A System and Source Joint Estimation Method for EM Emission Modeling of Clock Circuits

HAO Xuchun*(1), XIE Shuguo(1), ZHANG Weidong(1)

(1) School of Electronic and Information Engineering, Beihang University, Beijing, China, 100191

Abstract
A system and source joint estimation method for EM emission modeling is proposed in this paper. Based on the priori knowledge, i.e., the transfer function of system is smooth and the form of source signal was already known, the paper using joint estimation method to solve modeling problems when measured only the emission amplitude spectra at some ports. Experimental results verify the method could estimate the source parameters accurately, and the errors are acceptable while using the estimated transfer function to predict EM emission spectra.

1. Introduction
High-density packaging is widely applied to PCB design, leading to more electromagnetic interference problems between PCBs, particularly within enclosures [1]. Hence, electromagnetic (EM) emission modeling [2] is becoming critically important for the design, study and optimization of microwave circuits and systems [3].

Lots of works has been done at chip-level. Integrated circuit electromagnetic modeling (ICEM) [4] method was a simple and intuitive way, which used the RLC (resistance inductance capacitance) circuit to fit the measured port characteristics [5]. In the description of the active integrated device model, the chip can be divided into two parts, which are passive Power Distribution Network (PDN) and internal source component. It is stated that the precision of the chip model is mainly decided by the accuracy of PDN. Various researches have been published on model PDN. In fact, for EM emission problems, the accuracy of internal source model is also critical. However, in most applications, such as clock circuits or IC, the internal source component cannot work alone. As its parameters can hardly be measured accurately, the EM source model is difficult to be established.

As for PCB-level modeling [6-8], current methods ignore the specific physical implementation details of the modeling object, and treat it as a black box [9]. Generally, to calculate an accurate system transfer function, the exact parameters of both input signal and output signal are required. The traditional way of PCB-level modeling is to use a simplified mathematical model to describe the transfer function of the modeled system by testing its input and output port characteristics. A common testing method is using a vector network analyzer to measure S-parameters of the circuit-under-test, as is showed in Figure 1. Several modeling techniques were presented to calculate system models in the past years. Limited by algorithm mathematical models, most of the modeling methods need both amplitude and phase information of all ports, to keep their accuracy in a high level.

However, for EM emission problems, the input signal is generally an internal EM source in the circuit, which makes it difficult to acquire S-parameters of the circuit system directly. Restricted by the test conditions, the only information we could acquiring is the amplitude information of output signal measured by spectrum analyzer, as is showed in Figure 2. Under these conditions, the traditional modeling method could not be used to construct EM emission successfully. Furthermore, as the source module cannot be removed from the circuit or work alone, the parameters of EM source cannot be acquired easily. As system transfer function is codetermined by both its input and output port characteristics, estimating the EM source model accurately shares the same importance with modeling the system transfer function when solving EM emission modeling problems. The inaccuracy of source estimation also affects the estimation accuracy of the system transfer function.

To solve the above mentioned problem, a new EM emission modeling method is proposed in this paper, which based on several common and reasonable priori knowledge of the system and its EM source, and using joint estimation of system transfer function and EM source parameters, to solve modeling problem when measured only output amplitude spectra.

2. Signal Model of a clock-type source
The clock-type source is always the main resource to generate electromagnetic emission of microwave circuits. Hence, under the above mentioned demands, a clock-type source is taken as a normal example, to explain the process of joint estimation method.

The time domain waveform of a clock-type source can often be approximate by a rectangle wave. The time-
domain expression of an ideal rectangle wave can be representing as below:

\[
x(t) = \begin{cases} 
A_o, & n \cdot f_0 + d < t \leq (n + 1) \cdot f_0, n \in \mathbb{N}^+ \\
-A_o, & (n + 1) \cdot f_0 < t \leq (n + 2) \cdot f_0 
\end{cases}
\] (1)

where \( x(t) \) represents the time-domain expression of a rectangle wave, \( A_o \) is the amplitude, \( f_0 \) is the repetition frequency, and \( d \) is the duty cycle.

Do Fourier transform to (1), the magnitude-frequency characteristics of \( x(t) \) were gotten as below:

\[
X(f) = \sum_{n=-\infty}^{\infty} X_n \delta(f - nf_0) = \sum_{n=-\infty}^{\infty} A_o dc \delta(n \pi dc \delta(f - nf_0))
\] (2)

It can be find from (2) that the magnitude-frequency characteristics of a rectangle wave can be determined by three parameters, which are amplitude \( A_o \), repetition frequency \( f_0 \), and duty cycle \( dc \). As the value of amplitude \( A_o \) is known in the most applications, and the shape of the output spectrum is mainly determined by \( f_0 \) and \( dc \), the following discussed method would focus on the process of repetition frequency and duty cycle estimating.

