Large MIMO Systems with COST 2100 Channel Model

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***Abstract*—The problem of user scheduling with reduced overhead of channel estimation in the uplink of Massive multiple-input multiple-output (MIMO) systems has been considered. A realistic COST 2100 channel model has been considered. In this paper, we first propose a new user selection algorithm based on knowledge of the geometry of the service area and of location of clusters, without having full channel state information (CSI) at the BS. We then show that the correlation in geometry-based stochastic channel models (GSCMs) arises from the common clusters in the area. The analytical results are then verified by simulations. It is shown by analysing the capacity upper-bound that the capacity at high SNR strongly depends on the position of clusters in the GSCMs and users in the system.**

Keywords: Massive MIMO, geometry-based stochastic channel models, COST 2100 channel model, user scheduling, zero-forcing, cluster localization.

I. Introduction

Massive multiple-input multiple-output (MIMO) is a promising technique to achieve high data rate [1]. This paper considers an uplink multiuser system where the transmitter is equipped with M antennas and serves K_s decentralized single antenna users ($M \gg K_s$). In the uplink mode, the BS estimates the uplink channel and use a linear receivers to separate the transmitted data. The BS receiver uses the estimated channel to implement the zero-forcing (ZF) receiver which is suitable for Massive MIMO systems [2]. To investigate the performance of MIMO systems, an accurate small scale fading channel model is necessary. Most standardized MIMO channel models such as IEEE 802.11, the 3GPP spatial model, and the COST 273 model rely on clustering [3]. Geometry-based stochastic channel models (GSCMs) are mathematically tractable models to investigate the performance of MIMO systems [4]. The concept of clusters has been introduced in GSCMs to model scatterers in the cell environments [4]. In [5], the authors use clusters to characterize an accurate statistical spatial channel model (SSCM) in millimeter-wave (mmWave) bands by grouping multipath components (MPCs) into clusters. MmWave communications suffers very large path losses, and hence requires large antenna arrays in compensation. [4]. This paper investigates the throughput in the uplink for the Massive MIMO with carrier frequency in the order of 2 GHz, but the principles can also apply to other frequency bands, including mmWave.

Given a map of the area of the micro-cell, we perform efficient user scheduling based only on the position of users and clusters in the cell. In GSCMs, MPCs from common clusters cause high correlation which reduces the rank of the channel. In this paper, we investigate the effect of common clusters on the system performance. The performance analysis shows the significant effect of the distinct clusters on the system throughput. We prove that to maximize the capacity of system, it is required to select users with visibility of the maximum number of distinct clusters in the area. The proposed scheme significantly reduces the overhead channel estimation in Massive MIMO systems.

The rest of the paper is organized as follows. Section II describes the system model. Section III presents performance analysis of the proposed user scheduling with no estimated CSI. Numerical results are presented in Section IV. Finally, Section V concludes the paper. Note that in this paper, uppercase and lowercase boldface letters are used for matrices and vectors, respectively.

The notation $\mathbb{E}(\cdot)$ denotes expectation. $|\cdot|$ stands for absolute value. Conjugate transpose of vector $\mathbf{x}$ is $\mathbf{x}^H$. $\mathbf{X}^\dagger$ denotes the pseudo-inverse of matrix $\mathbf{X}$.

II. System Model

We consider uplink transmission in a single cell Massive MIMO system with M antennas at the BS and $K > M$ single antenna users. The $M \times 1$ received signal at the BS when $K_s\,(K_s \ll M)$ users have been selected from the pool of K users, is given by

$$\mathbf{r} = \sqrt{p_k}\mathbf{H}\mathbf{x} + n, \tag{1}$$

where $\mathbf{x}$ represents the symbol vector of K_s users, p_k is the average power of the kth user and $\mathbf{H}$ denotes the aggregate $M \times K_s$ channel of all selected users. The BS is assumed to have CSI only of the selected users. We are interested in a linear ZF receiver which can be provided by evaluating the pseudo-inverse of $\mathbf{H}$, the aggregate channel of all selected users according to

$$\mathbf{W} = \mathbf{H}^\dagger = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H. \tag{2}$$

