as

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{13} \\ S_{31} & S_{33} \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} S_{22} & S_{24} \\ S_{42} & S_{44} \end{pmatrix} \tag{2}$$

to avoid confusion in presenting and deriving equations. Moreover, S-parameters of the 2-port device, which represent mutual coupling is represented as

$$\begin{pmatrix} x_3 & x_1 \\ x_2 & x_4 \end{pmatrix} = \begin{pmatrix} S_{55} & S_{56} \\ S_{65} & S_{66} \end{pmatrix} \tag{3}$$

where its port 1 is associated with a_5, b_5 nodes and its port 2 is associated with a_6, b_6 nodes in the signal flow graph presented in Figure 1.

2.1 2-Port S-Parameters of an Antenna

In order to use the proposed model, 2-port S-parameters of each antenna at each frequency point of interest must be determined.

If the radiation efficiency of an antenna is known or readily available, the method described in [1] can be used to calculate the 2-port S-parameters of an antenna. Whereas, if the radiation efficiency of an electrically small antenna is unknown, the radiation efficiency can be calculated using the improved wheeler cap method in [2]. Subsequently, the 2-port S-parameters can be determined by the method in [1]. Furthermore, if the radiation efficiency of an electrically small antenna is unknown and the antenna radiates omni-directional spherical waves, 2-port S-parameters can be determined by Spherical Wheeler Cap (SWC) method in [3]. This method was also used in [4].

2.2 Theoretical Derivation of Radiation Efficiency of Each Antenna of a System

The S-parameters of the coupling 2-port device are unknown and have to be calculated. The signal flow graph presented in Figure 1 was solved and mathematical expressions were obtained for

$$S_{11,VNA} = C_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{-\begin{bmatrix} A_{11} + (A_{12}^2 - A_{11}A_{22})x_3 - A_{11}B_{22}x_4 \\ +B_{22}(A_{12}^2 - A_{11}A_{22})(x_1x_2 - x_3x_4) \end{bmatrix}}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]}, \tag{4}$$

$$S_{22,VNA} = C_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{-\begin{bmatrix} B_{11} + (B_{12}^2 - B_{11}B_{22})x_4 - B_{11}B_{22}x_4 \\ +B_{22}(A_{12}^2 - A_{11}A_{22})(x_1x_2 - x_3x_4) \end{bmatrix}}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]}, \tag{5}$$

$$S_{21,VNA} = C_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{-[A_{12}B_{12}x_1]}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]}, \tag{6}$$

$$S_{12,VNA} = C_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{-[A_{12}B_{12}x_2]}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]} \tag{7}$$

where $S_{11,VNA}$, $S_{22,VNA}$, $S_{21,VNA}$ and $S_{12,VNA}$ can be directly obtained using measurements or simulations. Due to reciprocity, $C_{12} = C_{21}$ thus (7) = (6). Subsequently, $x_1 = x_2$. Solutions to the 2-port coupling device S-parameters were determined to be,

$$x_1 = x_2 = \frac{A_{12}B_{12}C_{12}}{\begin{bmatrix} (B_{12}^2 - B_{11}B_{22})(A_{12}^2 - A_{11}A_{22}) \\ +B_{22}(A_{12}^2 - A_{11}A_{22})C_{22} \\ +A_{22}(B_{12}^2 - B_{11}B_{22})C_{11} \\ +A_{22}B_{22}(C_{11}C_{22} - C_{12}^2) \end{bmatrix}}, \tag{8}$$

$$x_3 = \frac{-\begin{bmatrix} A_{11}B_{12}^2 - A_{11}B_{11}B_{22} \\ +(B_{11}B_{22} - B_{12}^2)C_{11} + A_{11}B_{22}C_{22} \\ -B_{22}C_{11}C_{22} + B_{22}C_{12}^2 \end{bmatrix}}{\begin{bmatrix} (B_{12}^2 - B_{11}B_{22})(A_{12}^2 - A_{11}A_{22}) \\ +B_{22}(A_{12}^2 - A_{11}A_{22})C_{22} \\ +A_{22}(B_{12}^2 - B_{11}B_{22})C_{11} \\ +A_{22}B_{22}(C_{11}C_{22} - C_{12}^2) \end{bmatrix}}, \tag{9}$$

$$x_4 = \frac{-\begin{bmatrix} B_{11}A_{12}^2 - A_{11}B_{11}A_{22} \\ +(A_{11}A_{22} - A_{12}^2)C_{22} + B_{11}A_{22}C_{11} \\ -A_{22}C_{11}C_{22} + A_{22}C_{12}^2 \end{bmatrix}}{\begin{bmatrix} (B_{12}^2 - B_{11}B_{22})(A_{12}^2 - A_{11}A_{22}) \\ +B_{22}(A_{12}^2 - A_{11}A_{22})C_{22} \\ +A_{22}(B_{12}^2 - B_{11}B_{22})C_{11} \\ +A_{22}B_{22}(C_{11}C_{22} - C_{12}^2) \end{bmatrix}}. \tag{10}$$

