SA-Based Orbital Design Method for GEO-BiSAR Resolution Improvement

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***Abstract*—Compared with the monostatic geosynchronous synthetic aperture radar (GEO-SAR) system, GEO bistatic SAR (GEO-BiSAR) shows great advantages in finer spatial resolution and higher signal-to-noise ratio (SNR) with less system complexity. Many scholars have done researches on resolution of GEO-BiSAR, their models consist of a GEO transmitter and a low Earth orbit (LEO) receiver. The methods of improving the resolution by using two or more receivers are rare. In this paper, the system consists of a geosynchronous illuminator and two LEO receivers. The spectrum shift of SAR images could be introduced by multi-view angles of acquired images in the same coverage. We can widen the bandwidth of SAR images by combining the spectrum of two images to improve the resolution. The formation configuration design of two receivers is the crux of the matter. Then in this paper, an orbital design method of two receivers based on simulated-annealing (SA) algorithm is presented.**

Keywords*—*GEO-LEO bistatic synthetic aperture radar (BiSAR), baseline vector, resolution improvement factor, spectral shift, orbital design, SA algorithm

I. Introduction

Due to the characteristics of wide coverage and short revisit time, GEO SAR has aroused more attention of scholars in recent years. And GEO-BiSAR has advantages over GEO monostatic SAR as it can observe the regions more flexibly, obtain finer resolution and require much lower transmitting power [?], all of which have led the GEO-BiSAR to be a focus of research. Resolution is the main indicator concerning the imaging performance of the GEO-BiSAR. In [?] [?], based on the generalized ambiguity function method, range and azimuth resolution are calculated. Furthermore, [?] uses genetic algorithm (GA) and acquires several solutions of desired resolution.

According to the theory of spectral shift proposed by Prati and Rocca, Milan Polytechnic University [?], there are relative frequency offsets between different echo data. The key to improve the resolution is increasing system bandwidth directly or indirectly [?]. Rather than increase system complexity and power consumption greatly to broaden the bandwidth, we can obtain the data with wider bandwidth and get the image with higher resolution by using two or more measurements on the same target. Gatelli used wavenumbers to explain the principle of spectral offset [?]. In [?], the influence of spectrum offset in improving the resolution has been proved. The above-mentioned research is based on the monstastic models.

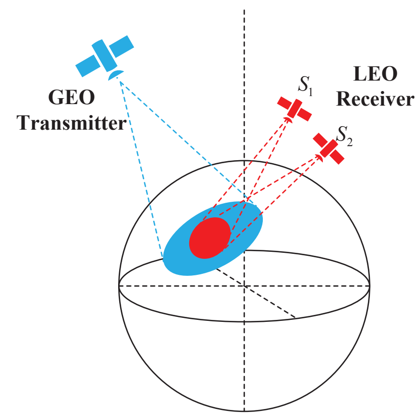

Fig. 1. Imaging geometry of GEO-LEO BiSAR

In this paper, the principle of the resolution improvement is applied to the GEO-Bistatic SAR model. And we suppose the resolution is fixed, then use an added receiver to the original GEO-BiSAR model, the configuration geometry of GEO-LEO BiSAR is shown in Fig.1. In order to measure the ratio of resolution improvement, we introduce the definition of resolution improvement factor. The baseline vector is used as an intermediate variable to acquire the relationship between resolution improvement factor and orbit element differences. Then the configuration can be accessed.

II. RESOLUTION IMPROVEMENT ANALYSIS FOR GEO-LEO BISAR

A. *Resolution Improvement Based on Spectrum Shift*

Consider a GEO-LEO BiSAR system containing a GEO transmitter and two LEO receivers. With a GEO satellite as the transmitter, wide-swath illuminated area is available for the LEO receivers with shorter revisit cycle and finer resolution [?]. The observation geometry of GEO-LEO BiSAR in centroid orbit coordinate (COC) is shown in Fig. 2.

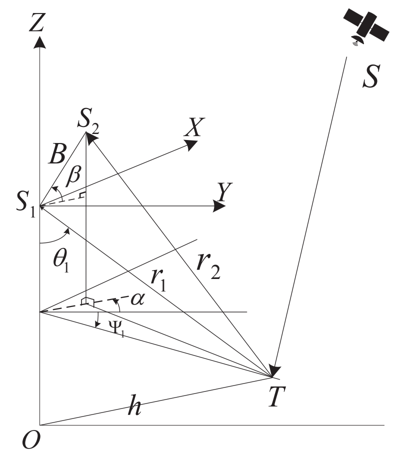

Fig. 2. Imaging geometry of GEO-LEO BiSAR in COC

Different receivers with different receiving angles result in the data acquired by different receiving satellites having spectrum shift in range and azimuth directions. We can obtain the spectrum of the coherent scattering coefficient with appropriate spectrum shift and phase compensation. The spectrum can be broaden through the coherent superposition, and a finer resolution can be achieved.

