



## High Resolution Extraction of Radar Micro-Doppler Signature Using Sparse Time-frequency Distribution

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### Abstract

In order to achieve high resolution TFD analysis of time-varying signal, the principle framework of sparse time-frequency distribution (STFD) is established combing the advantages of classical TFD and sparse representation. Then two special cases of STFD are proposed, i.e., short-time sparse Fourier transform (ST-SFT) and short-time sparse fractional Fourier transform (ST-SFRFT). They are used for signature extraction of radar micro-Doppler (m-D) signature. It is verified by real radar data that the proposed methods can achieve high-resolution of m-D signal in time-sparse domain, and has the advantages of anti-clutter.

### 1. Introduction

Doppler signature extraction and analysis of moving target are quite important for radar target detection and recognition [1]. The traditional Fourier spectrum cannot exhibit time-varying Doppler signature with low spectrum resolution. Recently, the micro-Doppler (m-D) theory has attracted extensive attention worldwide for accurate description of a target's motion [2]. The m-D features reflect the unique dynamic and structural characteristics, which are useful for target recognition and classification. The motion of a marine target is rather complex especially for marine target and maneuvering target [3]. Therefore, how to extraction the signature of m-D signal it the key step for the following target detection and estimation.

Time-frequency distributions (TFDs) provide an image of frequency contents as a function of time, which reveals how a signal changes over time. However, the classic TFDs, such as short-time Fourier transform (STFT) or Wigner-Ville distribution (WVD), suffer from the poor time-frequency resolution or cross-terms [4]. Moreover, it is difficult to separate the complex background (clutter) from the weak m-D signal in time or frequency domain.

In the last decade, sparsity has been proved as a promising tool for a high-resolution solution. Sparse transform is proposed to increase resolution in different transform domains [5], such as the sparse FFT [6] and sparse FRFT (SFRFT) [7], et al.. M-D signal can be approximated as sum of frequency-modulated (FM) signals and it can be considered to be sparse in the TF plane [8]. In this paper, the merits of TFD and sparse representation are combined together and a novel method, i.e., sparse TFD (STFD) is

proposed for high-resolution representation in the sparse time-frequency domain.

### 2. Radar M-D Signal Model

Suppose there is a target at the RLOS distance  $R_s(t_m)$ , where  $t_m$  is the slow-time measuring the time among pulses within a coherent processing interval. Then after pulse compression (PC), the returned signal has the following form.

$$s(t, t_m) = A_r \text{sinc} \left[ B \left( t - \frac{2R_s(t_m)}{c} \right) \right] \exp \left( -j \frac{4\pi R_s(t_m)}{\lambda} \right) \quad (1)$$

where  $A_r$  is the complex amplitude of the compressed signal,  $\text{sinc}(x) = \sin(\pi x) / \pi x$ ,  $B$  is the bandwidth, the delay is  $\tau = 2R_s(t_m) / c$ ,  $c$  is the speed of light, and  $\lambda$  is the wavelength.  $R_s(t_m)$  can be expanded with Taylor series,

$$R_s(t_m) = r_0 - v_0 t_m - \frac{v'}{2!} t_m^2 - \frac{v''}{3!} t_m^3 - \dots \quad (2)$$

where  $r_0$  is the initial range,  $v'$  and  $v''$  are higher order terms of velocity. Therefore, the radar returns of the maneuvering target can be expressed by polynomial phase signal, and its Doppler is frequency modulated, i.e.,

$$f_d(t_m) = \sum_i a_i t_m^{i-1} \quad (3)$$

Considering the real applications and the Weierstrass approximation principle, the nonuniformly translational motion of a target can be well described by a linear frequency modulated (LFM) or cubic phase signal (CPS).

In case of rotational motion, the radar returns are written in the vector form, and the radial velocity is expressed as  $\mathbf{v}_r$ . Therefore, the RLOS is the integral of  $\mathbf{v}_r \cdot \mathbf{n}$  during the time  $T_n$ ,

$$R_s(t_m) = \int_0^{T_n} (\mathbf{v}_r \cdot \mathbf{n}) dt_m \quad (4)$$

The corresponding Doppler shift has the form of

$$f_t(t_m) = \frac{2}{\lambda} \cdot (\mathbf{v}_r \cdot \mathbf{n}) = \sum_{i=0}^l a_i t_m^i \quad (5)$$

Then the m-D shift resulting from the 3-D rotational motion also has the form

$$f_r(t_m) = \frac{2}{\lambda} (\boldsymbol{\omega}_t \times \mathbf{R}_p) \cdot \mathbf{n} \quad (6)$$

