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Analysis of Duobinary Encoding for CPM Signals

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Abstract

In this paper, we investigate the effect of using a duobinary encoder on binary continuous phase modulations (CPM). This technique transforms the binary symbols into ternary ones using a specific mapping and has been adopted for linear modulation schemes. We show that applying this encoder to binary CPM increases the spectrum efficiency of the signal and keeps a theoretical bit error rate performance close to the binary case.

1 Introduction

Continuous phase modulation (CPM) is a class of bandwidth efficient modulations widely used in wireless digital communications [1]. The fact that the transmitted signals have constant envelopes makes CPM very attractive for long distance wireless transmissions as power amplifiers can be used near their maximal throughput without distorting the signal. In addition to this, CPM signals are also very robust towards the Doppler effect that often occurs during a wireless transmission.

The increasing demand of having high data rates pushes the use of several techniques to boost the spectrum efficiency of CPM while keeping an acceptable theoretical bit error rate (BER) performance. Some of these techniques consist of using waveforms that overlap several bits periods like the Gaussian pulse shape that is used in GSM systems [2]. More evolved techniques consist of increasing the alphabet constellation and using different modulation indexes like the ARTM CPM modulation that is standardized in the IRIG recommendations for aeronautical telemetry (see [3] for more details). However, dealing with such modulations requires high complexity receivers.

In this paper, we focus on another way to increase the spectrum efficiency of a CPM signal while maintaining a low complexity implementation thanks to duobinary encoding. This technique has been originally used for linear modulations and allows to achieve important spectral efficiency gains [4]. After giving in section 2 the signal model of CPM, we investigate in section 3 whether this encoding technique has the same effect on the bandwidth efficiency of binary CPM as well as its BER performance.

2 CPM Signal Model

The complex envelope of a binary CPM signal is expressed as follows [1]

$$s(t;\bar{\gamma}) = \exp\left\{ j\sum_i \gamma_i q(t-iT)\right\}, \tag{1}$$

where T is the bit time duration and $\bar{\gamma} = \{\gamma_i, i \in \mathbb{Z}\}$ the binary information symbols. The function $q(t)$ is called the phase pulse and represents the time integral of the frequency pulse $g(t)$ whose time support is equal to LT. If $L = 1$, the signal has a full response frequency pulse. Otherwise, the signal has a partial response one. The phase pulse $q(t)$ is defined as

$$q(t) = \begin{cases} 0, & t \le 0 \\ 2h\pi \int_0^t g(\tau)d\tau, & 0 < t < LT \\ h\pi, & t \ge LT, \end{cases} \tag{2}$$

where h is the modulation index and $\int g(t)dt = \frac{1}{2}$. The spectrum efficiency of the signal can be controlled via the modulation index or the frequency pulse. For a given modulation index, full response systems suffer from low spectrum efficiency, but they offer the best bit error rate (BER) performance in an additive white Gaussian noise (AWGN) channel. As for partial response systems, the spectrum efficiency is better but the inter symbol interference (ISI) introduced by the the frequency pulse causes BER performance loss compared to full response systems. In this paper, we take as references two signals having half integer modulation index: the first one is a full response binary CPM known as minimum shift keying (MSK), and the second one is a partial response CPM with a Gaussian frequency pulse, known as Gaussian minimum shift keying (GMSK) [1]. We focus on GMSK having a bandwidth-time product parameter (BT) equal to 0.25. We introduce in the next section a precoder that can be used to increase their spectrum efficiency and we study its impact on the BER performance in an AWGN channel.

3 Duobinary Encoder for CPM signals

3.1 Duobinary Encoder

We assume that the binary symbols are encoded using a duobinary encoder before passing through the CPM modulator like shown in Fig. 1, i.e. for a random binary sequence $\gamma_i \in \{-1,+1\}$

$$\alpha_i = \frac{1}{2}(\gamma_i + \gamma_{i-1}), \tag{3}$$

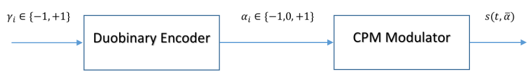

Figure 1. CPM modulator using a duobinary encoder

The duobinary encoder generates ternary symbols belonging to the alphabet $\{-1,0,+1\}$ and respects the following rules: a direct transition from $\alpha = -1$ to $\alpha = +1$ (or $\alpha = +1$ to $\alpha = -1$) never occurs. A zero state is necessary. This rule is of great interest since it smooths the phase transition of the CPM signal by creating an offset in its phase trajectory and thus it increases its spectrum efficiency.

3.2 Spectrum efficiency analysis

To illustrate the aforementioned propriety, we plot in Fig. 2 the spectrum of the classical MSK modulation as well as the spectrum of MSK whose alphabet is generated using the duobinary encoder, namely duobinary MSK. We can notice

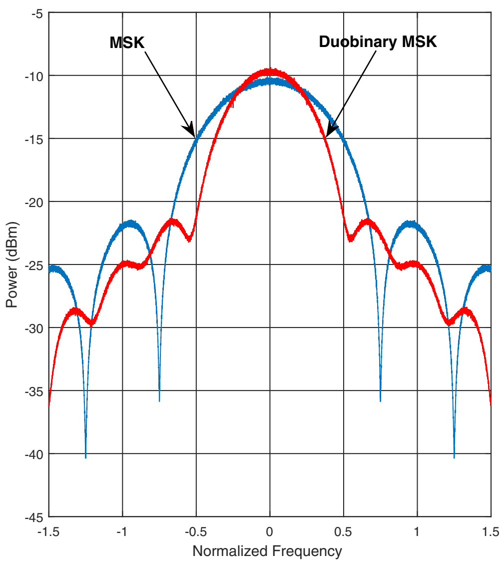

Figure 2. Spectrum of Binary MSK and duobinary MSK

that the main lobe is narrower and the secondary lobe is slightly lower than the binary case. The values of the spectrum efficiency for the different signals are given in table 1 and show that the occupied bandwidth decreases by 27.5% when we apply the duobinary encoder to MSK signal and by 13.6% for GMSK.