The wave number spectrum offset of signal received by S1 to the signal received by S2 in range and azimuth direction are [?] :

$$\Delta k_r = \frac{2\pi}{\lambda} \frac{B_\perp^r}{r} \cos\theta_c \tag{1}$$

$$\Delta k_a = \frac{2\pi}{\lambda} \frac{B_\perp^a}{r} \sin\varphi_c \tag{2}$$

$$\theta_c = \frac{\theta_1 + \theta_2}{2} \tag{3}$$

$$\varphi_c = \frac{\varphi_1 + \varphi_2}{2} \tag{4}$$

where λ is the wavelength of the transmitted signal, r is the distance from the target to the receiver S1, θ_i,ϕ_i are the incidence angles and conical angles of receiver S1 and receiver S2, respectively. $B_\perp^r$ is the vertical effective baseline in range direction, $B_\perp^a$ is the vertical effective baseline in azimuth direction, and they have the form:

$$B_\perp^r = B(sin\beta sin\theta_1 + cos\beta cos\theta_1 cos(\alpha + \varphi_1)) \tag{5}$$

$$B_\perp^a = Bcos\beta sin(\alpha + \varphi_1) \tag{6}$$

where B is the baseline length from receiver S1 to receiver S2, α represents angle of azimuth and β represents angle of pitch.

The original signal spectral width of two receivers are approximately equal, bandwidth in range direction and azimuth direction are given by

$$B_{k_r} = 2\pi/\rho_r \tag{7}$$

$$B_{k_a} = 2\pi/\rho_a \tag{8}$$

where ρ_r,ρ_a are the bistastic range and azimuth resolution, respectively.

Here, we define a resolution improvement factor, it represents the ratio of the expanded bandwidth to the original bandwidth. So the resolution improvement factor in range and azimuth direction have the form

$$\eta_r = (1 + \frac{\Delta k_r}{2\pi/\rho_r}) \tag{9}$$

$$\eta_a = (1 + \frac{\Delta k_a}{2\pi/\rho_a}) \tag{10}$$

The limited baseline in two directions can be obtained when $\Delta k_r = B_{k_r}, \Delta k_a = B_{k_a}$(In theory, when the spectrum shift equals to the original bandwidth, there does not exist spectrum overlapping), and the limited baseline in two directions are

$$B_{\perp c}^r = \lambda r/(\rho_r cos\theta_c) \tag{11}$$

$$B_{\perp c}^a = \lambda r/(\rho_a cos\theta_c) \tag{12}$$

So the resolution improvement factors can also be expressed as

$$\eta_r = (1 + \frac{B_\perp^r}{B_{\perp_c}^r}) \tag{13}$$

$$\eta_a = (1 + \frac{B_\perp^a}{B_{\perp_c}^a}) \tag{14}$$

B. Receivers Formation Configuration Analysis Based on Orbit Element Differences

The aim of the receivers formation configuration design for our system is to identify that under what receivers geometry can we achieve the finest resolution improvement. Based on the given analysis, the resolution improvement factor can be represented as a function of the baseline vector. In this section, we use baseline vector as an intermediate variable to deduce the relationship between resolution improvement factor and orbit element differences. Once the orbit elements of receiver S1 is determined, the finest formation configuration can be acquired.

The relationship between the baseline vector and its representation in S1 centroid orbit coordinate system are given by

$$\begin{cases} x(t) = B(\mathrm{t})cos\beta(t)cos\alpha(t) \\ y(t) = B(\mathrm{t})cos\beta(t)sin\alpha(t) \\ z(t) = B(\mathrm{t})sin\beta(t) \end{cases} \tag{15}$$

$$\begin{cases} B(t) = \sqrt{x^2(t) + y^2(t) + z^2(t)} \\ \alpha(t) = arctan\frac{y(t)}{x(t)} \\ \beta(t) = arcsin\frac{z(t)}{B(t)} \end{cases} \tag{16}$$

