

On the Use of Magnetic Current Loop Source Model in Lightning Electromagnetics

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Abstract

The paper deals with the use of antenna theory and the concept of magnetic current loop (MCL) source in lightning electromagnetics. The applications are related to lightning channel modeling and transient analysis of grounding system. The formulation is based on the corresponding Pocklington integro-differential equation, while the numerical solution is carried out by means of the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM).

1. Introduction

Thin wire models based on the integral equation(s) of Pocklington or Hallen type find various applications in electromagnetic compatibility (EMC), such as lightning channel modeling, grounding system analysis, electromagnetic field coupling to transmission lines, etc. Transmitting (antenna) mode requires modeling of a certain excitation (voltage or current source) at input terminals, and the excitation area is generally treated as electrically small. Nevertheless, the feed gap modeling has been shown to appreciably influence the near field behaviour, e.g. the antenna admittance calculation, while no significant impact on the far field is noticed. Basically, there are two principal approaches to source modeling problems; delta-gap source and magnetic frill (MF) source [1, 2]. Some other approaches have also been reported in literature, e.g. [3], [4], but they were applied to a lesser extent. The analysis recently reported in [5] features the use of magnetic current loop source in modeling excitation of thin wire antennas of finite length. Similar ideas have been discussed in [6, 7].

This paper deals with the application of the magnetic current loop (MCL) approach reported in [5] to modeling of lightning channel and grounding electrodes. Note that MF and MCL source models are both based on the magnetic current density existing at the source area and generating an axial excitation electric field. Some illustrative computational examples pertaining to lightning return stroke channel and horizontal grounding electrode modeling are given in the paper, as well.

2. Theoretical background

While MF source model is well known approach for modeling excitation of thin wire antennas in transmitting

mode [1], MCL model was just recently introduced for thin wire antenna applications [5]. This section outlines the full wave model featuring the MCL source for a lightning channel current and a horizontal grounding electrode.

2.1 Lightning channel model

The lightning channel is represented by a straight monopole antenna vertically mounted on a perfectly conducting (PEC) ground, as depicted in Fig 1.

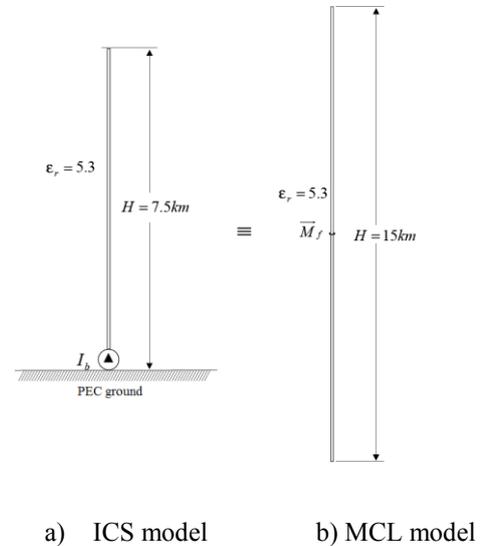


Figure 1. Antenna model of the lightning channel

The unknown current distribution along the lightning channel $I(z')$ is governed by the Pocklington integro-differential equation for a lossy straight thin wire [8]

$$E_z^{inc}(z) = -\frac{1}{j4\pi\omega\epsilon_{eff}} \int_{-L}^L I(z') \cdot \left[k^2 + \frac{d^2}{dz'^2} \right] g_0(z, z') dz' + Z_S \cdot I(z) \quad (1)$$

where $E_z^{inc}(z)$ is the excitation function (incident field), $g_0(z, z')$ denotes the Green function for free space [8], k is the propagation constant and Z_S stands for the loading impedance to account for the losses along the wire.

In the case of an ideal current source (ICS) excitation at the channel base (Fig. 1a), the incident field is set to zero and the ICS is incorporated into the formulation as the forced condition within the numerical solution, while in the case of voltage source excitation or MCL (Fig. 1b), $E_z^{inc}(z)$ is expressed in terms of input voltage and feed-gap

width. The lightning channel is represented by a 7.5-km long monopole antenna (15-km long equivalent dipole antenna) with a wire radius $a=0.05\text{m}$, excited by a magnetic loop at the channel base. The wire is immersed in a virtual homogeneous dielectric space¹ with $\epsilon_r=5.3$, to adjust the signal propagation velocity to realistic values [9, 10] (in this work $v=0.43c$). PEC and finitely conducting wire, respectively, are studied. The latter model assumes the channel losses $R=0.07\Omega/\text{m}$. The source voltage in the MCL model is adjusted to ensure the specified channel base current. The input current waveform is assumed to have a peak value of 11 kA and a maximum steepness of 105 kA/ μs [11, 12]. The transient lightning channel current at different locations along the channel is shown in Figs 2 and 3 (0m, 100m, 500m, 1km, 3km and 5km) for two wire models.

