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# Characterization of Dispersion Code Multiplexing (DCM) in Incoherent Radio Channels

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## Abstract

We characterize dispersion code multiplexing (DCM) in incoherent line-of-sight (LOS) channels. First, we conceptually and mathematically describe an LOS-DCM system. Next, we statistically analyze and model multiple access interference (MAI) in this system. Finally, we investigate the system performance in terms of bit error probability (BEP).

## 1 Introduction

Real-time analog signal processing (R-ASP) is a most promising paradigm in radio technology [1]. R-ASP processes ultra-wideband signals with negligible latency and low power consumption. The core of a R-ASP system is the phaser, a component exhibiting specified group delay responses, $\tau(\omega)$ [1]. The group delay response of phasers may vary based on the application, being, for instance, linear in real-time Fourier transformation [2], stair-case in frequency sniffing [3], or Chebyshev in Dispersion Code Multiplexing (DCM) [4].

DCM was introduced in [4], using Chebyshev group delay phaser responses for multiplexing. This scheme, that may be regarded as the counterpart of optical fiber Bragg grating based multiplexing [5], may lead to ultra-fast millimeter-wave communication systems. However, its performance has not been rigorously analyzed so far and not even discussed in the case of incoherent channels as occurring in multiple access systems. This paper fills up that gap by providing a mathematical description of the essence of the DCM concept and a statistical analysis of DCM for incoherent channels.

## 2 DCM Concept

Figure 1 shows an $N\times N$ DCM communication system. The channels are assumed to be incoherent.

At transmitter $\mathrm{TX}_i$, a phaser encodes the information signal $s_i(t)$, which is intended to be decoded by a matched phaser at receiver $\mathrm{RX}_i$. For this purpose, the transfer functions of the encoding and decoding phasers should be phase-conjugated, namely $H_{\mathrm{RX}_i}(\omega)=H^*_{\mathrm{TX}_i}(\omega)$. In other words, $\mathrm{TX}_i$ and $\mathrm{RX}_i$ are assigned the complementary dispersion codes

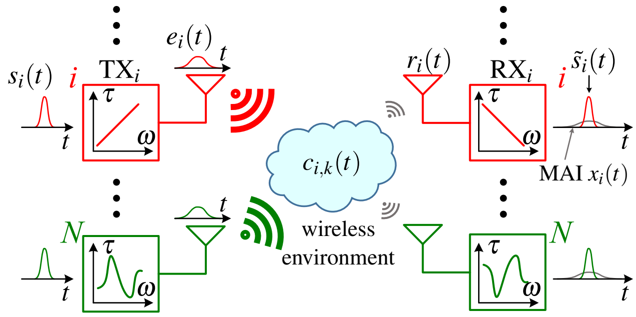


**Figure 1.** $N\times N$ DCM communication system, shown here in a multiple access (MA) configuration, in wireless indoor environment.

$$\tau_{\mathrm{TX}_i}(\omega)=-\tau_{\mathrm{RX}_i}(\omega)=-\frac{\partial\angle H_i}{\partial\omega}. \tag{1}$$

Assuming noiseless wireless channels, the signal arriving at $\mathrm{RX}_i$ may be expressed as

$$r_i(t)=\sum_{k=1}^{N}e_k(t)*c_{i,k}(t)=\sum_{k=1}^{N}[s_k(t)*h_{\mathrm{TX}_k}(t)]*c_{i,k}(t), \tag{2}$$

where "$*$" denotes convolution, $e_k(t)$ is the encoded signal, $h_{\mathrm{TX}_k}(t)=\mathscr{F}^{-1}\left[H_{\mathrm{TX}_k}(\omega)\right]$ is the corresponding encoding phaser impuse response, and $c_{i,k}(t)$ is the channel impulse response from $\mathrm{TX}_k$ to $\mathrm{RX}_i$. Here, we only consider LOS channel, corresponding to

$$c_{i,k}(t)=a_{i,k}\delta(t-\tau_{i,k}), \tag{3}$$

where $a_{i,k}$ and $\tau_{i,k}$ represent the attenuation and delay, respectively, and which may be different for different $k$-$i$ pairs, i.e. incoherent. The delay $\tau_{i,k}$ may be uniformly distributed in the interval $[0,d_{\max}/c]$, where $d_{\max}$ is the maximum communication distance and will be chosen to be 3 meters in the later simulation, $c$ is the speed of light, and $a_{i,k}$ may be log-normally distributed [6], corresponding to

$$\tilde{a}_{i,k}=20\log_{10}(a_{i,k})\ \propto\ \text{normal}\left(0,\sigma_{\mathrm{a}}^2\right)\ (\mathrm{dB}), \tag{4}$$

with zero mean and standard deviation $\sigma_{\mathrm{a}}$ dB. Polarization incoherency between the TX and RX antennas is naturally taken into account in this attenuation randomness of this distributions.

