Cooperative Game-Theoretic Power Allocation Algorithm for Target Detection in Radar Network

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Abstract

This paper investigates the problem of power allocation for radar network in a cooperative game-theoretic framework such that the low probability of intercept (LPI) performance can be improved. Taking into consideration both the transmit power constraint and the minimum signal-to-interference-plus-noise ratio (SINR) requirement of each radar, a cooperative Nash bargaining power allocation game (NBPAG) based on LPI is formulated, whose objective is to improve the LPI performance by optimizing the transmit power allocation in radar network for a predefined S-INR threshold. First, a novel SINR-based network utility function is defined as a metric to evaluate power allocation. Then, the existence and uniqueness of the Nash bargaining solution (NBS) are proved analytically. Finally, an iterative Nash bargaining algorithm is developed that converges quickly to a Pareto optimal equilibrium for the cooperative game. Theoretic analysis and simulations are provided to evaluate the effectiveness of the proposed algorithm.

1 Introduction

Distributed radar network systems have been attracting continuously growing attention recently and on a path from theory to practical use owing to their advantage of signal and spatial diversities. As the notion of low probability of intercept (LPI) design is an essential part of military operations in hostile environments, LPI performance is a primary issue that needs to be taken into account in designing radar network systems [1]. Shi et al. investigate the LPI optimization in radar network for the first time [2], and it has triggered a resurgence of interest in radar network.

The main task of this paper is to improve the LPI performance for radar network by optimizing power allocation in a cooperative game-theoretic framework. In [3], a distributed algorithm based on non-cooperative game theory is presented to capture the tradeoff between detection capabilities and power consumption for a radar network. The authors in [4] propose a non-cooperative game based power allocation scheme for a multistatic multiple-input multiple-output (MIMO) radar network, which minimizes the total transmission power while maintaining a specific signal-to-interference-plus-noise ratio (SINR). While in a non-cooperative game model, rational but selfish players maximize their own individual utilities in a self-interested manner, which will inevitably increase the mutual interference to other players. The objective of a non-cooperative game is to find an Nash equilibrium (NE) solution, where each player has no chance to increase its utility unilaterally. Unfortunately, the sum of the individual utilities might not be maximized at the NE point [5]. Cooperative game theory can provide an expressive and flexible framework for modelling collaboration in multi-agent systems, in which players are motivated to cooperate with one another to enhance the network utility function. In [6], the optimal power allocation problem in distributed MIMO radar network is modeled as a cooperative game and the solution is obtained by exploiting the concept of Shapley value. However, to the best of authors’ knowledge, no literature has addressed the LPI based cooperative game-theoretic strategy for power allocation in radar network.

In this paper, a novel cooperative Nash bargaining power allocation game (NBPAG) based on LPI is formulated. In the rest of this paper we introduce the system model and cooperative game formulation in Section II. In Section III the proposed NBPAG is elaborated. Simulation results are provided in Section IV and are followed by conclusion in Section V.

2 System Model and Game Formulation

Consider a radar network comprised of $N_t$ netted radars, as illustrated in Figure 1. The $i$th radar receives the echoes from the target due to its transmitted signals as well as the signals from the other radars, both scattered off the target.

Figure 1. Rada networks system model.
and through a direct path. We assume that all the radars detect the target in the same frequency band. The transmitted signals from different radars may be correlated because of the absence of radar transmission synchronization. Each radar can independently detect the target and send its received signals to the fusion center which takes a decision once the information coming from all the radars is collected. Define the propagation gains of the corresponding paths as:

\[
\begin{align*}
 h_{i,j}^t &= \frac{G_i G_j \sigma_{RCS}^t \lambda^2}{(4\pi)^3 R_i^4}, \\
 h_{i,j}^r &= \frac{G_i G_j \sigma_{RCS}^r \lambda^2}{(4\pi)^3 R_i^4 R_j^4},
\end{align*}
\]