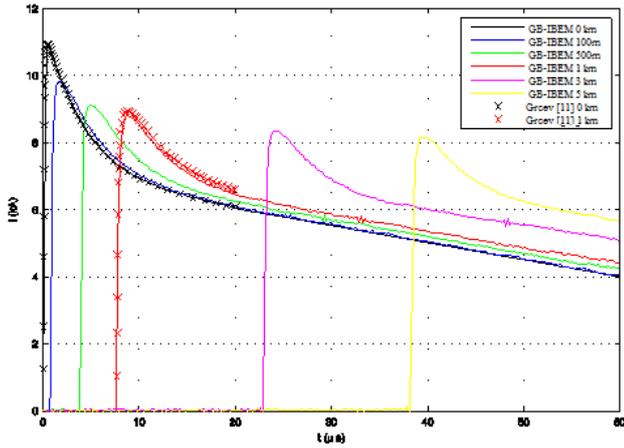


Figure 2. Lightning return-stroke channel current versus time for different heights (0, 100 m, 500 m, 1 km, 3 km, 5 km). The channel is considered as a PEC wire. The results at 1 km height are compared with those of [11].

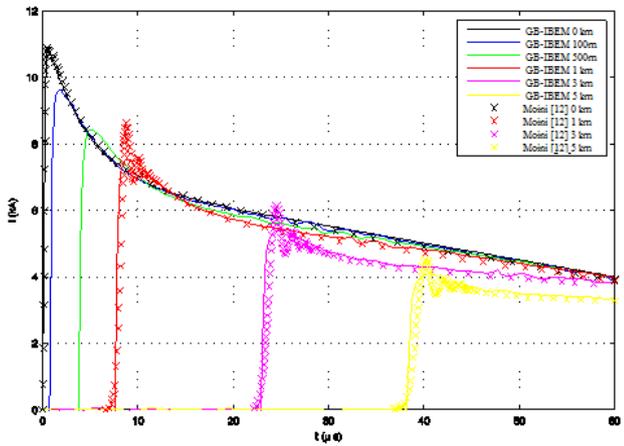


Figure 3. Lightning channel current versus time for different heights (0, 100 m, 500 m, 1 km, 3 km, 5 km).

¹ Note that the virtual dielectric space is only considered to determine the spatial-temporal distribution of the current along the return stroke channel. For the computation of the electromagnetic fields, the virtual dielectric space is removed and replaced by air.

The channel is represented by a finitely-conducting wire ($R=0.07\Omega/\text{m}$). The results at 1 km, 3 km and 5 km are compared with those of [12]. Figure 2 shows a slight attenuation of the current along the channel while much faster attenuation is visible in Fig 3, as the current moves upward along the channel. Some small ringing can be observed, due to numerical instabilities [11], [13]. A satisfactory agreement of the results obtained via MCL model with the results published in [11, 12] is achieved. Figs. 4 and 5 show the vertical electric field at 500 m and 5 km from the channel base for the PEC wire model.

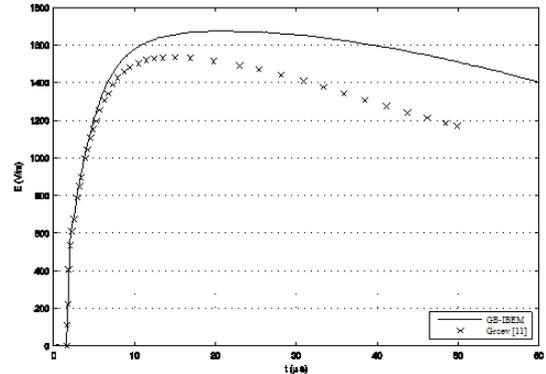


Figure 4. Vertical field at 500m (PEC wire model)

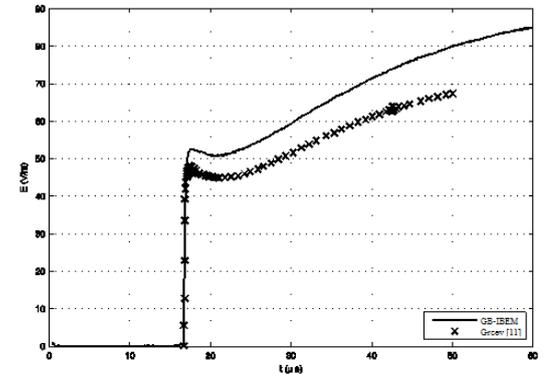


Figure 5. Vertical field at 5 km (PEC wire model)

The results are comparable with the results published in [11, 12]. The differences in late times are due to numerical instabilities arising from the current calculation [12]. The segment length in [12] is 15 m, while in this paper, 3-m long segments are used.

