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Copulas, Correlation and Dependence in Uncertainty Quantification of EMC

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Extended Abstract

Linear and more general higher-order statistical dependence is an aspect of statistical and stochastic analysis that is of increasing importance. Particularly for uncertainty quantification that involves several input variables and/or system parameters characterized by uncertainty or random fluctuation, the 'curse of dimensionality' makes the inclusion of multiple sources of uncertainty rapidly prohibitive in practical calculations and estimation. However, not all input parameters need not necessarily to be treated as statistical independent. In deterministic settings, this is a familiar situation, e.g., encountered in Kramers-Kronig type relationships. On the other hand, in stochastic physical and engineering problems, statistical dependence often relies on empirical determination.

In this paper, the feasibility of using copulas in unearthing relationships between random EM quantities is investigated. Based on analysis of experimental data, the dependence structure (known as copula) that overlays the marginal distributions is determined. It is shown that, in particular, power and energy are sensitive to in-phase/quadrature dependence, and that this dependence can be characterized explicitly for both Cartesian and vector fields.

In relation to EMC risk analysis and estimation, extreme values are associated with exceptionally high values (relevant to immunity testing) or low values (e.g., in fading in communications). It is found that outliers of fields or energy can be identified as local concentrations in the copula density near the center or vertices of its domain U^n, where they are more likely to give rise to catastrophic events and outliers. Available analytic copulas models appear to perform rather poorly in the application studied (fields and energy inside reverberation chambers). By contrast, empirical copulas and their density [1, eqns. (10)-(11)] tend to give better agreement. For the numerical evaluation, kernel density estimation is useful for the quadrature of the integral for the total energy or intensity involving the copula density. On the other hand, estimation of the remaining parameter of the distribution or its density is more difficult.

Detailed results on an analytic formalism and experimental results for copulas for energy and power density are given in [1].

Reference

[1] L. R. Arnaut., "Copulas, outlier and rogue states of nonelliptic fields and energy in electromagnetic reverberation," *IEEE Trans. Electromagn. Compat.*, **58**, 2, April 2016, pp. 371-384, doi: 10.1109/TEMC.2015.2511664.