where \( h_{i,j}^t \) represents the propagation gain for the radar \( i \)-target radar \( j \) path, \( h_{i,j}^r \) represents the direct radar \( i \)-radar \( j \) path. \( G_i \) is the radar main-lobe transmitting antenna gain, \( G_j \) is the radar main-lobe receiving antenna gain, \( \sigma_{RCS}^t \) is the radar cross section (RCS) of the target with respect to the \( i \)th radar, \( \sigma_{RCS}^r \) is the RCS of the target between the \( i \)th radar and \( j \)th radar, \( \lambda \) denotes the wavelength, \( R_i \) denotes the distance from the \( i \)th radar to the target, \( R_j \) denotes the distance from the \( j \)th radar to the target, and \( d_{i,j} \) denotes the distance between the \( i \)th radar and \( j \)th radar. All the channel gains are assumed to be fixed during observation.

Here, the generalized likelihood ratio test (GLRT) is used to determine the appropriate detector [4]. The probabilities of miss detection \( P_{MD,i}(\delta, \gamma) \) and false alarm \( P_{FA,i}(\delta) \) are:

\[
\begin{align*}
P_{MD,(i)}(\delta, \gamma) &= 1 - \left(1 + \frac{\delta}{1 - \delta} \frac{1}{1 + N\gamma} \right)^{1-N}, \\
P_{FA,(i)}(\delta) &= (1 - \delta)^{N-1},
\end{align*}
\]

where \( \delta \) is the detection threshold, \( N \) is the number of received pulses in the time-on-target. \( \gamma \) denotes the SINR received at the \( i \)th radar, which can be given by:

\[
\gamma = \frac{h_{i,j}^t p_i}{\sum_{j=1, j \neq i}^{N_i} c_{i,j} \left(h_{i,j}^r p_j + h_{i,j}^r p_j \right) + \sigma^2},
\]

where \( p_i \) is the transmit power of the \( i \)th radar, and \( c_{i,j} \) denotes the cross correlation coefficient between the \( i \)th radar and the \( j \)th radar. The environment noise is an additive white Gaussian noise with zero mean and variance \( \sigma^2 \). (3) can equivalently be rewritten as \( \gamma = \frac{h_{i,j}^t p_i}{L_i} \), where the total interference and noise received at the \( i \)th radar is defined as \( L_i = \sum_{j=1, j \neq i}^{N_i} c_{i,j} \left(h_{i,j}^r p_j + h_{i,j}^r p_j \right) + \sigma^2 \).

**Cooperative Game Formulation.** In cooperative game, the netted radars adjust their transmitting strategies to maximize the network utility function. Consequently, the underlying LPI based NBPG model can be formulated as the following optimization problem:

\[
\begin{align*}
\max_{\{p_i, i \in \mathcal{N}\}} \sum_{i=1}^{N} u_i(p_i, \mathbf{p}_{-i}) = \sum_{i=1}^{N} h_{i,j}^t \ln(\gamma - \gamma_{\text{min}}),
\end{align*}
\]

where the utility function is \( u_i(p_i, \mathbf{p}_{-i}) = h_{i,j}^t \ln(\gamma - \gamma_{\text{min}}) \), and \( \mathbf{p}_{-i} = [p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_N] \). \( \gamma_{\text{min}} \) denotes the peak transmitting power of radar \( i \), and \( p_{\text{tot}} \) denotes the maximum total transmitting power of radar network, and \( \mathcal{N} = \{1, 2, \cdots, N\} \) denotes the finite set of radars. Here, the network utility function is selected as the sum of the novel SINR-based individual utilities of radars. It should be noted that \( h_{i,j}^t \) is a modifiability coefficient that is used to minimize the transmit power while guaranteeing the required target detection performance of each radar.

**3 Power Allocation and Distributed Implementation**

**Convergence Analysis.** To characterize the convergence properties of the NBPG, we first recall the existence and uniqueness theorems of NBS.

**Theorem 2 (Existence):** There is at least one NBS to the proposed NBPG in (4) if and only if, for \( \forall i \):  
(a) The strategy is a non-empty, convex and compact subset of some Euclidean space.  
(b) The utility functions are continuous and quasi-concave.