2.2 Horizontal grounding electrode

Standard full wave model of a horizontal grounding electrode proposed in [14] deals with a thin wire of length L and radius a , buried in a lossy ground at depth d and excited via an ICS, Fig. 6.

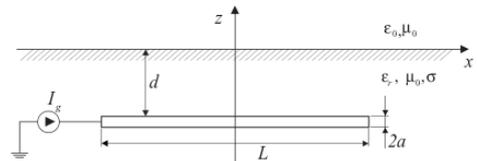


Figure 6. ICS excitation of the grounding electrode

The current distribution along the electrode $I(x')$ is governed by the following Pocklington equation [15]:

$$-\frac{1}{j4\pi\omega\epsilon_{eff}} \int_0^L I(x') \cdot \left[k^2 + \frac{d^2}{dx'^2} \right] G(x, x') dx' = \bar{E}^{exc}(x) \quad (2)$$

where $G(x, x')$ is the lossy medium Green function [15], k and ϵ_{eff} is the complex propagation constant and complex permittivity of the lossy ground, respectively. Using the classical model of the electrode featuring ICS excitation, shown in Fig 6, the right hand side of (1) vanishes and corresponding Pocklington equation becomes homogenous [14], while the ICS is incorporated into the formulation in the form of boundary conditions [14]. Though ICS model has been widely used for over three decades, its use might suffer from difficulties in defining the input voltage being necessary to obtain the input impedance of the grounding system [14]. The ICS model, as well as MF and MCL source models (both based on the magnetic current density existing at the source area and generating axial excitation electric field) are illustrated in Fig 7. While in antenna applications, the source is placed in the feed gap area with input terminals put close together, in grounding applications input terminals are between two distant points (one at the electrode, the other in the remote soil), thus causing a problem with the implementation of classical delta gap voltage source particularly in numerical sense. On the other hand, MF and MCL are planar perpendicular to the wire axis and can be readily applied to the open wire thus making them suitable for grounding applications.

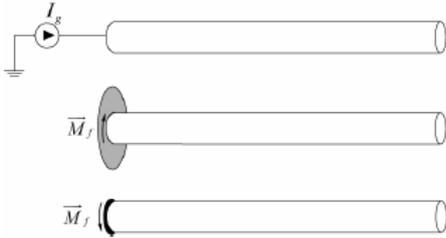


Figure 7. ICS, MF and MCL source at the open wire end

The relation for excitation field \bar{E}^{exc} for the MF model is

$$E_x^{exc}(x) = \frac{V_g}{\ln\left(\frac{b}{a}\right)} \left[\frac{e^{-jk\sqrt{a^2+(x-x')^2}}}{\sqrt{a^2+(x-x')^2}} - \frac{e^{-jk\sqrt{b^2+(x-x')^2}}}{\sqrt{b^2+(x-x')^2}} \right] \quad (3)$$

while, for MCL model it follows

$$E_x^{exc}(x) = V_g a^2 \left[jk + \frac{1}{\sqrt{a^2+(x-x')^2}} \right] \frac{e^{-jk\sqrt{a^2+(x-x')^2}}}{a^2+(x-x')^2} \quad (4)$$

where V_g is the imposed input voltage and a is the wire radius. In the MF source model, b represents the outer radius of the magnetic current ring. As this radius is not clearly defined in the antenna analysis, b is usually determined from the characteristic impedance of the transmission line exciting the antenna as presented in [1]

(e.g. for 50Ω impedance $b=2.3a$). Comparison of the input impedance spectra determined by different models is shown in Figs 8 and 9. The differences between the results increase with frequency with maximal difference below 2% and within 0.5% in the range relevant for lightning (0 Hz – 10MHz).

