



## A Research on SAGE Algorithm Based on Massive MIMO Channel Measurements

Qi Wang<sup>(1)</sup>, Bo Ai<sup>\*(1)</sup>, Ruisi He<sup>(1)</sup>, Jianzhi Li<sup>(1)</sup>, Zhangdui Zhong<sup>(1)</sup>, Nan Li<sup>(2)</sup>, and Hongfeng Qin<sup>(2)</sup>

(1) State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

(2) ZTE Corporation, China

Bo Ai<sup>\*</sup>(Corresponding Author): aibo@ieee.org

### Abstract

This paper investigates the channel parameter estimation algorithm by using the Space-Alternating Generalized Expectation-Maximization (SAGE). Two strategies, i.e., parallel interference cancelation (PIC) and serial interference cancelation (SIC), are implemented and validated in the simulated SV channel model. Based on the massive MIMO channel measurement results in an indoor hall scenario in the 6 GHz band, the channel parameters are jointly extracted. The similar results are obtained by using the SIC and the PIC, but the SIC demonstrates lower computational complexity. Finally, the performance of the SIC SAGE is improved by introducing a threshold when estimating the multipath delays. The proposed SAGE algorithm should be useful for extracting the channel parameters with higher accuracy.

### 1 Introduction

With rapidly increasing demands for higher mobile data [1], the research on the fifth generation (5G) mobile and wireless communication systems has drawn a lot of attentions recently [2, 3]. Innovative technologies in 5G communications include the massive multiple-input multiple-output (MIMO) [4], new spectrum allocations in the millimeter wave frequency bands [5], etc. As a foundation for any wireless communication, the research on propagation channels is important [6, 7]; it is also critical to establish reliable channel models to assist the high-layer design of communication systems. In order to achieve this, the initial channel parameters of multipath components (MPCs) should be extracted. These parameters includes the delays, angle of arrivals (AOAs), angle of departures (AODs), and complex amplitudes.

Various estimation algorithms have been applied to extract the channel parameters. Traditional algorithms include the spectral estimation algorithm such as the Bartlett [8], and the parametric subspace-based estimation algorithms for angular parameters such as the multiple signal classification (MUSIC) [9] and the estimation of signal parameter via rotational invariance techniques (ESPRIT) [10]. Based on the maximum likelihood (ML) method, the expectation-

maximization (EM) algorithm [11] is proposed to jointly extract the channel parameters. As an extension of the EM algorithm, the space alternating generalized expectation maximization (SAGE) [12] is proposed, which can reduce the computational complexity while obtain the high resolution channel parameters.

In this paper, the SAGE algorithm is firstly implemented to extract the delays, AODs, and complex amplitudes of MPCs in the simulated SV channel model. By using two different strategies, i.e., the parallel interference cancelation (PIC) and the serial interference cancelation (SIC), the SAGE algorithm is validated. Based on the massive MIMO channel measurement data in an indoor hall scenario at 6 GHz, the channel parameters of MPCs are extracted by using the PIC SAGE and SIC SAGE, and the results are further compared. Finally, the performance of the standard SAGE algorithm is improved by introducing a threshold when estimating the multipath delays.

The paper is organized as follows. The validation of the SAGE algorithm based on the SV channel model is presented in Section 2. The investigation on SAGE algorithm based on the massive MIMO channel measurement data is presented in Section 3. Section 4 concludes the paper.

### 2 Joint Estimation of Channel Parameters Using SAGE Algorithm

The multipath channel can be described by the channel impulse response (CIR)

$$h(\tau) = \sum_{l=1}^L \alpha_l \cdot \delta(\tau - \tau_l) \cdot \delta(\varphi - \varphi_l), \quad (1)$$

where  $\alpha_l$ ,  $\tau_l$ , and  $\varphi_l$  denote the complex amplitude, delay, and azimuth AOD of the  $l$ -th MPC, and  $L$  is the total number of MPCs.

