Advanced analysis of the transient impedance of the horizontal grounding electrode: from statistics to sensitivity indices

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Abstract

The paper deals with a stochastic analysis of influence of randomly distributed values of certain parameters on the value of the transient impedance of the horizontal grounding electrode. Deterministic model for the transient impedance calculation is based on the antenna theory and corresponding Pocklington integro-differential equation in the frequency domain. The Pocklington equation is solved using previously developed analytical technique, thus obtaining scattered voltage at the input terminal of the grounding electrode and, subsequently, transient impedance. Stochastic collocation analysis is applied by taking into account three different random parameters and performing subsequent analysis of their respective impact.

1. Introduction

Transient impedance represents one of the most important parameters in lightning protection system (LPS) design and its calculation is of paramount importance [1, 2]. Accuracy of the transient impedance calculation depends on the approximations used in developing the model as well as on a given solution method. On the other hand, limitations of a model stem from uncertainties of input parameters set where the statistical nature of complex systems (materials, electrical properties and/or geometry) has to be taken into account [3]. The natural complexity of electromagnetic environment also requires the extraction of the most influential parameters [4]. The uncertainties can arise due to inability to accurately measure certain properties of the system (e.g. soil conductivity) or due to stochastic nature of the phenomena of interest (e.g. rise time of the lightning pulse) [5, 6].
Current induced along the grounding electrode is governed by Pocklington integro-differential equation in the frequency domain [1, 7]. Transient impedance of a single grounding electrode is calculated using frequency domain analytical solutions of the governing equation for the current and the scattered voltage induced along the horizontal electrode, respectively [8]. These results are subsequently transformed into time domain via Inverse Fast Fourier Transform (IFFT).
Due to uncertain variations of parameters of interest, some techniques for an efficient integration of stochastic modelling have been developed [9]. This paper aims to demonstrate the ability of a precise analytical deterministic method to compute the transient impedance, combined with an efficient and accurate Stochastic Collocation method (SC) to integrate uncertainties with regard to parameters accuracy.
In second part of the paper, an outline of the deterministic model for the calculation of transient impedance is given. Stochastic collocation method used for the calculation of uncertainties is also given in this part. Third part of the paper presents some numerical examples with the extensive analysis of the obtained results. The concluding remarks are given in the final part of the paper.

2. Theoretical background

2.1 Deterministic antenna theory model

Horizontal, perfectly conducting grounding electrode of length L and radius a, embedded in a lossy medium at depth d and excited by an equivalent current source is considered [10]. The medium is characterized with electric permittivity ε and conductivity σ. Dimensions of the electrode are assumed to satisfy the thin-wire approximation conditions [7].

The current induced along the electrode $I(x')$ is governed by homogeneous Pocklington equation [7]

$$j\omega\frac{\mu_0}{4\pi}\int_0^L I(x')g(x,x')dx' - \frac{1}{j4\pi\omega\varepsilon_{eff}}\frac{\partial}{\partial x}\int_0^L \frac{\partial I(x')}{\partial x'}g(x,x')dx' = 0 , \quad (1)$$

where ε_{eff} is the complex permittivity of the medium [11].

Green's function $g(x,x')$ can be expressed as [8]

$$g(x,x') = \frac{e^{-\gamma R_1}}{R_1} - \Gamma_{ref}\frac{e^{-\gamma R_2}}{R_2} . \quad (2)$$

Propagation constant of the medium is defined as $\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$ and distances R_1 and R_2 correspond to distances from the source and the image to the observation point, respectively. Presence of the earth-air interface is taken into account via the reflection

coefficient arising from the Modified Image Theory (MIT) [12].

Equation (1) can be solved analytical and the solution is given as [8]

$$I(x) = I_g \frac{\sinh\left[\gamma(L-x)\right]}{\sinh(\gamma L)}. \quad (3)$$

The scattered voltage along the electrode is defined as an integral of the vertical component of the scattered electric field [11]. The Generalized Telegrapher's Equation for spatial distribution of the scattered voltage is given as [11]

$$V^{sct}(x) = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \int_0^L \frac{\partial I(x')}{\partial x'} g(x,x')\,dx'. \quad (4)$$

The expression for scattered voltage can be obtained by inserting (3) into (4), which yields

$$V^{sct}(x) = \frac{\gamma I_g}{j4\pi\omega\varepsilon_{eff}\sinh(\gamma L)} \int_0^L \cosh\left[\gamma(L-x)\right] g(x,x')\,dx'. \quad (5)$$

To calculate the transient impedance of the grounding electrode, the time domain value of scattered voltage has to be determined by applying the IFFT algorithm. Once the scattered voltage is obtained the transient impedance can be expressed as [1]

$$z(t) = \frac{v(0,t)}{i(0,t)}. \quad (6)$$

Functions for voltage and current at the right hand side of (6) represent their values at the beginning of the electrode, i.e. at $x = 0$, where $i(0,t)$ represents the current of the equivalent current source, e.g. a lightning strike current. This current is usually represented as a double exponential pulse [13].