3. Estimating Method

Assuming the transfer function of the system is \( H(f) \), the amplitude spectra of the output signal \( Y(f) \) can be written as below:

\[
Y(f) = X(f)H(f) = \sum_{n=-\infty}^{\infty} X_n H(nf_0) \delta(f - nf_0), n \in \mathbb{N}
\] (3)

where \( X_n = A_o dc \delta(n \pi dc \delta) \).

3.1 Repetition Frequency Estimating

(3) shows that the frequency spectra of rectangle wave are a bunch of discrete spectrum lines. The discrete frequency points set \( \{ F_{\text{sample}} \} \) could be described as below:

\[
\{ F_{\text{sample}} \} = n \cdot f_0, n = 0, 1, 2, \ldots
\] (4)

It can be easily found from (4) that elements in \( \{ F_{\text{sample}} \} \) are only related to the repetition frequency \( f_0 \) and all of them are integral multiples of \( f_0 \).

According to the assumption of smooth transfer function, it can be found that the amplitude spectra of the output signal \( Y(f) \) shares the same discrete frequency points set \( \{ F_{\text{sample}} \} \) with the source signal’s.

\[
\{ Y_{\text{sample}} \} = \{ X(\{ F_{\text{sample}} \})H(\{ F_{\text{sample}} \}) \}
\] (5)

It means that \( f_0 \) can be estimated by calculating the frequency interval between two adjacent spectrum lines \( \Delta f \) of the measured EM emission amplitude spectrum set \( \{ Y_{\text{sample}} \} \).

\[
\Delta f = F_{\text{sample}}(i + 1) - F_{\text{sample}}(i), \forall F_{\text{sample}}(i) \in \{ F_{\text{sample}} \}
\] (6)

Considering the parameter settings of spectrum analyzer, assume the frequency sampling interval is \( f_{\text{interval}} \), then

\[
m \cdot f_{\text{interval}} \leq n \cdot f_0 \leq (m + 1) \cdot f_{\text{interval}}, m, n \in \mathbb{N}^+
\] (7)

It can be inferred from (6) that the measured \( n \cdot f_0 \) can hardly be equaled with its true value, but the nearest \( n \cdot f_0 \). So the measured frequency interval between adjacent spectrum lines \( \Delta f \) is not unique in the most cases. Moreover, decided by the properties of system and source signal, it is difficult to measure the EM emission amplitude spectra at some harmonic frequencies for they were so weak that buried in the noise. In this situation, the frequency interval between adjacent lines might be several integer times of \( f_0 \).

The following three steps were required to solve this problem. First, acquire all the frequency intervals \( \{ \Delta f \} \) which is greater than a certain threshold.

Second, find the mode \( \Delta f \) of all elements in the set \( \{ \Delta f \} \). Finally, the estimated repetition frequency \( \hat{f}_0 \) equals with the average value of all the obtained \( \Delta f \) which in a neighborhood of \( \Delta f \), as is showed in (8).

\[
\hat{f}_0 = \text{average}(\{ \Delta f \}) \text{ } | \text{ } \Delta f_{\text{measured}} \text{ in a neighborhood of } \Delta f \text{ (9)}
\] (8)

3.2 Duty Cycle Estimating and Transfer Function Fitting

According to (8), let \( f_0 = \hat{f}_0 \) and, then (2) could be represented as below:

\[
\hat{X}(f, dc) = A_o dc \sum_{n=-\infty}^{\infty} \delta(f - nf_0), n \in \mathbb{N}
\] (9)

For a given \( dc \), (9) turns as:

\[
\hat{X}'(f) = A_o dc \sum_{n=-\infty}^{\infty} \delta(f - nf_0), n \in \mathbb{N}
\] (10)

Calculate the corresponding source amplitude spectrum set \( \{ \hat{X}_{\text{sample}} \} \).

\[
\{ \hat{X}_{\text{sample}} \} = \hat{X}'(\{ F_{\text{sample}} \}, dc)
\] (11)

Bringing (11) into (5), a sample set of the transfer function was gotten as:

\[
\{ H_{\text{sample}} \} = H(\{ F_{\text{sample}} \}) = Y(\{ F_{\text{sample}} \}) \cdot \hat{X}(\{ F_{\text{sample}} \}, dc)
\] (12)

which can be approximately regarded as the real amplitude of transfer function at the discrete frequency points set \( \{ F_{\text{sample}} \} \).