Then after using the detector, the received signal at the BS is

$$\mathbf{y} = \sqrt{p_k}\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n}. \tag{3}$$

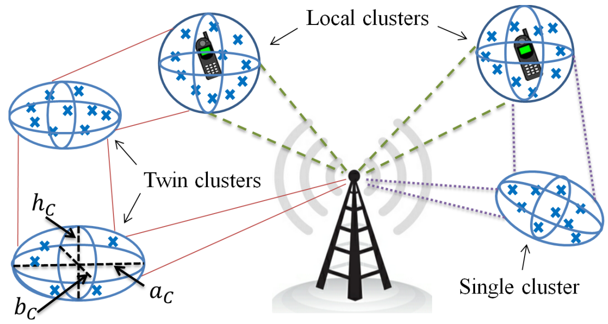

Fig. 1. The general description of the cluster model. The spatial spreads for Cth cluster are given.

Let us consider equal power allocation between users, i.e. $p = \frac{P_t}{k}$, in which P_t denotes the total power. The achievable sum-rate of the system is obtained as

$$R = \sum_{k=1}^{K_s} \log_2 \left(1 + \frac{p|\mathbf{w}_k \mathbf{h}_k|^2}{1 + \sum_{i=1, i \neq k}^{K} p|\mathbf{w}_k \mathbf{h}_i|^2} \right), \quad (4)$$

where $\mathbf{w}_k$ and $\mathbf{h}_k$ are respectively the kth rows of the matrix $\mathbf{W} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \cdots, \mathbf{w}_{K_s}^T]^T$, and the kth column of $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{K_s}]$.

A. Geometry-based Stochastic Channel Model

In GSCMs the double directional channel impulse response is a superposition of MPCs. The channel is given by [7]

$$h(t, \tau, \phi, \theta) = \sum_{j=1}^{N_C} \sum_{i=1}^{N_p} a_{i,j} \delta(\phi - \phi_{i,j}) \delta(\theta - \theta_{i,j}) \delta(\tau - \tau_{i,j}), \quad (5)$$

where N_p denotes the number of multipath components, t is time, τ denotes the delay, δ denotes the Dirac delta function, and ϕ and θ represent the direction of arrival (DoA) and direction of departure (DoD) respectively. Similar to [7], we group the multipath components with similar delay and directions into clusters. Three kinds of clusters are defined; local clusters, single clusters and twin clusters. Local clusters are located around users and the BS while single clusters are represented by one cluster and twin clusters are characterized by two clusters related respectively to the user and BS side as shown in Fig. 1. A local cluster is a single cluster that surrounds a user: single clusters can also occur in a different position. Twin clusters consist of a linked pair of clusters, one of which defines the angles of departure of multipaths from the transmitter, while the other defines the angles of arrival at the receiver [7]. There is a large number of clusters in the area, however just some of them can contribute to the channel. The circular visibility region (VR) determines whether the cluster is active or not for a given user. The MPC's gain scales by a transition function that is given by

$$A_{VR}(\bar{r}_{MS}) = \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{2\sqrt{2}L_c + d_{MS,VR} - R_C}{\sqrt{\lambda L_c}}\right), \quad (6)$$

where $\bar{r}_{MS}$ is the centre of the VR, R_C denotes the VR radius, L_C represents the size of the transition region and $d_{MS,VR}$ denotes the distance between users and the VR centre. For a constant expected number of clusters N_C, the area density of VRs is given by $\rho_C = \frac{N_C - 1}{\pi(R_C - L_C)^2}$. All clusters are ellipsoids in the environment and can be characterized by the cluster spatial delay spread, elevation spread and azimuth spread. Once the position of the BS and users are fixed, we need to determine the positions of the clusters in the area by geometrical calculations. For the local clusters, we consider a circle around the users and the BS, so that the size of the local cluster can be characterized by the cluster delay spread (a_C), elevation spread (h_C) and the position of MPCs [7]. For local clusters the cluster delay, azimuth and elevation spreads can be given by

$$a_C = \Delta\tau c_0 / 2 \quad (7a)$$
$$b_C = a_C \quad (7b)$$
$$h_C = d_{C,BS} \tan \theta_{BS}, \quad (7c)$$