2.2 Stochastic methodology: from statistics to sensitivity analysis

Stochastic Collocation (SC) method, as a part of more general spectral approaches [14], has been used in different areas of electromagnetic compatibility [15]. The main advantages of the method are non-intrusive nature and simplicity. Detailed theoretical background of SC method can be found in [14]. Nevertheless, for the sake of completeness it is outlined here.

The fundamental principle of SC method is the polynomial approximation of the considered output Z for N uncertain input parameters. The output of interest, Z is expanded over a stochastic space by using the Lagrangian basis functions as detailed in [14]

$$Z(t) = \sum_{i=1}^{n} Z_i L_i(t), \quad (7)$$

where $L_i(t)$ is the Lagrangian basis function. Based upon convenient properties of Lagrange polynomials and depending on the probability density function $pdf(t)$ assigned to the assumed random variable, the mean and the variance of output Z are readily derived

$$\begin{aligned} \langle Z(t) \rangle &= \sum_{i=1}^{n} Z_i \omega_i \\ Var(Z(t)) &= \sigma_Z^2 = \sum_{i=1}^{n} \omega_i Z_i^2 - Z^2 \end{aligned}, \quad (8)$$

where ω_i denotes the weight given with

$$\omega_i = \int_D L_i(t)\, pdf(t)\, dt. \quad (9)$$

The order of the approximation depends on number of sigma points n. The higher n implies the better approximation, but at the cost of computational effort. The dimensionality of the problem can be increased to the desired extent. In this paper higher dimensions are included via tensor product rule.

Furthermore, stochastic collocation provides means for obtaining the sensitivity analysis (SA) for a given model. In this paper, variance based sensitivity analysis is considered. The influence of the i-th input random parameters on output value Z can be obtained in the following manner [6]

$$I_i = \frac{V_i(Z)}{V(Z)}. \quad (10)$$

where $V_i(Z)$ is variance obtained from univariate case for the i-th input variable and $V(Z)$ is the variance of the corresponding multivariate case. This approach enables ranking of variables from least to most influential ones, assuming weak interactions between random variables.

3. Computational results

The values of transient impedance for the horizontal grounding electrode are calculated for following deterministic parameters; length and radius of the electrode L=10 m and a=5 mm, respectively, depth of the electrode d=0.5 m and relative permittivity of the soil ε_r=10. As the paper aims to investigate the influence of variability of soil conductivity σ, lightning front time T_F and lightning time-to-half T_H, these three input variables are modeled as random, each prescribed with uniform distribution as follows; $\sigma \sim$ U(1, 10) (in mS/m), $T_F \sim$ U(0.4, 4) (in μs), $T_H \sim$ U(50, 70) (in μs).

3.1 Univariate test cases

The convergence of the method was tested for 3, 5, 7 and 9 SC points, upon which was concluded that satisfactory convergence is ensured by using 7 SC point. Figures 1 to 3 present mean value ± standard deviation of transient impedance for three univariate cases, i.e. when only one parameter is considered as random. Comparing the

figures, it is evident that soil conductivity has the highest impact on the value of transient impedance, since the spread of the results around the mean value is largest.

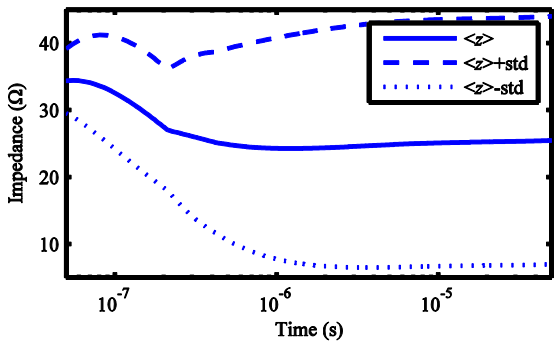

Figure 1. Mean value ± standard deviation of transient impedance for conductivity as a random variable.