**Proof:** One can observe from (9a) that the utility functions \( u_i(p_i, \mathbf{p}_{-i}) \) are continuous and concave with respect to \( p_i \). Taking the second derivative of \( u_i(p_i, \mathbf{p}_{-i}) \) with respect to \( p_i \), we can obtain \( \frac{\partial^2 u_i(p_i, \mathbf{p}_{-i})}{\partial p_i^2} = \left( \frac{1}{\gamma} \right)^2 \frac{h_{i,j}^t}{\sum_{j \neq i} h_{i,j}^r + \sigma^2} > 0 \), and \( \frac{\partial^2 u_i(p_i, \mathbf{p}_{-i})}{\partial p_i^2} = - \left( \frac{1}{\gamma} \right)^2 \frac{h_{i,j}^t}{\sum_{j \neq i} h_{i,j}^r + \sigma^2} < 0 \). Thus, \( u_i(p_i, \mathbf{p}_{-i}) \) is concave in \( p_i \). As a result, the utility functions are continuous and quasi-concave. This proves the existence of NBS in the proposed NBPG.

**Theorem 3 (Uniqueness):** The NBS to NBPG is unique for which the NBPG converges to.

**Proof:** For the uniqueness of the NBS in a cooperative game [5], it has been established that there is at most one NBS to the game for which the NBPG converges to if and only if the following four conditions are satisfied.

(a) \( A_i = \{p_i \in \mathcal{S} | f(p_i) = \mathbf{p} - p_i \geq 0\} \) is non-empty, where \( \mathcal{S} \) is the average transmission power.

(b) There exists \( p_i \in A_i \) that satisfies \( f(p_i) \geq 0 \).

(c) \( w_i(p_i, \mathbf{p}_{-i}) \) is continuous and quasi-concave.

(d) The game model is diagonally strictly concave on its strategy set \( \mathcal{S} \), that is, for any \( \mathbf{p}^{(0)} \neq \mathbf{p}^{(1)} \) with \( \mathbf{p}^{(k)} \in [p_i^1, \cdots, p_i^N] \in \mathcal{S} \) for \( k = 0, 1 \), and for \( t = [t_1, \cdots, t_N] \geq 0 \),
the inequality \( Q = (p^{(0)} - p^{(1)})^T [d(p^{(0)}, t) - d(p^{(1)}, t)] < 0 \) holds, where \( d(p, t) = [t_1, t_2, \ldots, t_{N_r}]^T \).

The proof of this theorem is omitted due to space limitations. A similar detailed proof can be found in [5].

**Iterative Nash Bargaining Algorithm.** Having proved the existence and uniqueness of the NBS, we now solve for this unique equilibrium by solving the constrained optimization problem (4) employing the method of Lagrange multipliers. The best response of the \( i \)th radar can be obtained as:

\[
P_i^{(n+1)} = \left[ \frac{p_i^{(n)}}{\eta_i^{(n)}} \gamma_{th} + \frac{h_i}{\eta_i^{(n)}} \gamma_i^{(n)} \right] + \xi_i^{(n)} - \psi_i^{(n)} \left[ p_i^{(n)} \right],
\]

where \( x_{th} = \max \{ \min(x, b), a \} \), \( n \) denotes the iteration index, \( \{ \eta_i^{(n)} \}_{i=1}^{N_r}, \{ \phi_i^{(n)} \}_{i=1}^{N_r}, \{ \xi_i^{(n)} \}_{i=1}^{N_r}, \) and \( \psi_i^{(n)} \) are the Lagrange multipliers. The sub-gradient method is employed to update \( \{ \eta_i^{(n)} \}_{i=1}^{N_r}, \{ \phi_i^{(n)} \}_{i=1}^{N_r}, \{ \xi_i^{(n)} \}_{i=1}^{N_r}, \) and \( \psi_i^{(n)} \) to ensure fast convergence:

\[
\begin{align*}
\eta_i^{(n+1)} &= \left[ \eta_i^{(n)} + s_i \left( \gamma_i - \gamma_{th} \right) \right]^+, \\
\phi_i^{(n+1)} &= \phi_i^{(n)} + s_i \left( \psi_i^{(n)} + \phi_i^{(n)} - \eta_i^{(n)} \right)^+, \\
\xi_i^{(n+1)} &= \left[ \xi_i^{(n)} + s_i p_i^{(n)} \right]^+, \\
\psi_i^{(n+1)} &= \left[ \psi_i^{(n)} + s_i (p_i^{(n)} - \sum_{i=1}^{N_r} p_i^{(n)}) \right]^+
\end{align*}
\]

where \( x^+ = x \) if \( x > 0 \), and \( x^{-} = a \) if \( x < 0 \). \( s_i \) is a small step size, \( n \in \{ 1, \cdots, N_{\text{max}} \} \), and \( N_{\text{max}} \) is the maximum number of iterations. The overall iterative procedure is detailed in Algorithm 1. The proposed Algorithm 1 will guarantee convergence by utilizing the subgradient method, which is known to converge under mild conditions.

**Algorithm 1:** Nash Bargaining for Adaptive Power Allocation.

1. **Initialization:** Initialize \( \gamma_{th}^{\min} \), \( \{ \eta_i^{(0)} \}_{i=1}^{N_r}, \{ \phi_i^{(0)} \}_{i=1}^{N_r}, \{ \xi_i^{(0)} \}_{i=1}^{N_r}, \) \( \{ \psi_i^{(0)} \}_{i=1}^{N_r} \), and \( \psi_i^{(0)} \) to some values; Initialize iterative index \( n = 1 \), the tolerance \( \varepsilon > 0 \).
2. **Repeat until:** \( p_i^{(n+1)} - p_i^{(n)} \) < \( \varepsilon \) or \( n = N_{\text{max}} \)
   - for \( i = 1, \cdots, N_r \), do:
     - Calculate \( p_i^{(n+1)} \) by solving (5);
     - Update \( \eta_i^{(n+1)}, \phi_i^{(n+1)}, \xi_i^{(n+1)} \) and \( \psi_i^{(n+1)} \) by (6);
   - end for;
3. **Set** \( n \leftarrow n + 1 \);
4. **Update:** Update \( p_i^{(n+1)} \) for all \( i \).

**4 Simulation Results**

In this section, we provide simulation results to access the performance of the proposed algorithm. A radar network with \( N_i = 4 \) spatially diverse radars is considered. We simulate a scenario where the radars are located at \([0, 0], [50, 0], [50, 50], [0, 50] \)km and \([0, 0], [50, 0], [50, 50], [0, 50] \)km. We assume that the target’s position is \([80, 60] \)km. The cross correlation coefficient between different radars is \( \rho_{ij} = 0.01 (i \neq j) \). The system parameters are set as follows: the radar antenna gains are \( G_i = 30 \)dB, \( G_i = G_i = -30 \)dB, the wavelength is \( \lambda = 0.03 \)m, the maximum transmit power of each radar is limited to \( P_{\text{tot}}^{\text{max}} = 5 \)kW, the number of received pulses is \( N = 512 \), \( P_{\text{DA}} = 2.7 \times 10^{-3} \), and \( P_{\text{PA}} = 10^{-6} \). The SINR can be computed using (2), which is \( \gamma = 10 \)dB for all radars, and the corresponding \( \delta_i \) is equal to 0.0267 for \( \forall i \). We initialize \( \sigma_i^{\text{RCS}} = 10^{-18} \), \( \epsilon = 10^{-15} \), \( \eta_i^{(0)} = 10 \), \( \phi_i^{(0)} = 10 \), \( \xi_i^{(0)} = 10 \) for all \( \forall i \), and \( \psi_i^{(0)} = 10 \). The step size \( s_i = 0.001 \).