For the MIMO channels, the CIR matrix can be expressed as

$$\mathbf{H}(\tau) = \sum_{l=1}^L \mathbf{a}(\varphi_l) \cdot \alpha_l \cdot \delta(\tau - \tau_l) \cdot \delta(\varphi - \varphi_l), \quad (2)$$

where  $\mathbf{a}(\varphi_l)$  is the steering vector, which can be easily obtained based on different arrangements of antenna arrays. In

(2), the unknown parameters are  $\theta = [\alpha_l, \tau_l, \varphi_l; l = 1, \dots, L]$ , which describes all the  $L$  MPCs and needs to be estimated. Thus, the observation that is used to estimate the channel parameters can be expressed as

$$\mathbf{X} = \sum_{l=1}^L s(\theta_l) + \mathbf{n}, \quad (3)$$

where  $s(\theta_l) = \mathbf{a}(\varphi_l) \cdot \alpha_l \cdot \delta(\tau - \tau_l) \cdot \delta(\varphi - \varphi_l)$ , and  $\mathbf{n}$  denotes the complex noise.

The SAGE algorithm is applied to jointly estimate the channel parameters in this paper. Two steps are usually included in this algorithm, i.e., the Expectation Step (E-step) and Maximization step (M-step). More details of the SAGE algorithm can be found in [12]. Note that two methods are widely utilized in the E-step. In the standard SAGE [12], the PIC strategy is used during the E-step. This is given as follows

$$\hat{x}_l = \mathbf{X} - \sum_{l' \neq l, l'=1}^L x(\hat{\theta}_{l'}), \quad (4)$$

where  $\hat{x}_l$  denotes the estimate of  $l$ -th MPC, and  $x(\hat{\theta}_{l'})$  denotes the updated estimate of  $l'$ -th MPC in the previous iteration.

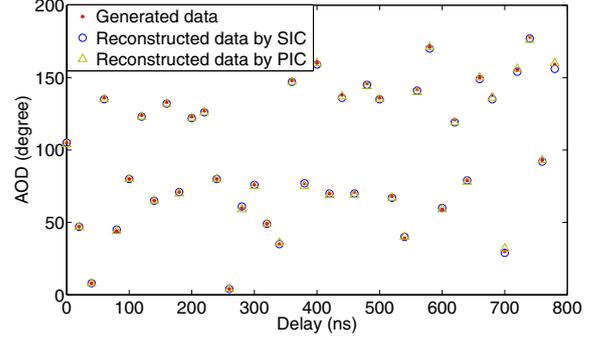
SIC is another strategy during the E-step. This can be expressed as follows

$$\hat{x}_l = \mathbf{X} - \sum_{l' < l, l'=1}^L x(\hat{\theta}_{l'}). \quad (5)$$

In this scheme, the MPCs are estimated and canceled successively from the received CIR in the descending order of their powers.

The performance of the SAGE algorithm by using different strategies are evaluated by the statistical SV model [13], and the AODs of MPCs are randomly generated between  $0^\circ$  to  $180^\circ$ . Totally eight clusters are generated in the simulated SV model, and the powers of the clusters decrease exponentially with delay. Five MPCs are generated within each cluster, and the powers of the MPCs follows another exponential distribution with delay. Furthermore, complex white Gaussian noise is added to the simulated channel model.

Fig. 1 gives the extracted results in SAGE by using the PIC and the SIC. From this figure, it can be concluded that both the methods can successfully extract all the MPC parameters with fairly low estimation errors, and the estimation difference between these two methods is not distinct. Comparing with the SIC, however, longer running time of the simulation for the PIC shows higher computational complexity. This is because the observation  $\mathbf{X}$  in the PIC strategy contains more MPCs when estimating the channel parameters. These two strategies will be further compared with the real-world channel measurements in Section 3.