The impulse response of the decoding phaser at $\text{RX}_i$ is $h_{\text{RX}_i}(t) = \mathscr{F}^{-1}\left[H^*_{\text{TX}_i}(\omega)\right] = h_{\text{TX}_i}(-t)$, which is the time-reversed (or matched, or conjugated) version of the encoding phaser impulse response. Then, the signal at the output of the decoding phaser at $\text{RX}_i$ is

$$\begin{aligned} z_i(t) &= r_i(t) * h_{\text{RX}_i}(t) \\ &= \sum_{k=1}^{N} [s_k(t) * h_{\text{TX}_k}(t) * h_{\text{TX}_i}(-t)] * c_{i,k}(t), \end{aligned} \tag{5}$$

using the associative property of the convolution product. Since $h_{\text{TX},i}(t) * h_{\text{TX},i}(-t) = \delta(t)$, which is the autocorrelation of $h_{\text{TX},i}(t)$, Eq. (5) splits into

$$z_i(t) = \tilde{s}_i(t) + x_i(t), \tag{6a}$$

$$\tilde{s}_i(t) = s_i(t) * c_{i,i}(t), \tag{6b}$$

$$x_i(t) = h_{\text{TX}_i}(-t) * \sum_{\substack{k=1 \\ k\neq i}}^{N} [s_k(t) * h_{\text{TX}_k}(t)] * c_{i,k}(t), \tag{6c}$$

where $\tilde{s}_i(t)$ and $x_i(t)$ are the recovered signal and the multiple access interference (MAI), respectively. Assume binary amplitude modulation (or on-off keying, OOK) with a specified threshold. If a "1" has been transmitted, then $z_i(t)$ should be higher then this threshold whereas it should be lower if a "0" has been transmitted.

## 3 System Characterization

### 3.1 Chebyshev Coding

An additional DCM requirement, apart from phase conjugation, is the availability of group delay diversity to distinguish and hence accommodate as many users as possible. Such dispersion can be provided by Chebyshev dispersion codes, with equal min-max for all users, as reported in [4] and implemented in [7]. The coding set for TXs and RXs correspond to

$$\mathbf{C}_{\text{TX}} = (\Delta\tau, \Delta f)[m_1, \cdots, m_i, \cdots, m_N], \quad \mathbf{C}_{\text{RX}} = -\mathbf{C}_{\text{TX}}, \tag{7}$$

where $\Delta\tau$ and $\Delta f$ are group delay swing and bandwidth, respectively. The corresponding code (group delay) function is

$$\tau_{\text{TX}_i}(\omega) = \frac{\Delta\tau}{2} T_{m_i}\left(\frac{\omega - \omega_0}{\Delta\omega/2}\right), \ \tau_{\text{RX}_i}(\omega) = \tau_0 - \tau_{\text{TX}_i}(\omega), \tag{8}$$

with $\tau_0$ being constant, $T_{m_i}(x)$ being the $m_i^{\text{th}}$ order Chebyshev polynomial of the first kind [8], $\omega_0 = 2\pi f_0$ is the center frequency, and $\Delta\omega = 2\pi\Delta f$. Figure 2 shows a decoded signal example at one of the RXs corresponding to an all-odd coding set, where the occurring time of the desired signal and the corresponding MAI are random, hence requiring a statistical system modeling.

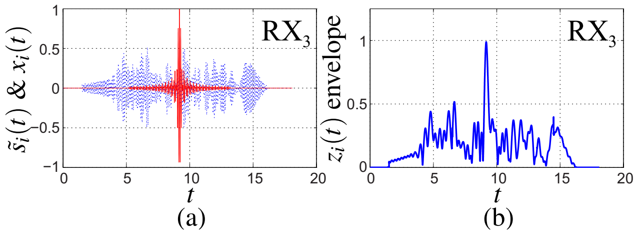


**Figure 2.** Normalized decoded signals corresponding to the coding set $\mathbf{C}_{\text{TX}} = (4\text{ ns}, 4\text{ GHz})[1, -1, 3, -3]$, and channel attenuation deviation $\sigma_{\text{a}} = 0$ dB. (a) Solid red: desired signal $\tilde{s}_i(t)$ [Eq. (6b)], dot blue: MAI $x_i(t)$ [Eq. (6c)]. (b) Envelope of $z_i(t) = \tilde{s}_i(t) + x_i(t)$.