Two target RCS models are adopted in this paper. The first model is uniform reflectivity, where \( \sigma_i^{\text{RCS}} = [1, 1, 1, 1] \text{m}^2 \). On the other hand, in order to evaluate the effect of the target RCS on the power allocation strategy, we also adopt the second RCS model \( \sigma_i^{\text{RCS}} = [1, 0.1, 5, 0.5] \text{m}^2 \).

Fig.1 testifies the convergence of the proposed algorithm. It is obvious from Figure.2 that the transmit power and SINR converge fast to the equilibrium values after 4-8 iterations. We can clearly notice that the proposed algorithm can meet the SINR requirement of each radar, which confirms that the NBPG method can overcome the near-far effect.

In order to disclose the effects of several factors on the power allocation results, Figure.2 (a) & (c) plot the power allocation results of the proposed algorithm. In Figure.2 (a), less transmit power is assigned to Radar 2 and Radar 3, as they are closer to the target. In other words, more power tends to be allocated to the radar farther from the target. While the results in Figure.2 (c) illustrate that the radars with smaller RCS are favorable over others, when it comes to power allocation. In the optimization process, higher transmit power is assigned to the radars with relative weaker propagation channels.

**Figure 2.** Convergence of the proposed algorithm: (a) Power convergence with \( \sigma_i^{\text{RCS}} \); (b) SINR convergence with \( \sigma_i^{\text{RCS}} \); (c) Power convergence with \( \sigma_i^{\text{RCS}} \); (d) SINR convergence with \( \sigma_i^{\text{RCS}} \).

To demonstrate the superior advantages of the proposed al-
algorithm further, we compare the proposed algorithm with a couple of benchmark algorithms for power allocation: the standard NBS for cooperative game, the Koskie and Gajic’s (K-G) algorithm [8], and the adaptive non-cooperative power control algorithm (ANCPCA) [9], as depicted in Figure 3. In Figure 3 (a) & (c), we notice that the ANCPCA transmits the most power due to the radars’ self-interested non-cooperative behavior in the game process, which is consistent with the results in [8]. To be specific, when one of the radars cannot reach or maintain its minimum SINR, it resorts to the only means of increasing the transmit power to guarantee the SINR requirement, as do other radars in a similar situation [5][8]. As a result, the LPI performance of radar network degrades. While for the NBPG algorithm, the netted radars can perceive the interference environment well and accordingly make the most appropriate transmit power adjustment decision.

From Figure 3 (b) & (d), one can see that the SINR value of NBPG algorithm and ANCPCA can meet the target detection requirement but the SINRs of the standard NBS method and K-G algorithm are not ideal which part of radars are below the SINR threshold. Thus, it is unfairness for all the radars because the standard NBS method and K-G algorithm cannot guarantee the SINR requirement of each radar. However, the NBPG approach can accommodate each radar’s transmit power to satisfy its SINR requirement, which improves its LPI performance. Generally speaking, those results demonstrate that the NBPG approach not only guarantees the SINR requirements of all the netted radars but also improves the LPI performance of radar network.

Figure 3. Comparisons of equilibrium values with various power allocation schemes: (a) Transmit power with $\sigma^2_{\text{RCS}}$; (b) Achieved SINR with $\sigma^2_{\text{RCS}}$; (c) Transmit power with $\sigma^4_{\text{RCS}}$; (d) Achieved SINR with $\sigma^2_{\text{RCS}}$.

5 Conclusion

We considered an LPI based distributed power allocation for radar networks in a cooperative game-theoretic framework. A novel SINR-based network utility function is developed as a metric to evaluate power allocation, which guarantees the existence and uniqueness of the NBS. In addition, an iterative Nash bargaining algorithm is utilized to play the game among the radars, which is shown to converge quickly to the NBS for power allocation. Simulation results are provided to demonstrate the performance of the proposed algorithm.

6 Acknowledgements

This research is supported by the National Natural Science Foundation of China (Grant No. 61371170, No. 61671239).

References