Suppose that the orbital elements of receiver S1 are $(a_s, e_s, i_s, \Omega_s, w_s, \tau_s)$, S1 is set to be the reference satellite, S2 is the tributary satellite .The position of S2 in S1 centroid orbit coordinate system can be represented as a linear function of orbit element differences [?]:

$$[x(t), y(t), z(t)]^T = A(\mathrm{t})[\sigma e, \sigma i, \sigma\Omega, \sigma w, \sigma\tau]^T \tag{17}$$

where

$$A(t) = \begin{bmatrix} a_e & 0 & a_p & a_s & n_s a_s \\ 0 & -a_q & a_l & 0 & 0 \\ a_q & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$\begin{cases} a_e = 2a_s \sin f_s(t) \\ a_p = a_s \cos i_s \\ a_q = a_s \cos f_s(t) \\ al = -a_s \sin f_s(t) sini_s \end{cases} \tag{19}$$

Here n_s represents the velocity of receiver S1. Since there is a one-to-one correspondence between true anomaly f and time to cross the ascending node τ, once $\delta\tau$ is determined, σf can be calculated.

In order to ensure companion flying, the orbital semi-major axis of receiver S2 a_2 should be the same as that of receiver S1 a_s. Based on the analysis above, the relationship between resolution improvement factor and orbit element differences has been established, and it has the form

$$[\delta e, \delta i, \delta\Omega, \delta w, \delta\tau]^T = A^{+}(t_0)[x(t_0), y(t_0), z(t_0)]^T \tag{20}$$

where $A^{+}(t_0)$ is the Moore-Penrose generalized reverse of $A(t_0)$.

III. SA-BASED RECEIVER FORMATION CONFIGURATION DESIGN

In order to assess the performance, J is put forward to be our objective function of this optimization problem, which is given by:

$$J = k_r\eta_r + k_a\eta_a \tag{21}$$

where k_r, k_a are the proportional coefficients. Obviously, J can be written as a function of baseline vector (B, α, β).

SA is a stochastic global search method that derived from solid annealing. Compared with traditional optimization algorithms, SA introduces a random factor. As it accepts a worse solution than current solution with an uncertain probability, it would run out of the local optimal solution to the global optimal solution, which leads to our choice of SA. The flowchart of SA-based baseline parameters optimization is shown in Fig.3 SA-based receiver orbit design operation is shown in the following part.

IV. EXPERIMENTAL RESULTS

In our system, 1 GEO transmitter and 2 LEO receivers are utilized. The simulating system parameters in the following simulation are shown in Table 1. Based on the simulating parameters, resolution improvement factor can be completely determined by B, α, β. Fig.4 shows the influences of azimuth angle and pitch angle on the vertical effective baseline in range and azimuth directions, when B^r and B^a are set to be the limited baseline in two directions, respectively. Here, we assume that both the initial range and azimuth resolution are $2m$. θ_1 is set to be $35°$ and ϕ_1 is set to be $95°$.

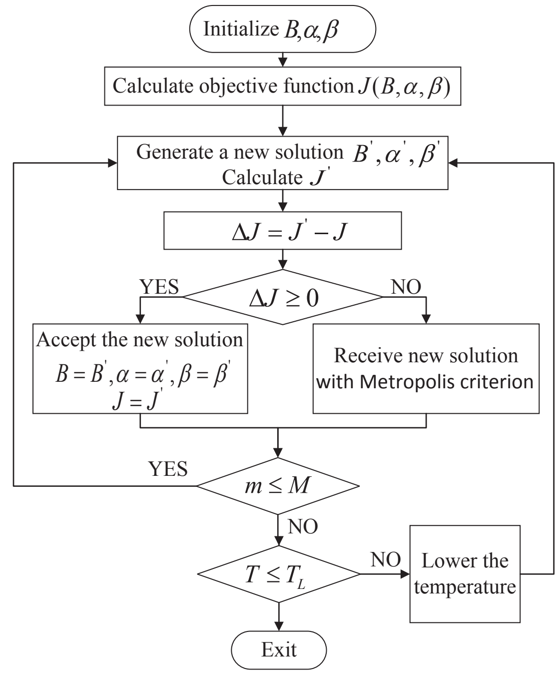

Fig. 3. Flowchart of SA-based baseline parameters optimization

TABLE I. SIMULATED PARAMETERS OF GEO-BISAR

Carrier frequency	1Ghz	Transmitter beam coverage area	315.2km
Transmitting signal bandwidth	80Mhz	Transmitter slant range	36122km
Transmitter orbit height	35786km	Receiver orbit height	800km
Transmitter incidence angle	$3°$	Receiver beam width	$10°$
Transmitter beam width	$5°$	Receiver slant range	944km
Transmitter speed	3.08km/s	Receiver speed	7.47km/s

It can be observed that the $B_\perp^r$ is the finest when α= $80°$ and β= $45°$, $B_\perp^a$ is the finest when α=$0°$, β=$0°$.