### 3.2 Bit Error Probability (BEP) Characterization

To characterize the DCM system using BEP, we first have to statistically characterize MAI. Assuming sampling time $t_s$ and the corresponding samples $x_{i,\ell} = x_i(\ell t_{\text{s}})$, one may approximate the actual statistical distribution of $x_{i,\ell}$ by the normal distribution [5]. To do so, we first normalize the decoded signals at each $\text{RX}_i$ respectively by each desired signal, such that the desired signal energy is 1. Then we statistically calculate the corresponding mean ($\mu_{\text{X},i}$) and standard deviation ($\sigma_{\text{X},i}$) for each $\text{RX}_i$. The standard deviation square (variance) $\sigma^2_{\text{X},i}$ is also the power of $x_i(t)$. Since $x_{i,\ell}$s for different $i$ are considered to be statistically independent [5], the overall mean ($\mu_{\text{X}}$) and overall variance ($\sigma^2_{\text{X}}$) are the mean of $\mu_{\text{X},i}$ and $\sigma^2_{\text{X},i}$, i.e. $\mu_{\text{X}} = \overline{\mu_{\text{X},i}}$ and $\sigma_{\text{X}} = \overline{\sigma_{\text{X},i}}$. Here, $\sigma^2_{\text{X}}$ is the averaged MAI power and the signal to interference (MAI) ratio is therefore $\text{SIR} = 1/\sigma^2_{\text{X}}$. Then, we use random variable $x$ to represent $x_{i,\ell}$ without loss of generality, the approximate distribution of $x$ follows

$$x \propto \ \text{normal}\left(0, \sigma^2_{\text{X}}\right), \tag{9}$$

where it is found that $\mu_{\text{X}} \equiv 0$ here, which may be the result of the fast oscillation in $x_i(t)$ and hence the global cancellation in the averaging process. We let the threshold be 0.5, when "1" is sent, so that the condition $1 + x < 0.5$ ($x < -0.5$) corresponds to error. Similarly, when "0" is sent, the condition $0 + x > 0.5$ corresponds to error. The BEP is found by integrating the probability density function corresponding to (9) over $x < -0.5$ and $x > 0.5$, which yields

$$P_{\text{BE}} = \text{Q}\left(\frac{\sqrt{\text{SIR}}}{2}\right), \tag{10}$$

where

$$\text{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-s^2/2} ds \tag{11}$$

exponentially decreases as its argument increases, i.e. increasing SIR exponentially decreases BEP.

Figure 3 plots BEP as function of user number $N$. The codes are chosen to be of odd Chebyshev orders [4], corresponding to $\mathbf{C}_{\text{TX}} = (\Delta\tau, \Delta f)[1, -1, 3, -3, \ldots, N]$ ($N$ is odd) or

$\mathbf{C}_{\mathrm{TX}} = (\Delta\tau, \Delta f)[1, -1, 3, -3, \ldots, N, -(N-1)]$ ($N$ is even). The BEP degrades with increasing $N$ as the result of MAI accumulation, corresponding to poorer SIR at larger $N$. However, such BEP degradation can be mitigated by increasing the group delay swing $\Delta\tau$ or bandwidth $\Delta f$, which indicates that time-bandwidth product ($\mathrm{TBP} = \Delta\tau\Delta f$) is the critical figure of merit in DCM.

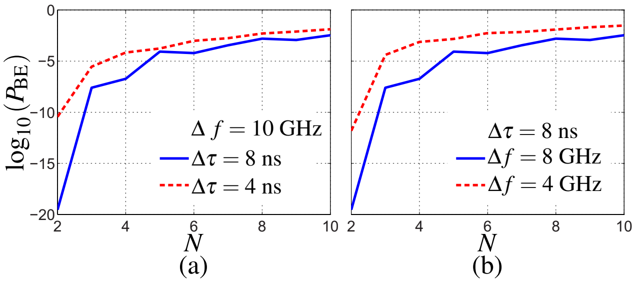


**Figure 3.** BEP for $N \times N$ DCM system with all-odd coding set and attenuation deviation $\sigma_{\mathrm{a}} = 3$ dB. (a) Fixed $\Delta f$ and varying $\Delta\tau$. (b) Fixed $\Delta\tau$ and varying $\Delta f$.

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