respectively, where c_0 denotes the speed of light, $d_{C,BS}$ is the distance between the cluster and the BS, $\Delta\tau$ refers to the delay spread and θ_{BS} is the elevation spread seen by the BS. The delay spread, angular spreads and shadow fading are correlated random variables and for all kinds of clusters are given by [6]

$$\Delta\tau_c = \mu_\tau (d/1000)^0.510^{\sigma_\tau Z_c/10} \quad (8a)$$
$$\beta_c = \tau_\beta 10^{\sigma_\beta Y_c/10} \quad (8b)$$
$$S_m = 10^{\sigma_s X_c/10}, \quad (8c)$$

where S_m is the shadow fading of cluster c, $\Delta\tau_c$ refers to the delay spread and β_c denotes angular spread. Moreover, X_c, Y_c and Z_c denote correlated random variables with zero mean and unit variance. Correlated random process can be computed by Cholesky factorization [6]. The MPCs' positions can be drawn from the truncated Gaussian distribution given by [7]

$$f(r) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_{r,o}^2}} \exp\left(-\left(\frac{(r - \mu_{r,o})}{\sqrt{2}\sigma_{r,o}}\right)^2\right) & |r| \leq r_T, \\ 0 & \text{otherwise}, \end{cases} \quad (9)$$

where r_T denotes the truncation value. For single clusters, the cluster delay, azimuth and elevation spreads can be given by $a_C = \Delta\tau c_0/2$, $b_C = d_{C,BS} \tan \phi_{BS}$ and $h_C = d_{C,BS} \tan \theta_{BS}$, respectively. To get the fixed positions of the single clusters, the radial distance of the cluster from the BS drawn from the exponential distribution [7]

$$f(r) = \begin{cases} 0 & r < r_{min}, \\ \frac{1}{\sigma_r} \exp(-(r - r_{min}/\sigma_r) & \text{otherwise}. \end{cases} \quad (10)$$

To determine the fixed position of the cluster, the angle of the cluster can be drawn from the Gaussian distribution with a standard deviation $\sigma_{\phi,C}$. For the twin clusters, for both the

BS and user side clusters we have

$$a_C = \Delta\tau c_0/2 \tag{11a}$$

$$b_C = d_{C,BS}\tan\phi_{BS}. \tag{11b}$$

For the BS side cluster the elevation spread can be given by $h_C = d_{C,BS}\tan\theta_{BS}$, while for the mobile station (MS) side cluster $h_C = d_{C,MS}\tan\theta_{MS}$. Fig. 1 gives an example of the geometry of the Cth cluster. For twin clusters, the distance between the cluster and the BS and the distance from the VR center and the user is given by $d_{C,BS}\tan\Phi_{C,BS} = d_{C,MS}\tan\Phi_{C,MS}$ [7]. The delay of a cluster is represented by [7]

$$\tau_C = (d_{C,BS} + d_{C,MS} + d_C)/c_0 + \tau_{C,link}, \tag{12}$$

where the geometrical distance between twin clusters is represented by d_C, $d_{C,MS}$ denotes the geometrical distance between the user and the center of the visibility region, $d_{C,BS}$ refers to the distance between the BS and the cluster, and finally $\tau_{C,link}$ is the cluster link delay between the twin clusters. Hence, the cluster power attenuation is given by [7]

$$A_C = \max(\exp[-k_\tau(\tau_C - \tau_0)], \exp[-k_\tau(\tau_B - \tau_0)]), \tag{13}$$

where k_τ denotes the decay parameter, and τ_B is the cut-off delay. We assume Rayleigh fading for the MPCs within each cluster. Hence, the complex amplitude of the ith MPC in the jth cluster in (5) is given by

$$a_{i,j} = \sqrt{L_p}A_{VR}\sqrt{A_C A_{MPC}}\ e^{-j2\pi f_c \tau_{i,j}}, \tag{14}$$

where L_p is the channel path loss, A_{MPC} is the power of each MPC which is characterized by the Rayleigh fading distribution and $\tau_{i,j}$ is the delay of the ith MPC in cluster j given by [7]

$$\tau_{i,j} = (d_{MPC_{i,j},BS} + d_{MPC_{i,j},MS})/c_0 + \tau_{i,C,link}. \tag{15}$$

By assuming a fixed OFDM subcarrier, we can drop the variable $\tau_{i,j}$ from (22).