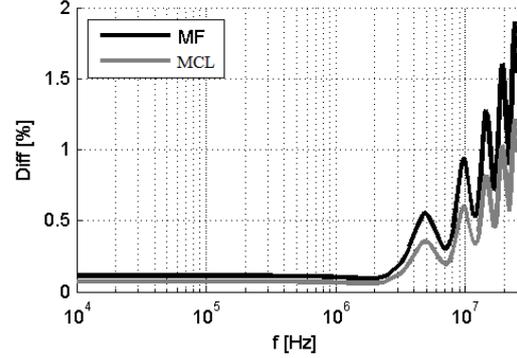


Figure 8. Percentage difference from ICS ($L=10\text{m}$; $a=0.005\text{m}$; $\sigma=0.001\text{S/m}$)

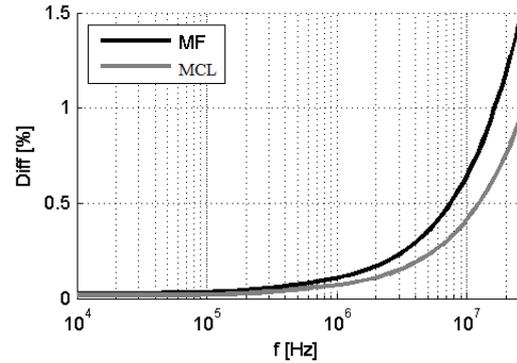


Figure 9. Percentage difference from ICS ($L=50\text{m}$; $a=0.005\text{m}$; $\sigma=0.001\text{S/m}$)

The time domain results are obtained for a channel-base current expressed by Heidler function [16], with: $I_0=1\text{A}$, $k=0.93$, $\tau_1=19\mu\text{s}$, $\tau_2=485\mu\text{s}$ [17] (modified to the unit amplitude). Fig. 10 shows the transient current at the center of the electrode obtained by different models. No appreciable discrepancies between the results could be observed.

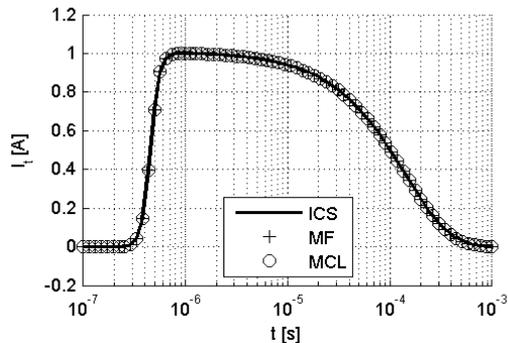


Figure 10. Current on the center of grounding wire ($L=50\text{m}$; $a=0.01\text{m}$; $\sigma=0.001\text{S/m}$)

Figs. 11 and 12 deal with the maximal transient input voltage which, for the case of the unit input current

corresponds to the impulse impedance. Fig 11 shows the percentage difference between the results obtained via MF source with different values of b/a and the results obtained using ICS for various values of the wire radius and ground conductivity. For a ratio b/a less than 10, the errors remain within 1% regardless of the value of the ground conductivity. Fig 12 shows the results obtained via different models for various conductivities. Again, MCL and MF provide accurate results for typical values of ground conductivity.

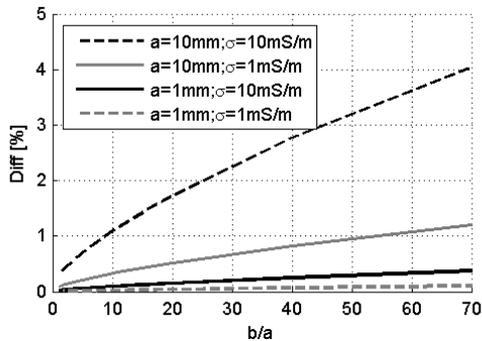


Figure 11. Percentage difference (MF results with respect to ICS results; $L=20\text{m}$)

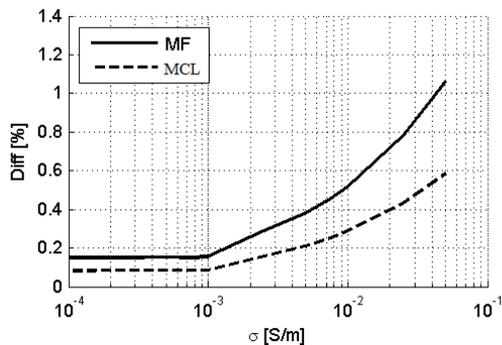


Figure 12. Percentage difference (with respect to ICS results; $L=20\text{m}$, $b/a=3$, $a=0.001\text{m}$)

5. Conclusion

The use of MCL to model the excitation for the lightning channel and horizontal grounding electrodes is presented in this work. The proposed source can be readily used for both lightning channel and grounding system analysis. The main advantage over the traditional ICS model in grounding applications is that no additional calculation of the input voltage is required for the input impedance assessment, since the input voltage is already set by the source itself. For lightning modeling, since generally it is the current which is specified at the channel base, still a current source representation might be more straightforward to use.

7. References

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