The  $\omega_t$  may result in Doppler frequency by the rotation motion and thus is called the effective rotation vector. The location of the point can be obtained from its initial position vector by multiplying by a rotation matrix  $\mathbf{R}_t$  with the rotation angles  $\theta$ , i.e.,  $\mathbf{R}_P = \mathbf{R}_t \mathbf{r}_0$ . Accordingly, the RLOS from the scatterer  $P$  to the radar is

$$R_s(t_m) = \int_0^{t_m} [(\omega_t \times \mathbf{R}_P) \cdot \mathbf{n}] dt_m. \quad (7)$$

From the above analysis, for the the m-D signal, which mostly has the form of nonuniformly translational and rotational motions, can be expressed as

$$x(t_m) = \sum_{i=1}^l a_i(t_m) e^{j\varphi_i(t_m)} \quad (8)$$

where  $\varphi(t_m) = 2\pi f_d(t_m) = \sum_k 2\pi a_k t_m^{k-1}$ .

### 3. M-D Signature Extraction via Sparse Time-frequency Distribution (STFD)

For the m-D signal as (8), its TFD can be written as

$$\rho_x(t, f) = \sum_{i=1}^l a_i^2(t) \delta[f - \dot{\varphi}_i(t) / 2\pi] \quad (9)$$

where  $\dot{\varphi}_i(t)$  is the estimation of Doppler frequency.

Therefore, the m-D signal can be regarded as sum of multiple instantaneous frequency components. In the time-frequency domain, the m-D signal can exhibit an obvious peak at the location of  $f = \dot{\varphi}_i(t)$  via sparse representation. And the time-frequency domain  $\rho_x(t, f)$  is the sparse representation domain of m-D signal. For example, for an LFM signal,  $a_1(t)=1$ ,  $\dot{\varphi}_1(t) = 2\pi(f_o + at)$ .

#### 3.1 Principle of STFD

Any signal can be represented in terms of basis or atoms  $g$ ,

$$\mathbf{x} = \sum_m \alpha_m g_m \quad (10)$$

where  $m$  is the number of atoms, and the coefficient  $\alpha_m$  denotes the similarity of the signal and the atoms.

It can be found by comparing (9) and (10) that the TFD is a special case of the sparse representation, i.e.,

$$\rho_x(t, f) = \sum_m \alpha_m(t) h(t) g_m(t, f) \quad (11)$$

where  $h(t)$  is a window function and  $g_m(t, f)$  is the atoms combined with frequency modulated signal.

For the condition without noise, the sparse representation of (11) can be regarded as the optimization problem and solved by  $l_1$ -norm minimization,

$$\min \|\rho_x(t, f)\|_1, \text{ s.t. } \mathcal{O}\{\rho_x(t, f)\} = b \quad (12)$$

where  $\mathcal{O}$  is the sparse operator with  $K \times N$  dimension. The above equation can be relaxed by the following constraint, i.e.,

$$\min \|\rho_x(t, f)\|_1, \text{ s.t. } \|\mathcal{O}\{\rho_x(t, f)\} - b\|_2 \leq \varepsilon. \quad (13)$$

When  $\varepsilon=0$ , (12) and (13) have the same form. Then the framework of STFD can be defined by the calculation from (11) to (13).

When  $\rho_x$  is the Fourier transform (FT) and  $b$  is the signal component in the FT domain, the STFD is named as short-time sparse FT (ST-SFT), i.e.,

$$\min \|\mathcal{F}(t, f)\|_1, \text{ s.t. } \|\mathcal{O}\{\mathcal{F}(t, f)\} - f\|_2 \leq \varepsilon \quad (14)$$

When  $\rho_x$  is the fractional FT (FRFT) and  $b$  is the signal component in the FRFT domain, the STFD is named as short-time sparse FRFT (ST-SFRFT), i.e.,

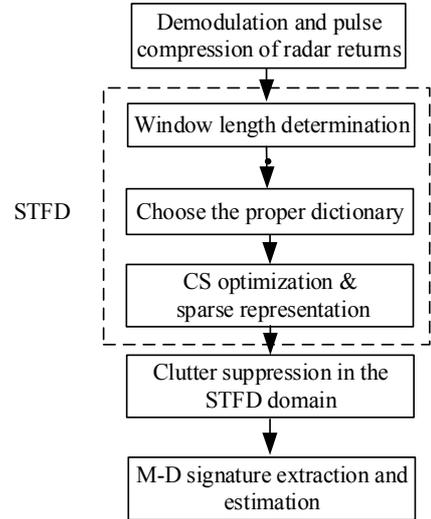
$$\min \|\mathcal{F}^\alpha(t, f)\|_1, \text{ s.t. } \|\mathcal{O}\{\mathcal{F}^\alpha(t, f)\} - f(\alpha, u)\|_2 \leq \varepsilon. \quad (15)$$

Therefore, the proposed STFD is the generalized form of the classical TFD.

