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# FUNDAMENTAL LIMITATIONS OF PML OMNIDIRECTIONAL ELECTROMAGNETIC ABSORBERS AS EM RADIATION SHIELDS: SUMMARY OF RECENT FINDINGS

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## Abstract

We first discuss fundamental limitations on the feasibility and performance of perfectly reflectionless, doubly-curved convex, and primitively causal absorbers of classical electromagnetic (EM) waves. The supporting analysis expands upon and solidifies the conclusions of past work giving evidence to the non-feasibility of such EM media. Second, given these fundamental limitations, we propose a technique to design polarization-independent, broadband doubly-anisotropic EM absorbers that, while exhibiting reflectivity (particularly strong forward scatter, i.e. "shadow" generation), exhibit very low backscatter and lateral scatter. The conclusions drawn from recent numerical studies motivate future explorations into the fabrication and deployment of said media in diverse applications, such as those concerning enhanced robustness of civilian automotive electronic systems to unintended external EM interference.

## 1. Introduction

Initially proposed for facilitating the efficient numerical simulation of classical EM wave propagation and scattering problems [15], the Perfectly Matched Layer (PML) Absorbing Boundary Condition (ABC) has since been generalized in a number of ways. Some diversifications, to name a few, are to 3-D problems, physics domains (e.g., thermodynamics), coordinate grids (e.g., cylindrical and spherical), PML boundary shapes (e.g., doubly-curved curvilinear), constitutive (*linear*) properties of the ambient medium into which the PML is embedded (frequency-dispersive, doubly bi-anisotropic, "double-negative", etc.), and numerical techniques (frequency and time-domain methods, for finite elements, finite volume, finite difference, etc.) [5,6,9,12]. Note that planar boundaries are a special, limiting case of a doubly-curved surface: namely, let its two principal radii of curvature (RoC) tend towards becoming infinitely large.

In its "coordinate-stretching" interpretation (frequency-domain), the PML can be seen as a region of space in which the spatial metric is distorted and made complex-valued [11]. Calling the normal coordinate of the boundary surface as $\zeta_3$ (the tangential, mutually orthogonal coordinates are $\zeta_1$ and $\zeta_2$ with metric factors $h_1$ and $h_2$, respectively), the PML *continuously* transitions $\zeta_3$ (from its boundary value $\zeta_0$) to its complex-valued, "stretched" form (exp[-i$\omega t$] time convention assumed) [11] $\bar{\zeta}_3(\zeta_3)=\zeta_0+\int_{\zeta_0}^{\zeta_3} s_\zeta(\zeta)\,d\zeta$. The metric stretching factor $s_\zeta(\zeta_3)=a_\zeta(\zeta_3)+i\sigma_\zeta(\zeta_3)/\omega$ has real ($a_\zeta$) and imaginary ($\sigma_\zeta$) parts responsible for wave-front stretching and wave energy absorption, respectively [1,4]. Note that other forms of $s_\zeta$, besides one based off the Drude dispersive model, have also been successfully utilized before too [16]. The tangential metric stretching factors are likewise "stretched" to $\bar{h}_1$ and $\bar{h}_2$ [11]. Alternatively, an equivalent formulation, casting the PML as a physical, doubly-anisotropic absorber in flat space, exists too [8]: $\bar{\boldsymbol{\epsilon}}_r=\bar{\mu}_r=\mathrm{Diag}\left[\frac{h_1\bar{h}_2}{\bar{h}_1h_2}s_\zeta,\frac{\bar{h}_1h_2}{h_1\bar{h}_2}s_\zeta,\frac{\bar{h}_1\bar{h}_2}{h_1h_2s_\zeta}\right]$, where $\bar{\boldsymbol{\epsilon}}_r$ and $\bar{\mu}_r$ are the electric permittivity and magnetic permeability tensors, respectively, and we assumed the PML is being matched to vacuum [9]; the more general form is shown elsewhere [5,11]. In the limit as $s_\zeta(\zeta_3)\to 1$, $\bar{\boldsymbol{\epsilon}}_r=\bar{\mu}_r\to 1$ (free space) [11].

## 2. Discussion and Conclusion

A long-standing limitation of the PML, however, appears to be its restriction to *concave* (i.e., from the source's viewpoint) doubly-curved surfaces distinguishing the boundary between the PML and its ambient medium [4]. This conclusion, concerning the PML's limitation, is based on extensive previous studies that mathematically and numerically demonstrated (at least for the *pre-dominantly* used class of PML media, with non-negative $a_\zeta$ and $\sigma_\zeta$) that *convex* PML/ambient medium boundaries lead to primitively non-causal PML media [4]. Alternatively, in a time-domain implementation where primitive causality (i.e., cause preceding effect) is explicitly enforced, the non-causal, absorptive PML instead behaves as a primitively causal but *gain* (i.e., active) PML that engenders late-time dynamical instability. Similar conclusions apply for two of the other three classes of PML media (non-negative $a_\zeta$ and non-positive $\sigma_\zeta$ being class two, and vice versa [class three]).

We recently found, however, that for the fourth meta-class of PML media (non-positive $a_\zeta$ and $\sigma_\zeta$), a convex reflectionless medium arises that, at first glance (i.e., using earlier analytical tests), appears to exhibit primitive causality and hence absorptivity too [2,3,7,14]. Time-domain numerical simulations revealed, however, that such a medium in fact exhibits *early-time* numerical instability. Earlier, analytical explorations explain such dynamic instability in the context of PMLs (of the first, pre-dominantly used variety) impedance-matched to "double-negative" media (i.e., negative electric permittivity and magnetic permeability), where the normal (to boundary surface) component of group and phase velocity are oppositely oriented [10,13].

Since usage of the first class of PMLs for double-negative media is (in terms of causal and absorptive/gain behavior, at least) is equivalent to using the fourth PML class for double-positive media (what we explore here), which we confirmed in recent studies [7,14], our observed early-time dynamic instability informed us that indeed none of the four PML classes were viable candidates for a convex-curvilinear, reflectionless, causal, and absorptive medium. This conclusion was reinforced, albeit with a purely theoretical subtlety, when we artificially "flipped" the signs of the components of the anisotropic constitutive material tensors [2,7]. Namely, this modified version of the fourth meta-class of PML media was, upon sign-flip, reflective and absorptive (confirmed in numerical results), however one could make it reflectionless by coating the absorber with a lossless material that has the effect of rendering the stretched radius, at the coating/absorber interface, equal to zero. However, such a coating is nothing more and nothing less than an "ideal" EM cloak, hence precluding external EM radiation from interacting with the absorber, a sufficient (although perhaps not necessary) condition for facilitating the existence of (provably, at least cylindrically and spherically-shaped) convex, doubly-curved reflectionless absorbers [2,7,17].

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