Radiation efficiency of an antenna (η) is defined as the ratio between the radiated power to the net accepted power [5]. Thus considering the two antenna system, the radiation efficiency of Antenna 1, when Antenna 1 is excited and Antenna 2 is terminated with a matched load is,

$$\eta_{Antenna1} = \frac{|b_3|^2 - |a_3|^2}{|a_1|^2 - |b_1|^2} \tag{11}$$

where,

$$a_3 = -\frac{\left(x_3 + B_{22}(x_1^2 - x_3x_4)\right)A_{12}a_1}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1^2 - x_3x_4) - 1]}, \tag{12}$$

$$b_3 = -\frac{(1 - B_{22}x_4)A_{12}a_1}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]}, \tag{13}$$

$$b_1 = -\frac{\begin{bmatrix} (A_{11} + (A_{12}^2 - A_{11}A_{22})x_3 - A_{11}B_{22}x_4)a_1 \\ +(A_{11}A_{22} - A_{12}^2)(x_3x_4 - x_1^2)B_{22}a_1 \end{bmatrix}}{[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1]}. \tag{14}$$

Similarly, the radiation efficiency of Antenna 2, when Antenna 2 is excited and Antenna 1 is terminated with a matched load is,

$$\eta_{Antenna2} = \frac{|b_4|^2 - |a_4|^2}{|a_2|^2 - |b_2|^2} \tag{15}$$

where,

$$a_4 = -\frac{\left(x_4 + A_{22}(x_1^2 - x_3x_4)\right) B_{12}a_2}{\left[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1\right]}, \quad (16)$$

$$b_4 = -\frac{(1 - A_{22}x_3)B_{12}a_2}{\left[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1\right]}, \quad (17)$$

$$b_2 = -\frac{\begin{bmatrix}(B_{11} + (B_{12}^2 - B_{11}B_{22})x_4 - B_{11}A_{22}x_3)a_2 \\ +(B_{11}B_{22} - B_{12}^2)(x_3x_4 - x_1^2)A_{22}a_2\end{bmatrix}}{\left[A_{22}x_3 + B_{22}x_4 + A_{22}B_{22}(x_1x_2 - x_3x_4) - 1\right]}. \quad (18)$$

3 Experimental Setup

The experiment was set up with two identical monopole antennas, where one antenna was attached to a 1 dB attenuator to make them non-identical. Each monopole is made of copper (height: 14 mm, diameter: 1.27 mm) and is surrounded by a layer of Teflon (relative permittivity: 2.1, height: 14 mm, inner diameter: 1.27 mm, outer diameter: 4.11 mm). An aluminum sheet (length: 300 mm, width: 300 mm, thickness: 1 mm) was used as the ground plane. The two monopoles were positioned 40 mm apart which is in between $\lambda/2$ and λ at all frequency points of interest. The prototype is presented in Figure 2. Antenna 2 was attached with the attenuator and the frequency points of interest are 4.2 GHz, 4.6 GHz, 5.0 GHz and 5.4 GHz.

Figure 2. The prototype of two monopoles on a ground plane

Another identical monopole on a aluminum ground plane was used to calculate the 2-port S-parameters of antennas. The prototype is presented in Figure 3. Wheeler cap radii were calculated according to [3, 4] and 3D printed, where their inner surfaces were coated with conductive paint. In order to calculate the A-matrix and B-matrix of (1) and (2), measurements were obtained with and without the 1 dB attenuator.

4 Simulation and Experimental Outcomes

For measurements and simulations Keysight 8719ES VNA and ANSYS HFSS were used respectively.

Figure 3. The prototype of single monopole on a ground plane along with the wheeler caps

4.1 Calculation of 2-Port S-Parameters of Antennas using Measurements

Both antennas used in these experiments were electrically small monopoles, where their radiation efficiencies were unknown. Thus, SWC method presented in [3, 4] was used to determine the 2-port S-parameters. All the steps in this calculation are not presented in this paper as the same steps discussed in [4] were followed and is not the main focus of this paper.

Table 1 and Table 2 presents the calculated S-parameters using measurements, for Antenna 1 and Antenna 2 at the frequency points of interest.

Table 1. Experimental outcomes for S-parameters of Antenna 1

Frequency	S-Parameters (dB $\angle$ deg)
4.2 GHz	$\begin{pmatrix} -11.04 \angle 59.12 & -0.44 \angle -12.20 \\ -0.44 \angle -12.20 & -10.15 \angle 96.15 \end{pmatrix}$
4.6 GHz	$\begin{pmatrix} -13.44 \angle -29.33 & -0.26 \angle -36.23 \\ -0.26 \angle -36.23 & -13.96 \angle 117.83 \end{pmatrix}$
5.0 GHz	$\begin{pmatrix} -10.48 \angle -89.92 & -0.47 \angle -58.12 \\ -0.47 \angle -58.12 & -10.25 \angle 156.05 \end{pmatrix}$
5.4 GHz	$\begin{pmatrix} -7.66 \angle -126.98 & -0.90 \angle -75.93 \\ -0.90 \angle -75.93 & -8.72 \angle 150.32 \end{pmatrix}$