### 3.2 Flowchart of the M-D Signature Extraction Method

Flowchart of the STFD-based M-D signature extraction and estimation method is shown in Fig. 1, which mainly consists of four steps, i.e.,

- 1) Perform demodulation and pulse compression of radar returns, which achieves high-resolution in range direction;
- 2) STFD, which is the most important procedure, consists of three parts, i.e., window length determination, choose the proper dictionary, CS optimization and sparse representation;
- 3) Clutter suppression in the STFD domain, which improves signal-to-clutter ratio of the m-D signal;
- 4) M-D signature extraction and estimation.



**Figure 1.** Flowchart of the STFD-based M-D signature extraction and estimation method.

It should be noted that the choice of dictionary can be determined according to the prior information of target to satisfy the sparsity condition, such as the types of observed target and different sea states. For the micromotion target whose main motion components are

the nonuniform translation, such as the speed boat, aircraft, et al., we can use the chirp signal as the dictionary of sparse representation

$$g_x(f_l, \mu_m) = \exp(j2\pi f_l t + j\pi \mu_m t^2) \quad (16)$$

where  $\mu$  is the chirp rate denoting the acceleration. For the micromotion target whose main motion components are the rotation or high mobility, its m-D signal exhibits periodical frequency modulated property. Also, we can use the QFM signal or periodical frequency modulated function as its dictionary.

$$g_x(\mu_m, k_n) = \exp\left(j\pi \mu_m t^2 + \frac{1}{3}\pi k_n t^3\right) \quad (17)$$

$$g_x(\omega_r) = \exp(j\omega_r t + \varphi_r) \quad (18)$$

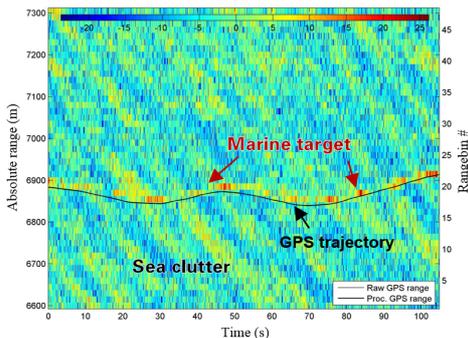
where  $k$  results from the jerk motion,  $\omega$  is the rotation angular frequency.

## 4. Simulation and Experimental Results

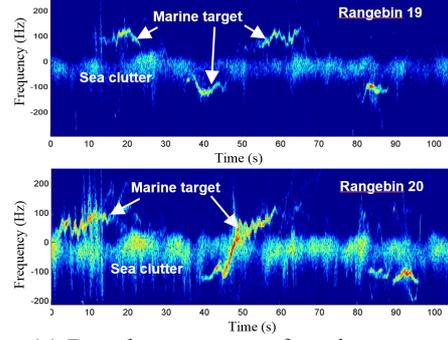
### 4.1 Description of the CSIR Dataset

The measurement trial was conducted with the Fynmeet dynamic RCS measurement facility at the Over-berg Test Range (OTB). The specifications of Fynmeet and the environment parameters can be found in [9]. The TFA17-014 dataset was chosen for validation and the cooperative WaveRider rigid inflatable boat (RIB) was deployed for m-D analysis and target detection.

The range-time intensity plot is presented in Fig. 2(a) covering nearly 50 range bins during 110s. The strong fluctuation caused by the local wind is clearly visible, which is repeated periodically. Due to the heavy sea clutter fluctuating downwind, it is rather difficult to separate the WaveRider RIB from sea clutter background. Furthermore, high Doppler resolution spectrograms of the 19th and 20th rangebins are plotted in Fig. 2(b). The WaveRider RIB had a narrow Doppler response, with a local disturbance of the sea surface visible when the boat was crashing through the crests of the waves. The time-varying Doppler character indicates a nonuniform motion and Doppler migration, which is a typical m-D signal. However, it is difficult to extract the weak m-D signal using traditional TFD method such as STFT.