Table 1. Spectrum Efficiency (bits/s)/Hz

MSK	Duobinary MSK	GMSK ($L=4, BT=0.25$)	Duobinary GMSK ($L=4, BT=0.25$)
0.833	1.15	1.163	1.346

We now compare the spectral efficiency gain introduced by the duobinary encoder for linear modulations to the one obtained for CPM modulations. Introducing the encoder to a binary linear modulation with a rectangular pulse shape divides the occupied bandwidth by two as shown in [4], which is therefore higher than the gain introduced by the encoder for MSK despite the fact that its frequency pulse is rectangular as well. The difference is due to the non linear nature of CPM modulations, i.e, CPM is not a linear function of the transmitted symbols. Using the pulse amplitude modulation (PAM) decomposition of the CPM signal as developed in [5, 6], it can be shown that duobinary MSK can be written as a sum of 2 PAM waveforms weighted by their pseudo-symbols. These pseudo-symbols however cannot be written as a result of a duobinary encoding process and thus the duobinary MSK spectrum cannot be two times narrower than the classical MSK spectrum.

3.3 Performance Analysis

The bit error probability P_b of the optimal receiver can be determined by using the concept of error events and minimum distances. The normalized Euclidean distance of CPM is defined as [1]

$$d^2 = \frac{log_2(M_s)}{2T} \int |s(t,\bar{\gamma}_I) - s(t,\bar{\gamma}_J)|^2 dt \tag{4}$$

where $log_2(M_s)$ is the number of information bits per symbol, which is equal to 1 for binary CPM $s(t,\bar{\gamma})$ as well as duobinary CPM $s(t,\bar{\alpha})$. Since the latter can be seen as a binary CPM signal with an equivalent phase pulse, which is equal to the half sum of the real phase pulse and its time shifted version, the error sequences may be determined based on the antipodal binary sequence $\bar{\gamma}$ that generates the ternary sequence $\bar{\alpha}$. For a modulation index equal to $1/2$, we consider the bit error sequences

$$\begin{aligned} \bar{\gamma}_I &= \ldots\ldots\ldots\ldots,\ \gamma_{k-1},\ \gamma_k, \ldots\ldots\ldots\ldots \\ \bar{\gamma}_J &= \ldots\ldots\ldots\ldots, -\gamma_{k-1}, -\gamma_k, \ldots\ldots\ldots\ldots \end{aligned}$$

where k is an arbitrarily chosen index. The computation of (4) for the four different cases outputs two distinct distances: the minimum distance d_0^2 is obtained when $\gamma_{k-1} = -\gamma_k$ whereas the largest distance d_1^2 is found when $\gamma_{k-1} = \gamma_k$. Hence, the bit error probability P_b of the optimal receiver is bounded by

$$P_b \leq \frac{1}{2} Q\left(\sqrt{d_0^2 \frac{E_b}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{d_1^2 \frac{E_b}{N_0}}\right), \qquad (5)$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} e^{-u^2/2} du. \qquad (6)$$

We give in table 2 the Euclidean distances for the different signals studied in this paper and we plot in Fig. 3 their performance bound. The dashed lines represent the performance bound of binary CPM and the continuous lines correspond to their duobinary versions. We can notice that introducing the duobinary encoder generates a BER performance loss of 0.32 dB for the MSK case and 0.64 dB for the GMSK case. The performance loss is coherent with the spectrum efficiency values described in table 1 since the more spectrum efficiency we have, the more ISI is generated. Moreover, it is important to mention that these bounds can be reached in an optimal or very near-optimal way using detectors based on a viterbi algorithm similar to the one presented in [7].

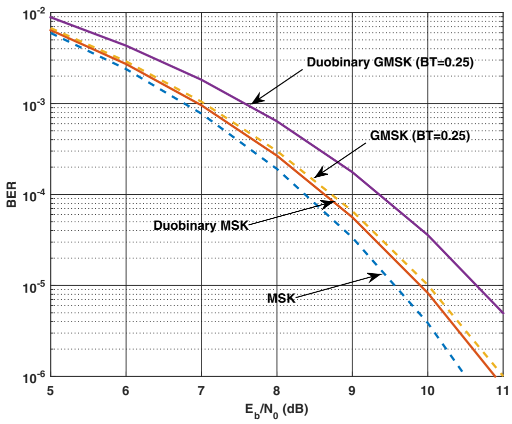

Figure 3. Performance bound of the different CPM signals

Table 2. Spectrum Efficiency (bits/s)/Hz

	MSK	Duobinary MSK	GMSK (BT=0.25)	Duobinary GMSK (BT=0.25)
d_0^2	2	1.73	1.69	1.446
d_1^2	2	2.36	2.37	2.79

4 Conclusion

We showed in this paper that applying the duobinary encoder to binary CPM signals can offer a new class of bandwidth efficient signals. MSK and GMSK modulations were used as illustrations, thus further tests can be done for other known frequency pulses such as the root raised cosine pulse in order to find the best trade off between spectrum efficiency and BER performance for future applications.

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