**Figure 1.** Extracted results in SAGE by using the PIC and the SIC based on the SV model.

### 3 Parameter Estimation Based on Massive MIMO Channel Measurements

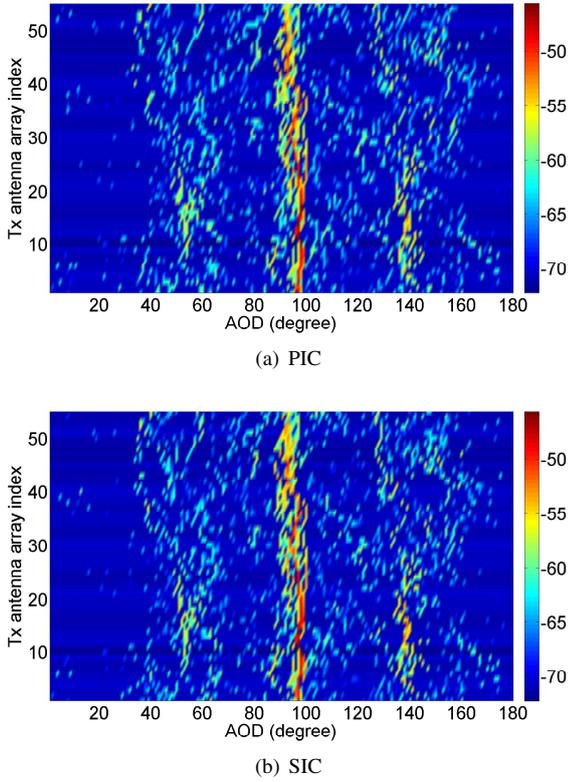
The real-world measurement results are applied to further test the performance of the SAGE algorithm. The massive MIMO channel measurements were performed at the Center Report Hall in Beijing Jiaotong University. Both the transmitter (Tx) and receiver (Rx) antennas were omnidirectional, and the numbers of antennas at the Tx and the Rx were 64 and 4, respectively. The measurements were conducted at 6 GHz with a bandwidth of 200 MHz, which means that the delay resolution of individual MPC was 5 ns. Details of the measurement campaign can be found in [14].

The SAGE algorithm described in Section 2 is used for the massive MIMO channel measurements at 6 GHz in the line-of-sight (LOS) condition. The parameters for the 40 MPCs with the strongest powers are extracted. The delay resolution in our SAGE algorithm is 5 ns, which equals to the value of the measurement system, and the angle resolution is defined as  $1^\circ$ . The extracted power angular spectrum (PAS) over the large-scale Tx array by using both the PIC and the SIC is illustrated in Fig. 2. However, the ground truth of the angular parameters can hardly be obtained from the measured data. Thus, to evaluate the accuracy of the estimation algorithms, the root mean square estimation error (RMSEE) between the estimated power delay profile (PDP)  $P_{estimate}$  and the measured PDP  $P_{measure}$  is calculated along the Tx antenna array, and the RMSEE at the  $m$ -th Tx position  $RMSEE_m$  is defined in (6), where  $n$  is the Rx antenna index, and  $N_{Rx} = 4$  is the number of Rx antennas. The parameter  $k$  ( $=0, 1, \dots, 511$ ) is the delay bin index, and  $N_\tau = 512$ .

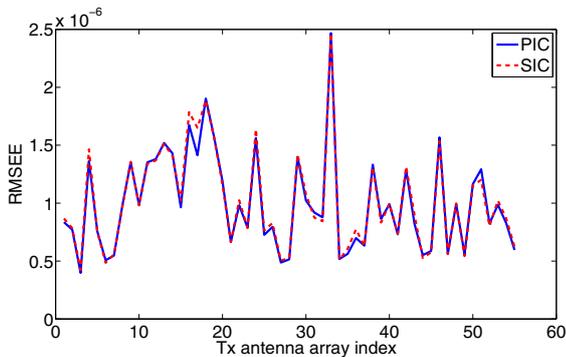
$$RMSEE_m = \sqrt{\frac{\sum_{n=1}^{N_{Rx}} \sum_{k=1}^{N_\tau} |P_{measure}(n, m, k) - P_{estimate}(n, m, k)|^2}{N_{Rx} \cdot N_\tau}}. \quad (6)$$

The RMSEEs by using the PIC and the SIC are presented in Fig. 3. From Figs. 2 and 3, it is observed that no distinct difference can be found for the two methods, as expected.

Considering the computational complexity analyzed in Section 2, the SIC is chosen during the E-step in the following simulation.