Table 2. Experimental outcomes for S-parameters of Antenna 2

Frequency	S-Parameters (dB $\angle$ deg)
4.2 GHz	$\begin{pmatrix} -13.30 \angle 39.12 & -1.37 \angle -23.77 \\ -1.37 \angle -23.77 & -10.79 \angle 99.84 \end{pmatrix}$
4.6 GHz	$\begin{pmatrix} -15.36 \angle -93.07 & -1.19 \angle -66.19 \\ -1.19 \angle -66.19 & -13.71 \angle 120.56 \end{pmatrix}$
5.0 GHz	$\begin{pmatrix} -11.55 \angle 172.92 & -1.39 \angle 74.04 \\ -1.39 \angle 74.04 & -10.43 \angle 155.79 \end{pmatrix}$
5.4 GHz	$\begin{pmatrix} -8.91 \angle 104.16 & -1.79 \angle 37.66 \\ -1.79 \angle 37.66 & -8.18 \angle 151.40 \end{pmatrix}$

4.2 Calculation of Radiation Efficiency of Each Antenna in the System

In order to calculate the radiation efficiency of each antenna of the system, values for C_{11} (4), C_{22} (5) and C_{12} (7) have to be known. Table 3 and Table 4 presents the values for above variables obtained using measurements.

Table 3. Measured C_{11} and C_{22} of the two antenna system

Frequency	(dB $\angle$ deg)	
	C_{11}	C_{22}
4.2 GHz	$-14.09 \angle 69.66$	$-16.38 \angle 52.59$
4.6 GHz	$-14.89 \angle -51.81$	$-16.75 \angle -114.23$
5.0 GHz	$-8.82 \angle -108.19$	$-10.25 \angle 157.61$
5.4 GHz	$-6.09 \angle -138.99$	$-7.75 \angle 93.75$

Table 4. Measured C_{12} of the two antenna system

Frequency	C_{12} (dB $\angle$ deg)
4.2 GHz	$-15.64 \angle 38.26$
4.6 GHz	$-15.51 \angle -33.69$
5.0 GHz	$-17.70 \angle -93.28$
5.4 GHz	$-18.80 \angle -144.83$

According to the proposed model, radiation efficiencies of Antenna 1 and Antenna 2 in the two antenna system were calculated using (11) and (15), respectively. Table 5 and Table 6 presents the simulated $\eta_{Antenna1}$ and $\eta_{Antenna2}$, and calculated $\eta_{Antenna1}$ and $\eta_{Antenna2}$ using measurements.

Table 5. Simulated and experimental outcomes of radiation efficiency of Antenna 1 in the system of two antennas

Frequency	Simulated η	Calculated η using measurements
4.2 GHz	99.44%	98.28%
4.6 GHz	99.48%	99.39%
5.0 GHz	99.45%	98.16%
5.4 GHz	99.37%	99.57%

Table 6. Simulated and experimental outcomes of radiation efficiency of Antenna 2 in the system of two antennas

Frequency	Simulated η	Calculated η using measurements
4.2 GHz	78.98%	78.25%
4.6 GHz	79.02%	79.27%
5.0 GHz	77.83%	77.04%
5.4 GHz	77.59%	74.17%

Considering Table 5 and Table 6, the simulated results and the calculated results based on the measurements have a reasonable agreement with each other. Thus, it can be concluded that the proposed model produces reasonably accurate results.

5 Conclusion

The proposed model successfully accounts for mutual coupling and calculates the radiation efficiency of each antenna in a system of two antennas. The equations derived can be directly applied to determine radiation efficiency of each antenna in a system of two antennas without electromagnetic calculations or simulations. This model could also be extended to a system of N number of antennas to model mutual coupling of larger systems.

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References

[1] J. T. Aberle, "Two-port representation of an antenna with application to non-foster matching networks," *IEEE Trans. Antennas Propag.*, vol. 56, no. 5, pp. 1218–1222, 2008.

[2] R. Johnston and J. McRory, "An improved small antenna radiation-efficiency measurement method," *IEEE Antennas Propag. Mag.*, vol. 40, no. 5, pp. 40–48, 1998.

[3] H. L. Thal, "A Circuit Model for Spherical Wheeler Cap Measurements," *IEEE Trans. Antennas Propag.*, vol. 59, pp. 3638–3645, oct 2011.

[4] P. Groves, P. Conroy, L. Belostotski, and M. Okoniewski, "Antenna 2-Port Electrical and Noise Parameters," *IEEE Antennas Wirel. Propag. Lett.*, vol. 1225, no. c, pp. 1–1, 2016.

[5] "IEEE Standard Definitions of Terms for Antennas," *IEEE Std 145-2013 (Revision IEEE Std 145-1993)*, pp. 1–50, 2014.