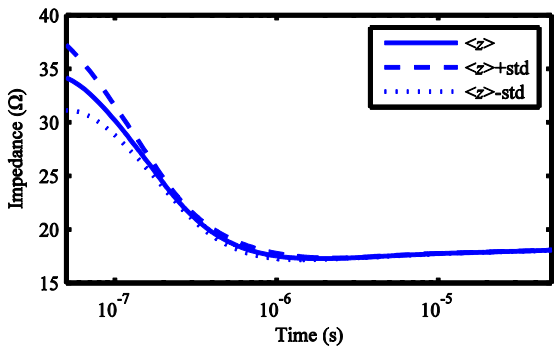

Figure 1. Mean value ± standard deviation of transient impedance for front time as a random variable.

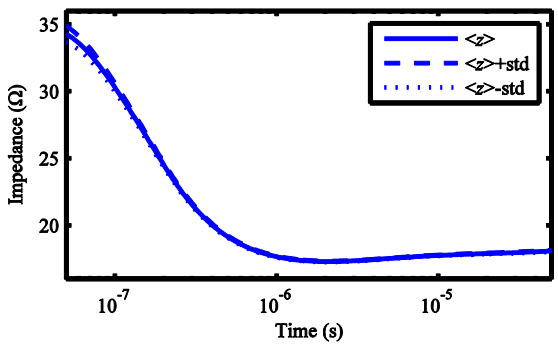

Figure 2. Mean value ± standard deviation of transient impedance for time-to-half as a random variable.

3.2 Multivariate test case

A multivariate test case, i.e. when all three parameters are considered random is presented in Figure 4. The assessment of the mean value and variance by using the SC full tensor model with 7x7x7 collocation points is given.

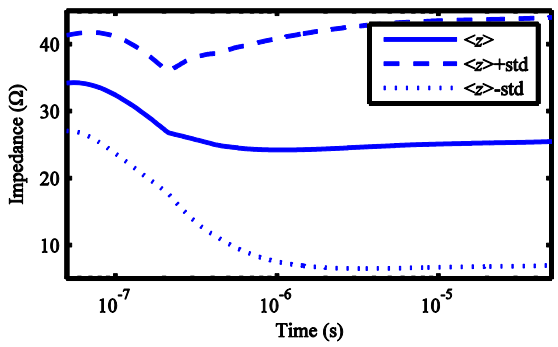

Figure 4. Mean value ± standard deviation of transient impedance for multivariate case.

Impact factor for each random variable calculated according to (10) is shown in Figure 5. It can be seen that the conductivity has the highest impact during the entire simulation period. When front time and time-to-half of the lightning pulse are considered it is evident that front time has higher impact at the early time period while time-to-half is more dominant in the steady state.

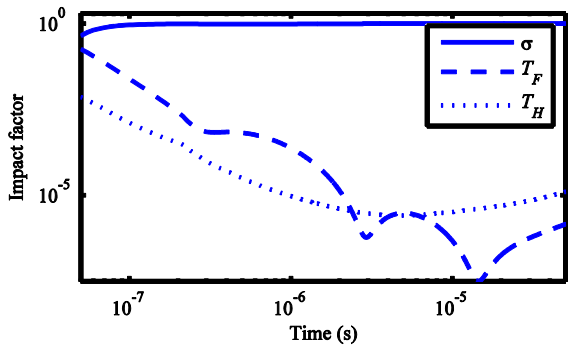

Figure 5. Impact factor for each random variable.

3.3 Maximum and steady state impedance

In Tables 1 and 2, analysis of maximum and steady-state impedance is shown, respectively.

Table 1. Maximum value of transient impedance.

	RV	$<Z_{max}>$ [Ω]	std(Z_{max}) [Ω]	I_i [%]
1RV case	σ	37.7991	12.8402	92.0
	T_F	34.1761	3.0164	5.0
	T_H	34.2745	0.6141	0.2
3RV case	σ, T_F, T_H	38.0147	13.3847	-

Table 2. Steady state impedance.

	RV	$<Z_0>$ [Ω]	std(Z_0) [Ω]	I_i [%]
1RV case	σ	25.4670	18.5321	99.9
	T_F	18.0702	0.0222	0.0
	T_H	18.0750	0.0660	0.0
3RV case	σ, T_F, T_H	25.4686	18.5335	-

Stochastic response of the maximum value of transient impedance is successfully obtained for defined stochastic model with 7 and 7x7x7 SC points for univariate and multivariate case, respectively. These results are presented in Table 1. Relevant information in Table 1 allows ranking of the random parameters from the most to the least influential. The highest impact is generated by soil conductivity. Nevertheless, the front time also has an observable influence. Time-to-half time has no significant impact on the maximum value of transient impedance. On the other hand, the results for steady state impedance presented in Table 2, show that significant influence comes only from the soil conductivity variations. The time variables have negligible impact on its value.