**Figure 2.** Extracted PAS over the large-scale Tx array by using different strategies.

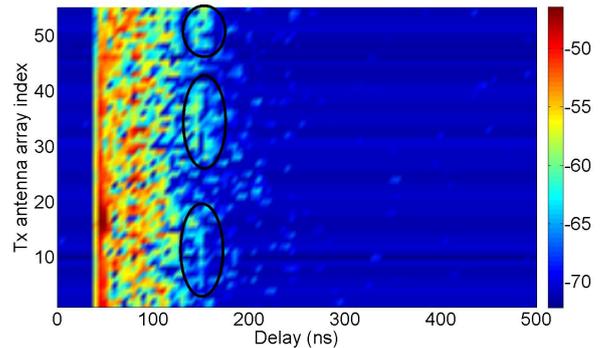


**Figure 3.** RMSEEs by using PIC and SIC.

In the standard SAGE by using SIC strategy, the estimated parameters of the  $l$ -th MPC should be subtracted from the observation of the measured data in each of the iteration, and the estimating process will be continue from the remaining data. Hence, the estimated delays of several MPCs may be the same, which is also reasonable from the propagation aspect as the reflections or scatterings from different paths may have the similar propagation distance. However, some low-power MPCs can always be extracted, and most of their delays are the same with the strong MPCs estimated in SAGE. This shows that the powers are over estimated in

some delays. To solve this problem, a threshold is defined for each estimated delay bin: if the difference between the sum of the estimated power and the measured power in a specific delay is smaller than the noise threshold, the data in the corresponding delay will be subtracted from the observation of the measured data in each of the iteration.

To evaluate the performance of the proposed SAGE algorithm, the PDPs over the large-scale Tx antenna array are utilized. First, the measured PDPs are presented in Fig. 4. The estimated PDPs by using the standard SAGE and the proposed SAGE are then illustrated in Fig. 5. By comparing the results in Fig. 4 and Fig. 5, we can see that most of the strong MPCs can be extracted by both the standard SAGE and the proposed SAGE. However, the MPCs with the delay range of 70 ns to 90 ns seem to be over estimated in the standard SAGE. Moreover, the MPCs marked with circles in Fig. 4 can not be extracted completely in the standard SAGE, whereas these MPCs can be found in the proposed SAGE, as shown in Fig. 5(b). Hence, it can be concluded that the performance of the proposed SAGE can be improved when estimating the parameters of MPCs.



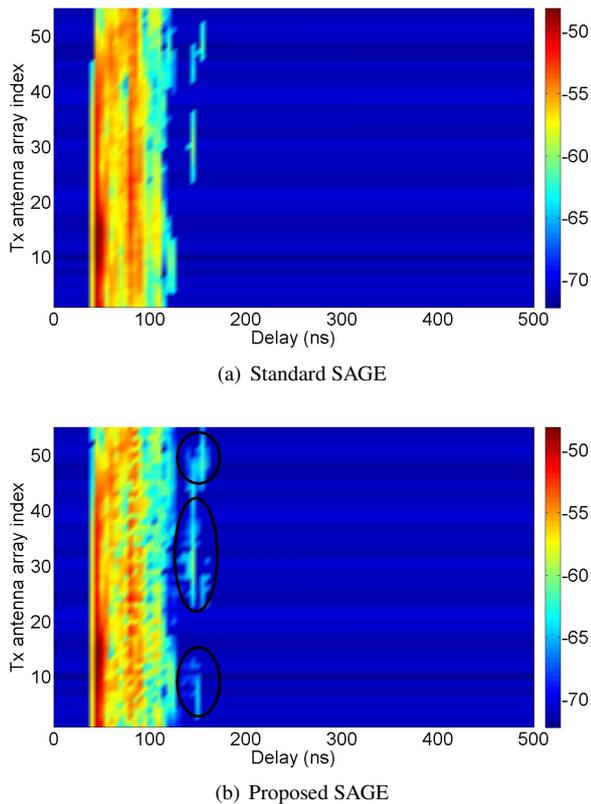
**Figure 4.** Measured PDPs over the large-scale Tx antenna array.