(b) Range line versus time plot

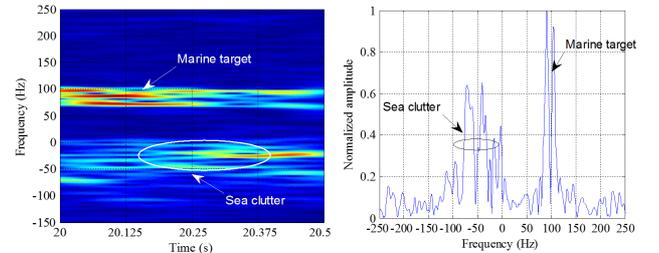


(c) Doppler spectrum of marine target

**Figure 2.** Description of the CSIR dataset. (Measurement trial at OTB in 2006, TFA17-014)

### 4.2 Experimental Results and Analysis

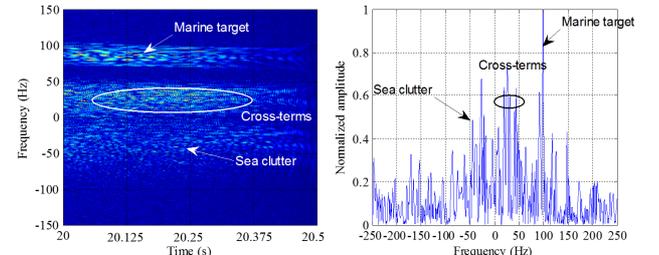
The target's returns are taken according the GPS information from the starting time  $t_0=20s$  for analysis. Fig. 3 and Fig. 4 compare the two popular TFD-based method for m-D signal analysis, i.e., the STFT and WVD method. Accordingly, their spectrograms are show aside as well. The two methods both can provide the TFD of the m-D signal, and the differences are the time-frequency resolution, ability of energy integration, and sea clutter suppression. The time-frequency resolution of STFT is poor for the nonstationary and time-varying m-D signal, which makes it difficult to obtain the accurate m-D signature. The normalized difference between sea clutter and the peak spectrum of marine target is 0.38 (not obvious). The terrible problem of cross terms for multi-component signals make it ambiguity for separating sea clutter and m-D signal. Therefore, it fails to extract the m-D signature of marine target using the traditional TFD methods.



(a) TFD via STFT

(b) STFT spectrum

**Figure 3.** STFT of the m-D signal ( $t_0=20s$ ).



(a) TFD via WVD

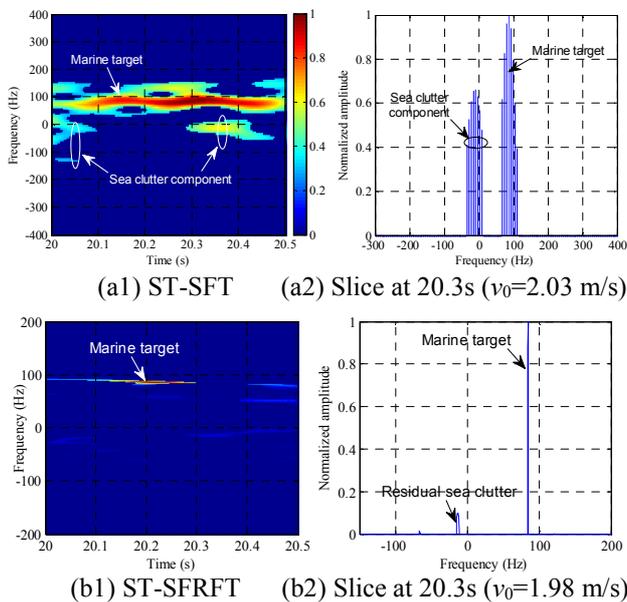
(b) WVD spectrum

**Figure 4.** WVD of the m-D signal ( $t_0=20s$ ).

Then the proposed STFD-based methods, i.e., ST-SFT and ST-SFRFT are used for high resolution m-D signature

extraction, which are shown in Fig. 5. Comparing the results of ST-SFT and WVD, it can be found that the m-D signature is more clear and narrow via ST-SFT due to the advantage of sparse representation. The sparsity of the m-D signal component is remained. Hence, not only the energy is accumulated, but also the high-resolution TFD of m-D signal is obtained. Accordingly, the estimation precision is more accurate ( $v_0=2.03$  m/s at 20.3s).

Since the m-D exhibits time-varying property, we perform the STFD via chirp dictionary and get the results via ST-SFRFT (Fig. 5(b)). Then we obtain very sharp peak in the ST-SFRFT domain, and at the same time, the sea clutter is greatly suppressed since it is not sparse in the ST-SFRFT domain. The normalized difference between sea clutter and m-D signal is about 0.9. Then the accuracy and detection ability are greatly improved.



**Figure 5.** M-D signature extraction via different STFDs ( $t_0=20$ s).

In real applications, we can establish different STFDs as needed according to (13). From the above analysis, the proposed STFD-based methods can achieve high-resolution m-D signature extraction and they are very useful for weak target detection and recognition.

## 5. Conclusions

In this paper, the principle framework of sparse time-frequency distribution (STFD) is established combining the advantages of TFD and sparse representation. They are useful for signature extraction and estimation for time-varying signal especially m-D signal. Marine radar data is used for validation. It is proved that the proposed methods can achieve high-resolution of m-D signal in time-sparse domain with good clutter suppression ability. It can be expected that the STFD would provide a novel solution for radar clutter suppression and weak moving target detection.

## 6. Acknowledgements

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