In order to access the finest resolution improvement radio, in other words, to maximize function J, the finest baseline vector is needed. SA can be utilized to obtain the global optimal solutions of the function. Here, we set both k_r, k_a to be 0.5 as we take the resolution improvement in range direction is as important as in azimuth direction. Initial temperature and minimum limit temperature of SA algorithm are set to 100 and 3, annealing factor is set as 0.95, iteration time is set to 1000. The optimum result obtained by SA is $B_{opt} \approx 3.13km$, $\alpha_{opt} = -49.1°$, $\beta_{opt} = 20.29°$ when J reaches its maximum 1.70.

It can be seen from (15) that the coordinate value of receiver S2(x_0, y_0, z_0) in S1 centroid orbit coordinate system can be calculated. According to (20) the optimal formation configuration is obtained, and the orbit elements are shown in Table2.

Also, based on the above analysis, we can set different values of k_r, k_a for different mission design. Table 3 and Table

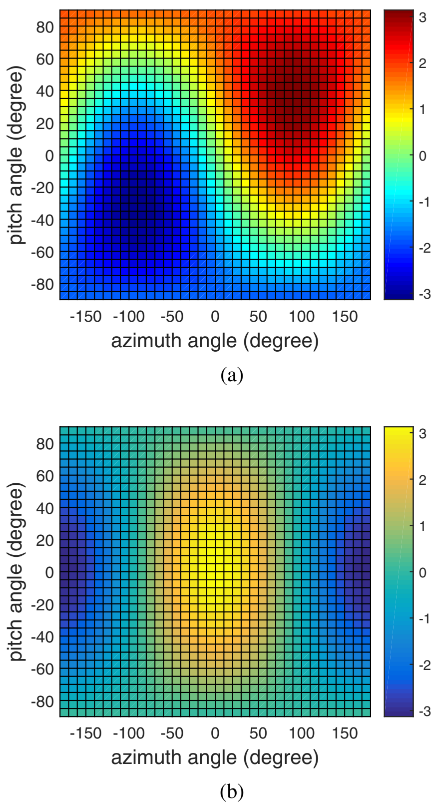

Fig. 4. vertical effective baseline with respect to α and β. (a) vertical effective baseline in range direction,(b)vertical effective baseline in azimuth direction

TABLE II. OPTIMIZATION FORMATION PARAMETERS $(k_r = 0.5, k_a = 0.5)$

Receiver satelites	Orbit elements $(a, e, i, \Omega, \omega, f)$
Receiver S1	[7171.017, 0.002, 53.10213, 60.8203584, 13.8156473, 0]
Receiver S2	[7171.017, 2.1517e-3, 53.10244, 60.8203612, 13.8156520, 2.10e-6]

TABLE III. OPTIMIZATION FORMATION PARAMETERS $(k_r = 0.7, k_a = 0.3)$

Receiver satelites	Orbit elements $(a, e, i, \Omega, \omega, f)$
Receiver S1	[7171.017, 0.002, 53.10213, 60.8203584, 13.8156473, 0]
Receiver S2	[7171.017, 2.2585e-3, 53.10246, 60.8203625, 13.8156495, 9.97e-7]

TABLE IV. OPTIMIZATION FORMATION PARAMETERS $(k_r = 0, k_a = 1)$

Receiver satelites	Orbit elements $(a, e, i, \Omega, \omega, f)$
Receiver S1	[7171.017, 0.002, 53.10213, 60.8203584, 13.8156473, 0]
Receiver S2	[7171.017, 2.0083e-3, 53.10214, 60.8203630, 13.8156571, 3.41e-5]

4 show the final orbit parameters when k_r, k_a are set to be $(0.7, 0.3)$ and $(0, 1)$.

V. Conclusion

In this paper, we proposed an orbital design method of two receivers based on simulated-annealing algorithm to improve the resolution of GEO-BiSAR. The spectrum shift of SAR images could be introduced by the multi-view angles of acquired images to broaden the bandwidth, thus improve the resolution. Baseline vector is used as an intermediate variable to establish the relationship between resolution improvement factor and orbit element differences, and then we can acquire the formation configuration of two receivers. In the experiment, we set three different value sets of k_r, k_a, but in the further study, the proportional coefficients k_r, k_a can be adjusted flexibly according to the mission design to meet the desired performance.

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