## 4 Conclusion

This paper investigates the SAGE algorithm, which can jointly extract the delays, angles, and complex amplitudes of MPCs. The standard SAGE is validated by using both the PIC and SIC strategies in the simulated SV channel model. Based on the real-world massive MIMO channel measurement results, the SIC SAGE and PIC SAGE show the similar extracted results. Considering the lower computational complexity, the SIC SAGE is therefore recommended. The performance of the standard SIC SAGE is further improved by defining a threshold when estimating the delays of MPCs. These results will be useful to accurately obtain the initial channel parameters before channel modeling in 5G communication systems.

## 5 Acknowledgements

This work is supported in part by the National Natural Science Foundation of China under Grant U1334202, the



**Figure 5.** Estimated PDPs over the large-scale Tx array by using different methods in SAGE.

Natural Science Base Research Plan in Shanxi Province of China under Grant 2015JM6320, and Key Project from Beijing science and Technology Commission under Grant D151100000115004, the National Natural Science Foundation of China under Grant 61501020, the State Key Laboratory of Rail Traffic Control and Safety under Grant RCS2016ZJ005, the China Postdoctoral Science Foundation under Grant 2016M591355, the Fundamental Research Funds for the Central Universities (No. 2016JBZ006), the Special Project of Cultivation and Development of Science and Technology Innovation Base in 2015.

## References

- [1] B. Ai, K. Guan, M. Rupp et al., "Future railway services-oriented mobile communications network," *IEEE Communications Magazine*, vol. 53, no. 10, pp. 78-85, 2015.
- [2] A. Gohil, H. Modi, and S. Patel, "5G technology of mobile communication: A survey," in *Proc. ICISSP*, March 2013, pp. 288-292.
- [3] B. Ai, X. Cheng, T. Klärner et al., "Challenges toward wireless communications for high-speed railway," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 5, pp. 2143-2158, October 2014.
- [4] L. Lu, G. Y. Li, A. L. Swindlehurst et al., "An overview of massive MIMO: Benefits and challenges," *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 5, pp. 742-758, 2014.
- [5] S. Sun, T. S. Rappaport, T. A. Thomas et al., "Investigation of Prediction Accuracy, Sensitivity, and Parameter Stability of Large-Scale Propagation Path Loss Models for 5G Wireless Communications," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 2843-2860, May 2016.
- [6] R. He, B. Ai, G. Wang et al., "High-Speed Railway Communications: From GSM-R to LTE-R," *IEEE Vehicular Technology Magazine*, vol. 11, no. 3, pp. 49-58, September 2016.
- [7] R. He, O. Renaudin, V.-M. Kolmonen et al., "Characterization of quasi-stationarity regions for vehicle-to-vehicle radio channels," *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 5, pp. 2237-2251, May 2015.
- [8] M. Bartlett, "An Introduction to Stochastic Processes with Special References to Methods and Applications," Cambridge University Press, New York, 1961.
- [9] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. AP-34, pp. 276-280, March 1986.
- [10] R. Roy and T. Kailath, "ESPRIT: Estimation of signal parameters via rotational invariance techniques," *IEEE transactions on acoustics, speech, and signal processing*, vol. 37, pp. 984-995, July 1989.
- [11] M. Feder and E. Weinstein, "Parameter estimation of superimposed signals using the EM algorithm," *IEEE transactions on acoustics, speech, and signal processing*, vol. 36, pp. 477-489, Apr. 1988.
- [12] B. H. Fleury, P. Jourdan, and A. Stucki, "High-resolution channel parameter estimation for MIMO applications using the SAGE algorithm," in *Proc. IZSBC*, February 2002, pp. 30-1-30-9.
- [13] A. A. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *IEEE Journal on Selected Areas in Communications*, vol. 5, no. 2, pp. 128-137, 1987.
- [14] J. Li, B. Ai, R. He et al., "Measurement-based characterizations of indoor massive MIMO channels at 2 GHz, 4 GHz, and 6 GHz frequency bands," in *Proc. 2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, Nanjing, China, 2016, pp. 1-5.