Using \( N \) order polynomial with least squares method to fit \( \{ H_{\text{sample}} \} \), the analytical transfer function of the system \( H'(f) \) was obtained as:

\[
H'(f) = \sum_{i=0}^{N} h_i \cdot f^i
\] (13)

where \( h_i \) means the k-order-coefficient calculated with least squares method.

Bringing \( f = \{ F_{\text{sample}} \} \) into (13), a new set \( \{ H_{\text{estimate}} \} \) was gotten:
\[ \{H'_{\text{estimate}}\} = H'(\{F'_{\text{sample}}\}) \] (14)

which represents the estimated amplitude of transfer function at \( \{F'_{\text{sample}}\} \).

A error function \( \text{error}' \) was built as the 2-norm of the difference between \( \{H'_{\text{calculate}}\} \) and \( \{H'_{\text{sample}}\} \), which can be represented as:

\[
\text{error}' = \|H'_{\text{estimate}} - H'_{\text{sample}}\|_2
\] (15)

which is used to measure the accuracy of the estimation.

Do traversal of the above process under different value of \( dc \), and we could get a series value of \( \text{error}' \), which means \( \text{error} \) is a function of \( dc \). Based on the assumption of smooth transfer function, estimate the duty cycle \( dc_0 \) as which made \( \text{error} \) achieve the minimum:

\[ dc_0 = dc_{\text{min(error)'}} \] (16)

and the corresponding \( H'(f) \) is the transfer function of the system \( \hat{H}(f) \).

At this point, the system transfer function \( (\hat{H}(f)) \) and source parameters \( (f_0 \text{ and } dc_0) \) of EM emission modeling have been estimated jointly via the output amplitude spectrum set \( \{Y'_{\text{sample}}\} \).

4. Experimental Verification

Several simulations and experiments were taken to verify the correctness of the above mentioned system and source joint estimation method for EM emission modeling.

4.1 Source Parameters Correction Verifying

As Figure 3 and Figure 4 show, a passive PCB transmission line was treated as the circuit-under-test; a signal generator was used to produce input rectangle wave with various parameter as the source of the circuit-under-test; and a spectrum analyzer was used to measure the amplitude spectra of the output signal.

![Figure 3. Block diagram of instruments connection](image)

![Figure 4. Experimental scenario with signal generator as the source](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter settings</th>
<th>Estimated results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_0 / \text{MHz} )</td>
<td>( dc_0 / % )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
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</tr>
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<td>47</td>
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</table>

The author changed the signal generator’ settings, and used the above mentioned joint estimating method to identify the source parameters to verify the feasibility of it. The parameter settings and the estimated results were showed in Table 1. It can be seen from the experimental results that the input parameters \( f_0 \) and \( dc_0 \) almost perfectly match their true values.

4.2 Applicability of Crystal Oscillator

To check the applicability of the situation that crystal oscillator as the source component, the author did the following experiments. The block diagram and the experimental scenario of instruments connection is showed in Figure 5 and Figure 6.

<table>
<thead>
<tr>
<th>No.</th>
<th>( \hat{f}_0 / \text{MHz} )</th>
<th>( \hat{dc}_0 / % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>crystal oscillator 1</td>
<td>4.999</td>
<td>49.5</td>
</tr>
<tr>
<td>crystal oscillator 2</td>
<td>7.375</td>
<td>49.2</td>
</tr>
</tbody>
</table>

![Figure 5. Block diagram of instruments connection with crystal oscillator generate the input signal](image)

![Figure 6. Experimental scenario with crystal oscillator as the source](image)
the system-under-test is built. Experimental results verify the method could estimate the input parameters accurately, and the forecast errors are acceptable while using the estimated transfer function to predict output spectra.

Owing to the joint estimation method was first proposed to solve EM emission problem in PCB-level modeling, the author simply used the most basic rectangle wave and linear passive circuit networks for validation. Therefore, the follow-up works mainly contain the following two aspects: firstly, establish a more complex and universal source model, that make the estimated source closer to the actual source module in PCB; secondly, use a more advanced fitting method to construct a more accurate system transfer function.

6. Acknowledgements
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7. References