For the non-line-of-sight (NLoS) case of the micro-cell scenario, the path loss expression can be given by

$$L = 26\log_{10} d + 20\log_{10}(4\pi/\lambda), \tag{16}$$

where d and λ denote the distance (in m) and the wavelength (in m), respectively.

III. Geometry-based User Scheduling

If perfect CSI is available at the BS, and assuming Gaussian input, the ergodic capacity is given by

$$C = \mathbb{E}[\log_2\det\left(\mathbf{I} + \frac{P_t}{K_s}\mathbf{H}\mathbf{H}^H\right)], \tag{17}$$

where the term $\frac{P_t}{K_s}$ is due to the equal-power allocation, I is an identity matrix, and $C(K)$ denotes the clusters seen by the kth user and $\alpha = -2\pi\frac{d}{\lambda}$, where d denotes the spacing between two antenna elements. As in [10], we need to define a threshold which can determine the minimum power that a cluster may have relative to the total cluster powers to be considered.

For ease of mathematical tractability, we analyse the capacity of a correlated three-user uplink using an upper bound. In the case of a large number of antennas at the BS, the capacity upper bound can be achieved in the case of distinct clusters. Note that in the case of a large number of transmit antennas, the elements of $\mathbf{H}\mathbf{H}^H$ converge to the correlation matrix so that $\mathbf{R} \approx \mathbf{H}\mathbf{H}^H$. Hence, we have

$$C = \mathbb{E}[\log_2\det\left(\mathbf{I} + \frac{P_t}{K}\mathbf{H}^H\mathbf{H}\right)] \approx \log_2\det(\mathbf{I} + \frac{P_t}{K}\mathbf{R}), \tag{18}$$

where R is the channel correlation matrix and is given by

$$\mathbf{R} = \mathbb{E}[\mathbf{H}^H\mathbf{H}] = \begin{bmatrix} 1 & r_{12} & r_{13} \\ r_{12}^* & 1 & r_{23} \\ r_{13}^* & r_{23}^* & 1 \end{bmatrix}, \tag{19}$$

where $r_{12} = \mathbb{E}[\mathbf{h}_1^H\mathbf{h}_2] = \zeta_{12}e^{j\beta_{12}}$, $r_{13} = \mathbb{E}[\mathbf{h}_1^H\mathbf{h}_3] = \zeta_{13}e^{j\beta_{13}}$, and finally $r_{23} = \mathbb{E}[\mathbf{h}_2^H\mathbf{h}_3] = \zeta_{23}e^{j\beta_{23}}$. By substituting the terms r_{12}, r_{13} and r_{23} into (18), the capacity maximization problem in a three-user scenario can be formulated as

$$C = \max_{\zeta_{12},\zeta_{13},\zeta_{23}\beta_{12},\beta_{13},\beta_{23}} \log_2\big[(1+p)^3 - p^2(\zeta_{12}^2+\zeta_{13}^2+\zeta_{23}^2) \tag{20}$$
$$+p^3\big(2\zeta_{12}\zeta_{13}\zeta_{23}\sin(\beta_{12}-\beta_{13}+\beta_{23}) - \zeta_{12}^2 - \zeta_{13}^2 - \zeta_{23}^2\big)\big],$$

where $p = \frac{P_t}{K}$ and P_t denotes the total power. To maximize (20), the gradient search (GS) method results in $\zeta_{12} = \zeta_{13} = \zeta_{23} = 0$ for different values of β_{12}, β_{13} and β_{23}, which is the case when common clusters do not occur between the users in the cell. In the case of distinct clusters between user m and user n, we have

$$\zeta_{mn} = \mathbb{E}\big[\sum_{j\in C(n)}\sum_{i=1}^{N_p} a_{i,j} \sum_{l\in C(m)}\sum_{g=1}^{N_p} a_{g,l}^*\big] = 0, \tag{21}$$

where $a_{i,j}$, the amplitude of the i MPCs in cluster j is given by (22). The equation (21) yields $\zeta_{mn} = 0$, which maximizes the capacity given by (20).