4. Concluding remarks

Stochastic analysis of random variation of parameters that have influence on the value of transient impedance is presented in this paper. Transient impedance of the horizontal grounding electrode is calculated using the deterministic model based on analytical solution of the frequency domain Pocklington equation. Statistical analysis is undertaken by using Stochastic Collocation method. Influence of three parameters is evaluated; soil conductivity, front time and time-to-half of the lightning pulse waveform. The analysis is performed for the entire time interval as well as for the maximum and steady state values of the impedance. As it was expected, the soil conductivity is the parameter with the highest impact to transient impedance variations. However, it is also observed that front time of the lightning pulse has an appreciable impact in the early time behavior, while time-to-half value is more significant in the later times. Additional analysis has been carried out on maximum and steady state values. The future work will consider other variables in the statistical analysis to detect additional significant influences of other parameters within transient impedance calculation.

5. References

[1] L. Grcev and F. Dawalibi, "An electromagnetic model for transients in grounding systems," *IEEE Transactions on Power Delivery,* vol. 5, no. 4, pp. 1773-1781, November 1990.

[2] S. Visacro, "A Comprehensive Approach to the Grounding Response to Lightning Currents," *IEEE Transactions on Power Delivery,* vol. 22, no. 1, pp. 381-386, 2007.

[3] G. Spadacini, F. Grassi and S. Pignari, "Statistical estimation of the electromagentic noise induced by field-to-wire coupling in random bundles of twisted-wire pairs," in *Asia-Pacific Symposium on Electromagnetic Compatibility 2015*, Taipei, 2015.

[4] L. Arnaut, "Optimizing Low-Frequency Mode Stirring Performance Using Principal Component Analysis," *IEEE Transactions on Electromagnetic Compatibility,* vol. 56, no. 1, pp. 3-14, 2014.

[5] Working Group 33.01 (Lightning), "Guide to procedures for estimating the lightning performance of transmission lines," CIGRE, 1991.

[6] S. Šesnić, S. Lallechere, D. Poljak, P. Bonnet and K. El Khamlichi Drissi, "A Stochastic Analysis of the Transient Current Induced along the Thin Wire Scatterer Buried in a Lossy Medium," *International Journal of Antennas and Propagation,* vol. 2016, pp. 1-12, 2016.

[7] D. Poljak, S. Sesnic and R. Goic, "Analytical versus boundary element modelling of horizontal ground electrode," *Engineering Analysis with Boundary Elements,* vol. 34, pp. 307-314, 2010.

[8] S. Šesnić and D. Poljak, "Antenna model of the horizontal grounding electrode for transient impedance calculation: Analytical versus boundary element method," *Engineering Analysis with Boundary Elements,* vol. 37, pp. 909-913, 2013.

[9] F. Paladian, P. Bonnet, S. Lallechere, A. Xemard and C. Miry, "On the numerical analysis of lightning effect on power installations," in *Int. Colloquium on Lightning and Power Systems*, Lyon, 2014.

[10] S. Lalléchère, S. Sesnic, P. Bonnet, K. El Khamlichi Drissi, F. Paladian and D. Poljak, "Sensitivity analysis of the time transient currents induced along thin wires buried in lossy and uncertain environments," in *EuCAP 2017*, Paris, 2017.

[11] D. Poljak, V. Doric, F. Rachidi, K. E. K. Drissi, K. Kerroum, S. V. Tkachenko and S. Sesnic, "Generalized Form of Telegrapher's Equations for the Electromagnetic Field Coupling to Buried Wires of Finite Length," *IEEE Transactions on Electromagnetic Compatibility,* vol. 51, no. 2, pp. 331-337, May 2009.

[12] T. Takashima, T. Nakae and R. Ishibashi, "Calculation of Complex Fields in Conducting Media," *IEEE Transactions on Electrical Insulation,* vol. 15, no. 1, pp. 1-7, February 1980.

[13] L. D. Grcev and F. E. Menter, "Transient Electromagnetic Fields Near Large Earthing Systems," *IEEE Transaction on Magnetics,* vol. 32, no. 3, pp. 1525-1528, May 1996.

[14] P. Bonnet, F. Diouf, C. Chauvière, S. Lalléchère, M. Fogli and F. Paladian, "Numerical simulation of a reverberation chamber with a stochastic collocation method," *Comptes Rendus Physique - New Approaches in Electromagnetic Compatibility,* vol. 10, pp. 54-64, 2009.

[15] B. Jinjun, Z. Gang, W. Lixin and A. Duffy, "Uncertainty analysis in EMC simulation based on Stochastic Collocation Method," in *2015 IEEE International Symposium on Electromagnetic Compatibility (EMC)*, Dresden, 2015.