If the positions of the clusters and users are known at the BS, the BS can use the large scale fading components to

$$v_i^j = \sqrt{L_p^j}A_{VR,i}^j\sqrt{A_{C,i}^j}, \tag{22}$$

where L_p^j denotes the channel path loss for user j, $A_{VR,i}^j$ is the MPC power attenuation which is a function of the distance between user j and the centre of the visibility region related to the jth cluster and is given by (6), and $A_{C,i}^j$ denotes the cluster power attenuation given by (13) for the user j and the ith cluster. We first select the user which has the highest geometry-based attenuation, i.e. $|\sum_{j=1}^{N_c} v_k^j|, \forall k$. Then, the proposed scheme selects the users which have highest number of distinct clusters with already selected users.

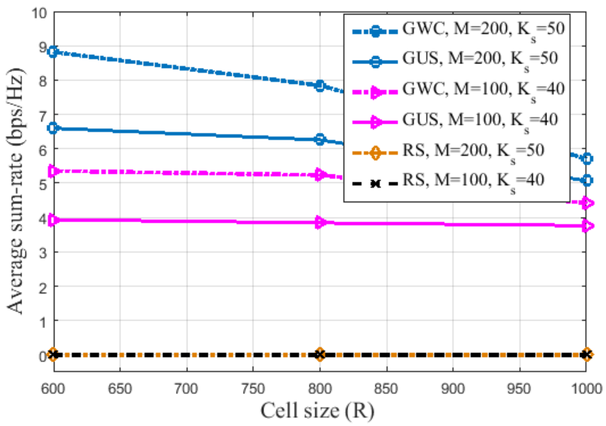

Fig. 2. The average sum-rate vs. the cell size for different values of $M = 200$, $M = 100$, $K_s = 50$, and $K_s = 40$. We set the total number of users in the cell $K = 400$.

IV. Numerical Results

A. Simulation Parameters for COST 2100 Channel Model

We evaluate the throughput of the system, averaging over 50 iterations. A square cell with a side length of $2 \times R$ has been considered so that we call R the cell size and also assume users are uniformly distributed in the cell. As in [2], we assume that there is no user closer than $R_{th} = 0.1 \times R$ to the BS. We simulate a micro-cell environment for the NLoS case and set the operating frequency $f_C =$ 2GHz. The external parameters and stochastic parameters are extracted from chapter 6 of [6] and chapter 3 of [7]. The BS and user heights are assumed to be $h_{BS} = 5$ and $h_{MS} = 1.5$, respectively. In (II-A) $N_C = 3$, $R_C = 50$, and $L_C = 20$. Moreover, we consider $N_P = 6$ paths per cluster.

B. Simulation Results

For this network setup, the average sum-rate is evaluated for the three scenarios. In the GUS scheme, it has been proposed that the receiver BS selects users that maximize the number of distinct clusters in the cell. We evaluate the average throughput of the GWC scheme [8]-[9] and random selection (RS) of users. For the case of GWC, similar to [9], we set the optimal channel direction constraint to achieve the best performance for GWC, so the complexity of GWC is much higher than GUS.

Fig. 2 depicts the average sum-rate with total number of receive antennas at the BS $M = 100$ and $M = 200$, and two values of the number of selected users $K_s = 40$ and $K_s = 50$ while adopting the proposed scheme with ZF receiver. As expected, since GWC exploits perfect CSI, it has the best throughput. As seen in Fig.2, the performance of the proposed algorithm is slightly lower than the case in which the BS exploits full CSI and performs GWC. Interestingly, for bigger cells, the superiority of the proposed scheme is more obvious in terms of achieving performance close to that of the GWC scheme.

V. Conclusions

In this paper, we have investigated geometry-based user scheduling (GUS) by considering large MIMO assumptions. By applying knowledge of the location of clusters and users and the geometry of the system, we suppose that the BS does not need to estimate the channels of all users and selects users based only on the location of users and clusters in the area. The proposed algorithm can be an efficient scheme to reduce the complexity of user scheduling in Massive MIMO systems.

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